Simple Maps in Accelerator Simulations

Steve Peggs & Ubaldo Iriso (PhD Thesis)

"History"

Simple maps for e-clouds

Coupled clouds: evidence & maps

Summary

BC (Before Computers)

Poincare, 1890s, knew that chaos existed, but …

Dynamical systems were reduced to differential equations – which could be “solved”

Linear maps – matrices – were well studied and understood

What about solving the simplest possible nonlinear maps, eg the “logistic map”? Breeding jellyfish

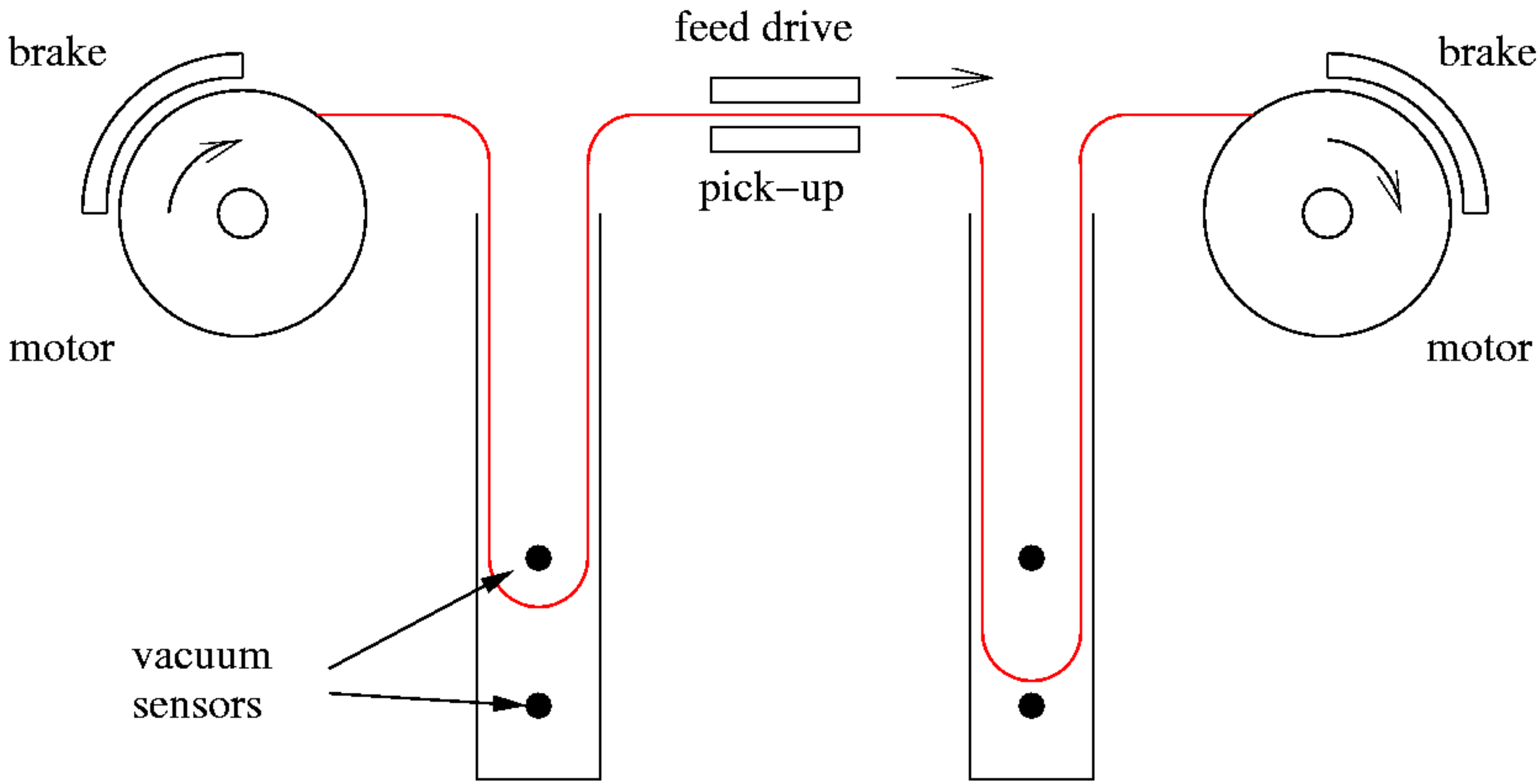
$$Y_{n+1} = \alpha Y_n (1 - Y_n)$$

Motion about the fixed point shows rich behavior!

“Some systems are intrinisically discrete in time”? Or …

1970's – eg, home built tape drive

Write THIS set of differential equations ! Is it stable?



"Making time advance in discrete steps introduces false artifacts"?

1980's – accelerator tracking

"Real men use differential equations", eg when tracking through multiple sextupoles

OK, the sextupoles are thin, but can

- expand each delta function as an infinite Fourier series

- throw away all but one Fourier term

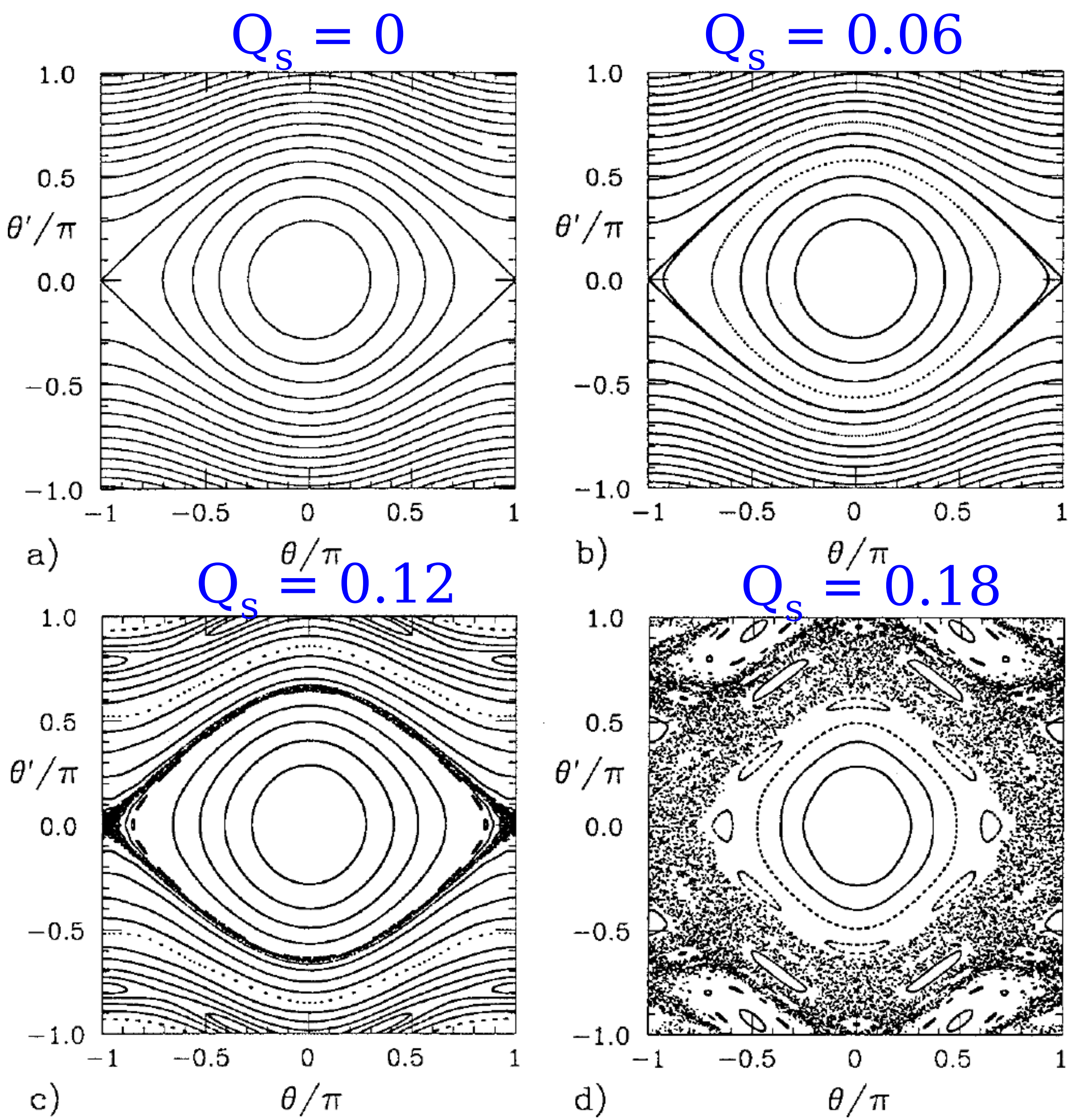
- derive first order Hamiltonian (and "solve")

Doesn't always work so well ...

Sometimes gravity is pulsed, and the gravity pendulum becomes the "standard map" RF system

1992: "I don't know what language we will be using in the year 2000, but its name will be FORTRAN" (not C++) !

1990's – accelerator tracking



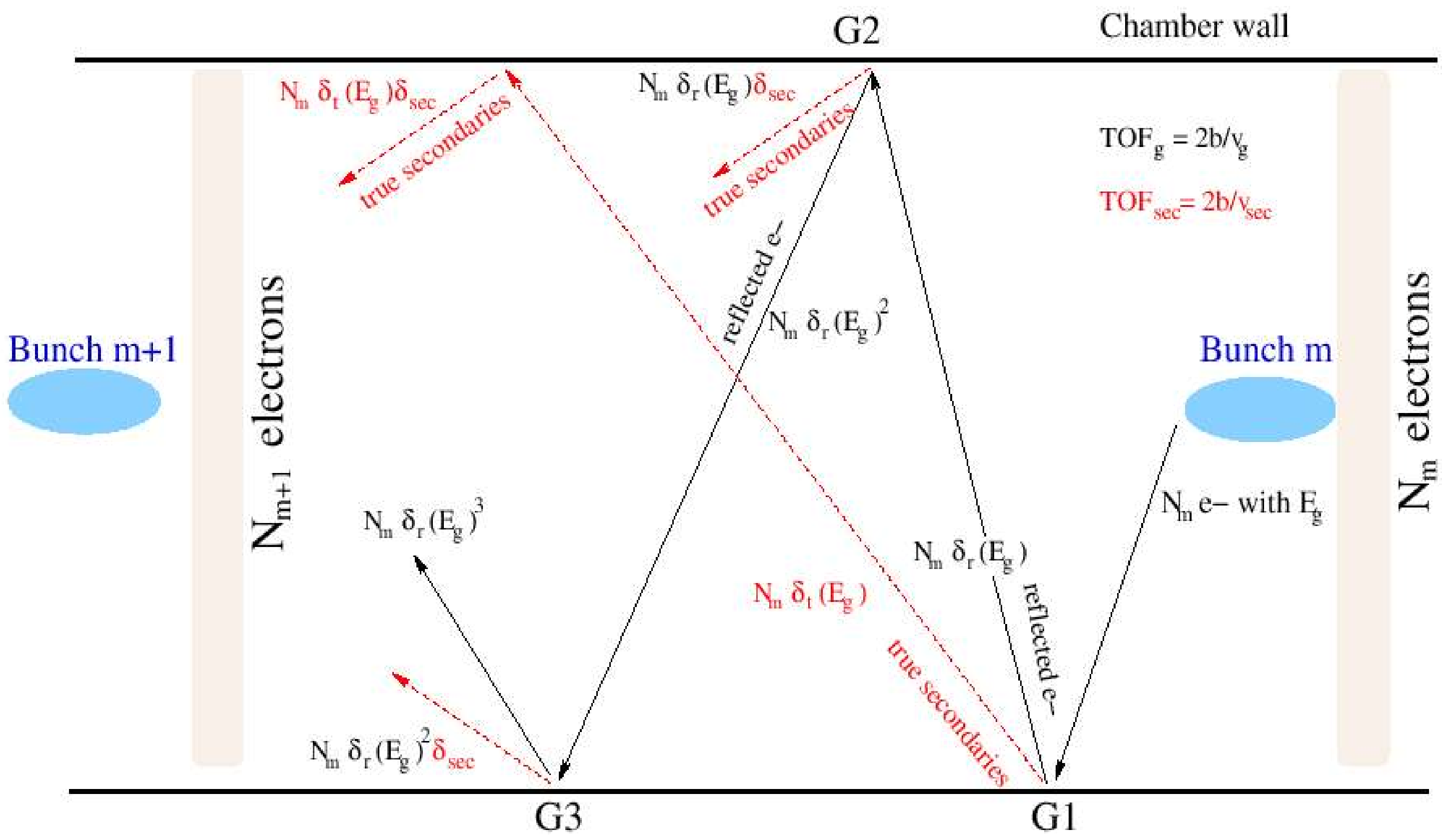
Accelerators don't just require discrete time representation but actually contain discrete (thin) elements Eg the standard map

One can construct symplectic single turn maps from brute force simulations

1) Maps are fast !

2) Even if their construction is slow

2000's – electron clouds



Violent transients – the electron energy spectrum relaxes enormously after the “shock” of bunch passage

Inelastic collisions – ~300 eV e-spectrum --> ~5 eV

Simple maps for e-clouds

Brute force simulations

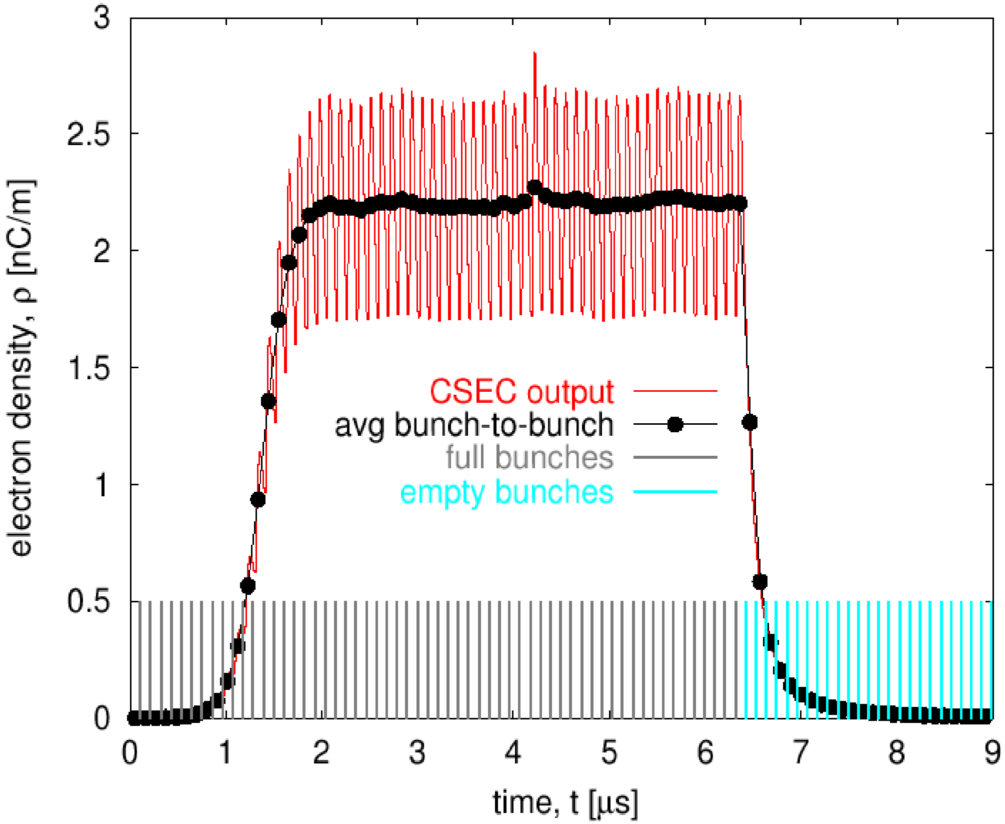
(CSEC, ECLOUD, CLOUDLAND, POSINST, WARP ...)

Compute the forces and fields to track the macroelectrons at each time step of ~1 ns to ~10 ns

Results depend on many input parameters (more than 8 for the Secondary Emission Yield alone).

The real interest is in the parametric behavior (eg, vs bunch length) and NOT cloud build up dynamics

~ 1 h to 1 d runs



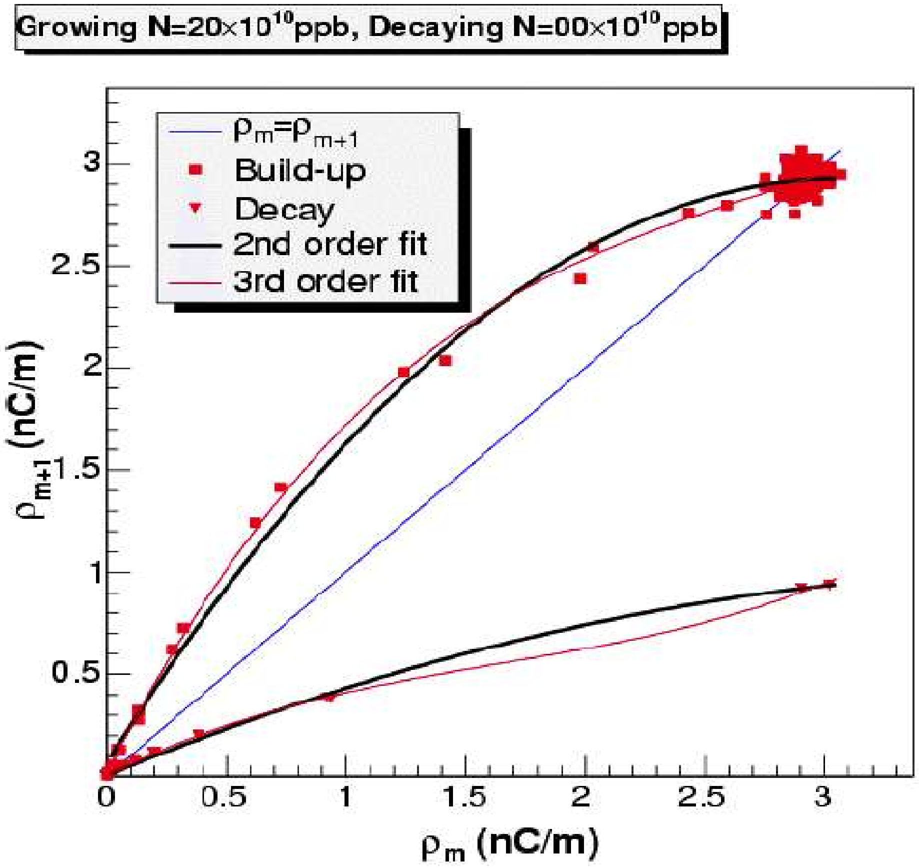
Cubic map

Follow the bunch-by-bunch evolution of the electron density ρ_m (natural time step: one bunch)

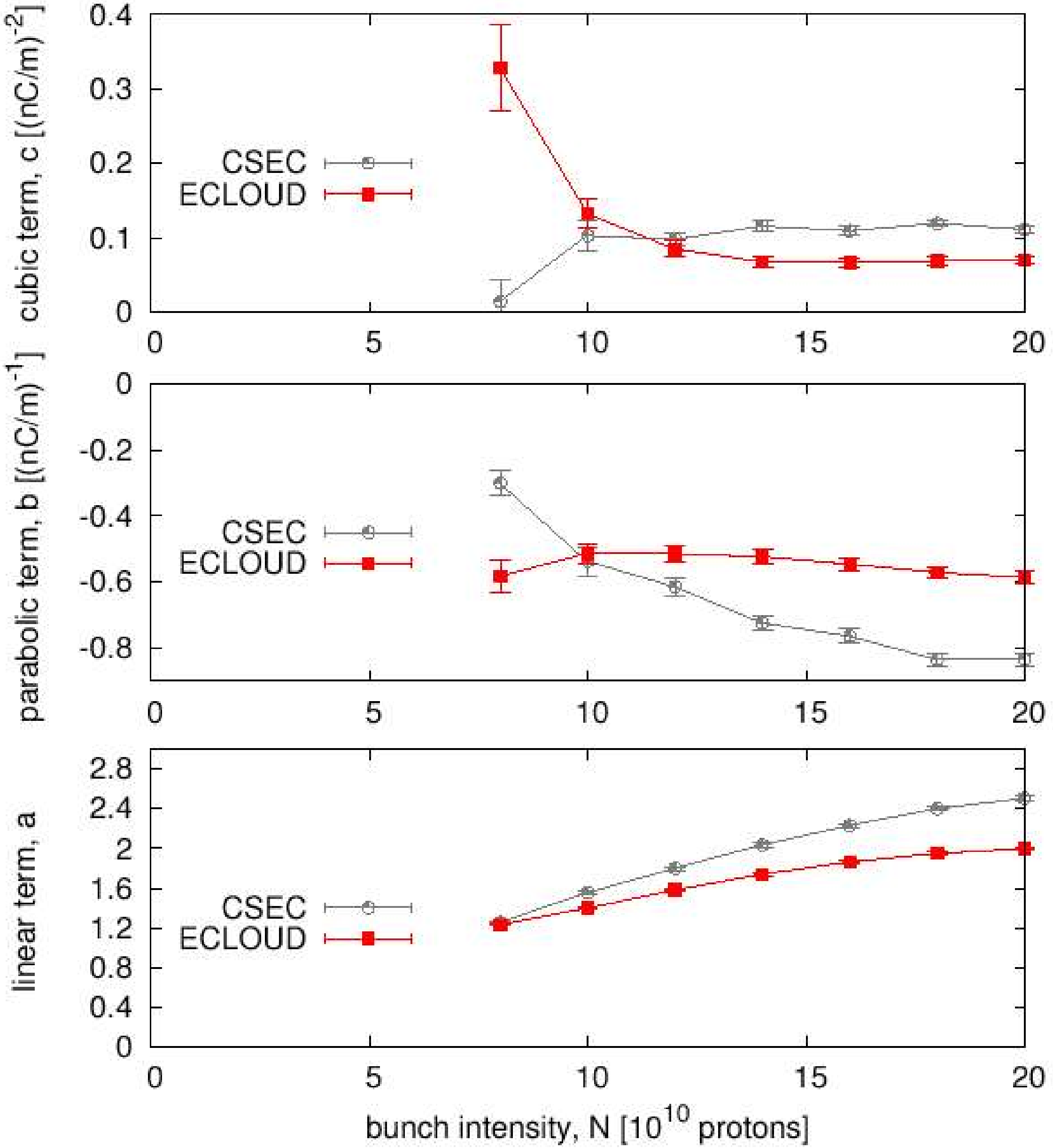
$$\rho_{m+1} = a\rho_m + b\rho_m^2 + c\rho_m^3$$

Map coefficients (a,b,c) depend on the model parameters (N, σ, ...)

Stability (saturation) occurs on the 45º line



Empirical determination of (a,b,c)



Slow codes must still be used to find parametric dependence of (a,b,c), eg versus N (LEFT)

But then cloud evolution (eg with a variety of bunch patterns and intensities) is very fast

~1 ms not ~1 h runs

A "map" application

Question: What is the best way to arrange M bunches of intensity N in a train of H possible locations? RHIC with (M,H)=(68,110) has ~10^{30} possible patterns!

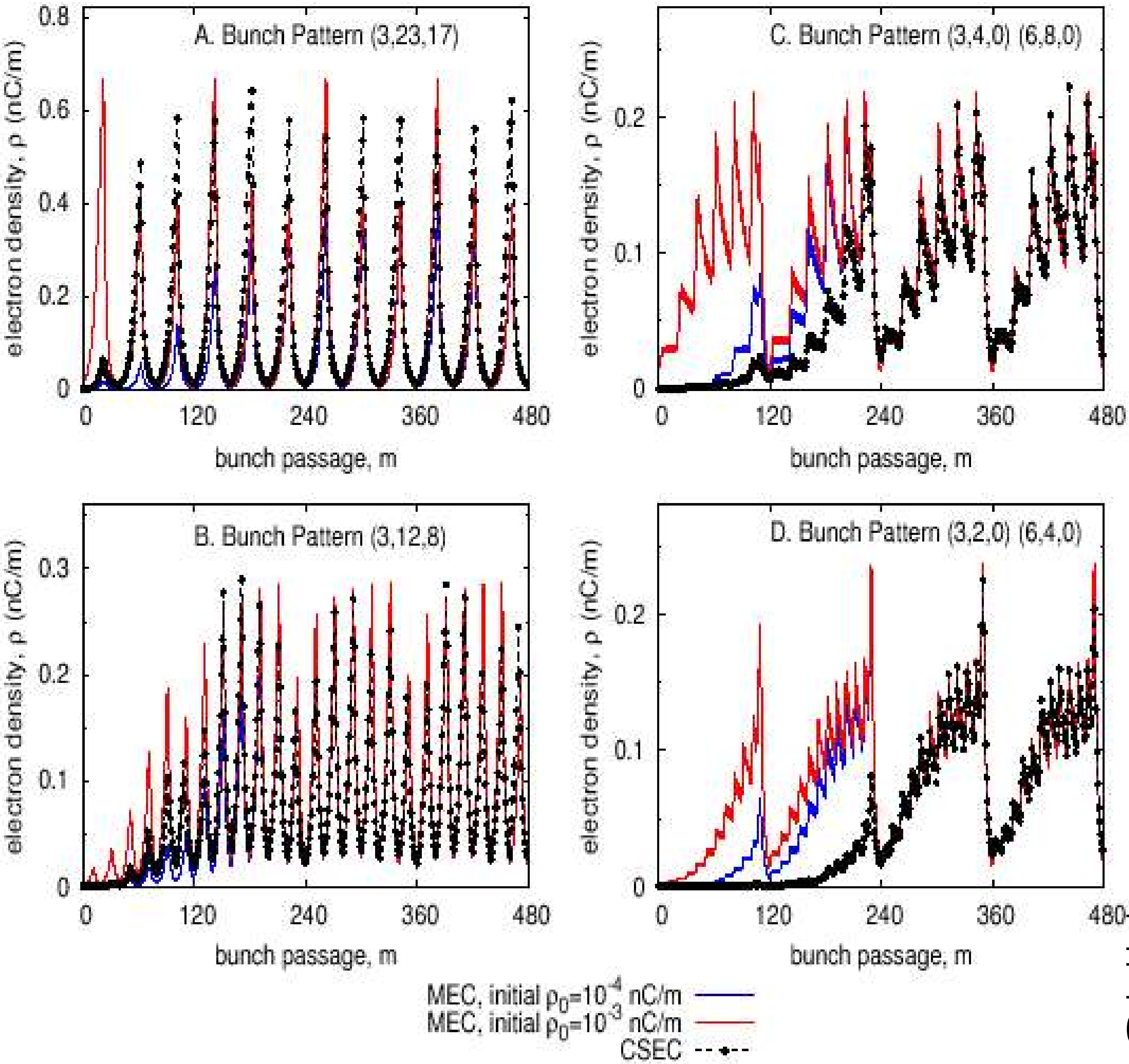
Answer: When the cloud is weak, only the linear term a(N) matters.

For RHIC (short bunches) it turns out that 4 a coefficients are required: off-to-off, off-to-on, on-to-off, and on-to-on. Electron cloud formation is supressed if

$$\left(\frac{a_{10}\, a_{01}}{a_{11}\, a_{00}} \right)^i < 1$$

where i is the number of transitions - the sparsest or the densest pattern is the most stable!

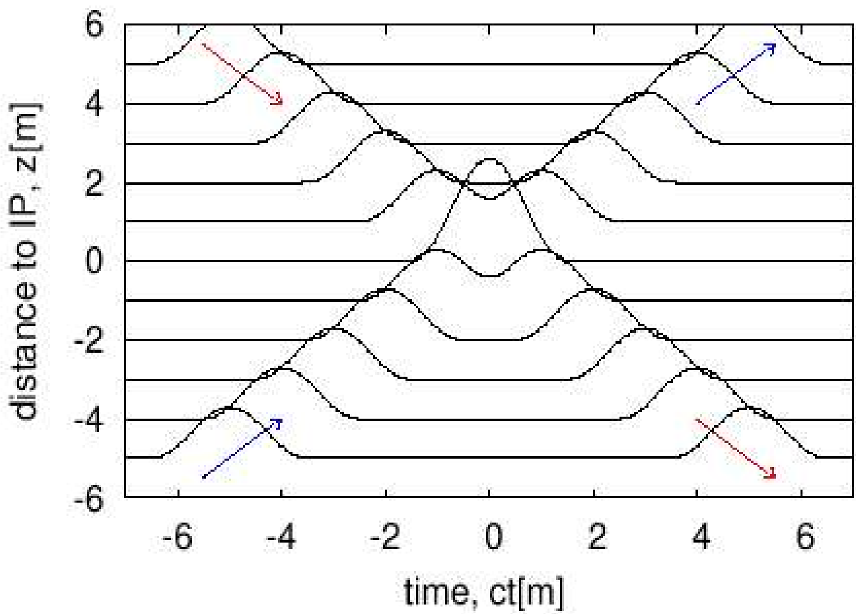
Good agreement with CSEC for various patterns

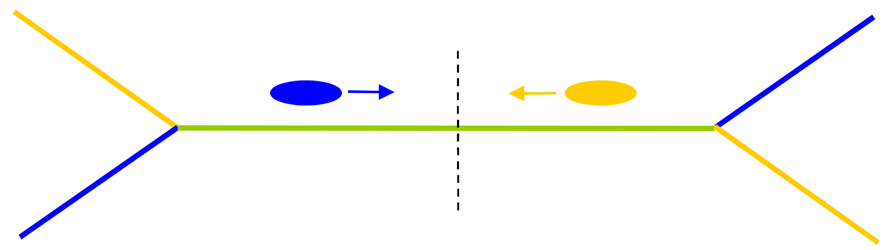


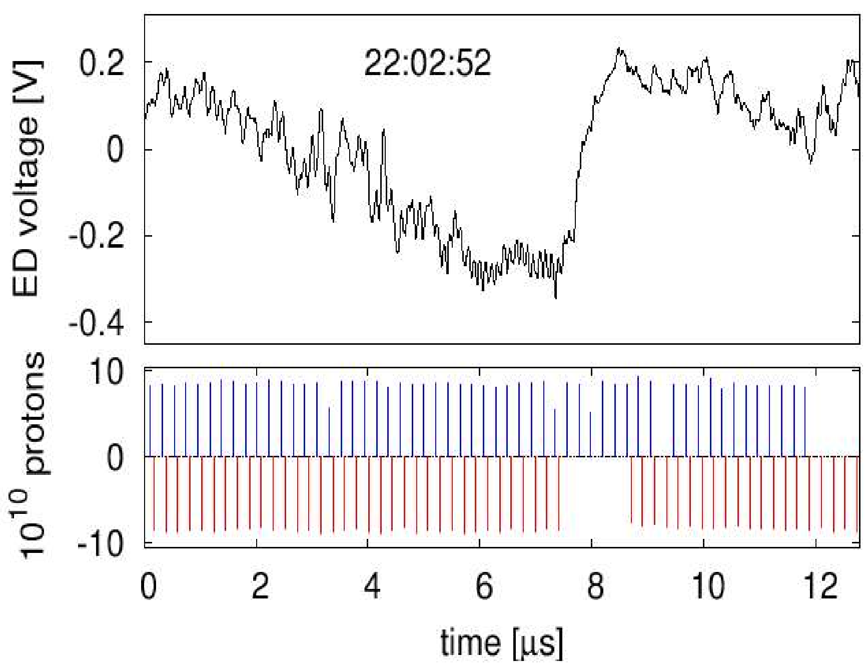
Iriso & Peggs, PRST-AB, 8, 024403, 2005

Evidence for coupled e-clouds & i-clouds

Electron clouds in RHIC IR12







Common beam pipe: the combination of blue & yellow bunches creates “shorter bunch spacings”

Store the Blue beam: no e-cloud.

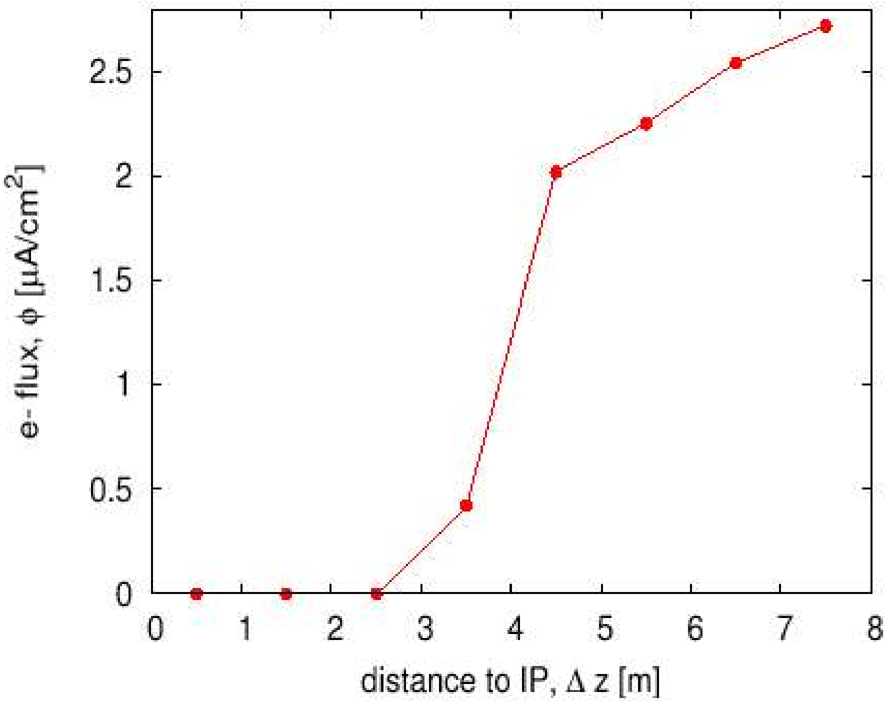
Inject the Yellow beam: then get e-clouds

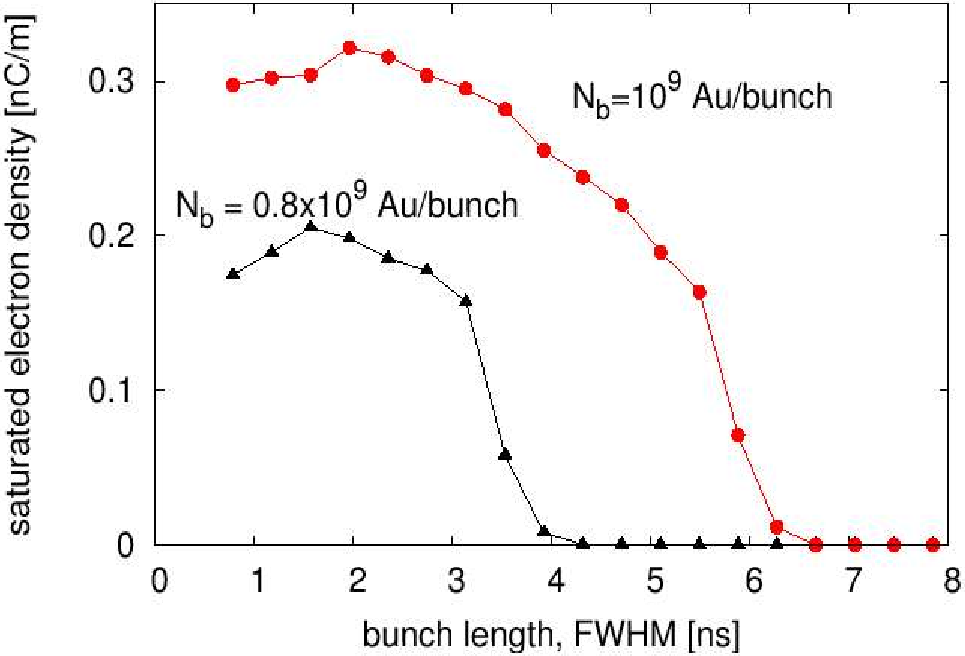
Common warm beam pipes can have “unique” properties

Simulated turn on across a threshold - **CSEC**

Crossing location

Bunch length



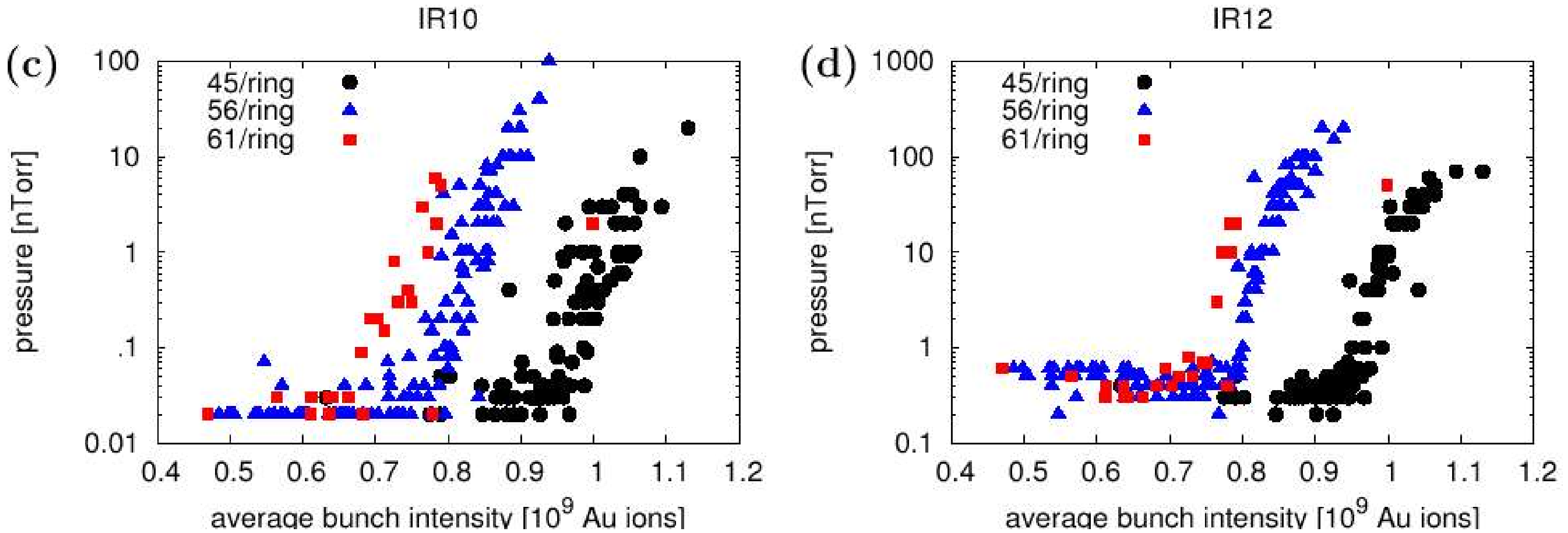


LEFT: e-flux vs bunch crossing location (Rumolo & Fischer, C-AD/AP/146)

RIGHT: e-cloud density vs bunch length (cf transition crossing & rebucketing)

Observed turn-on across an intensity threshold

Pressure vs average bunch intensity

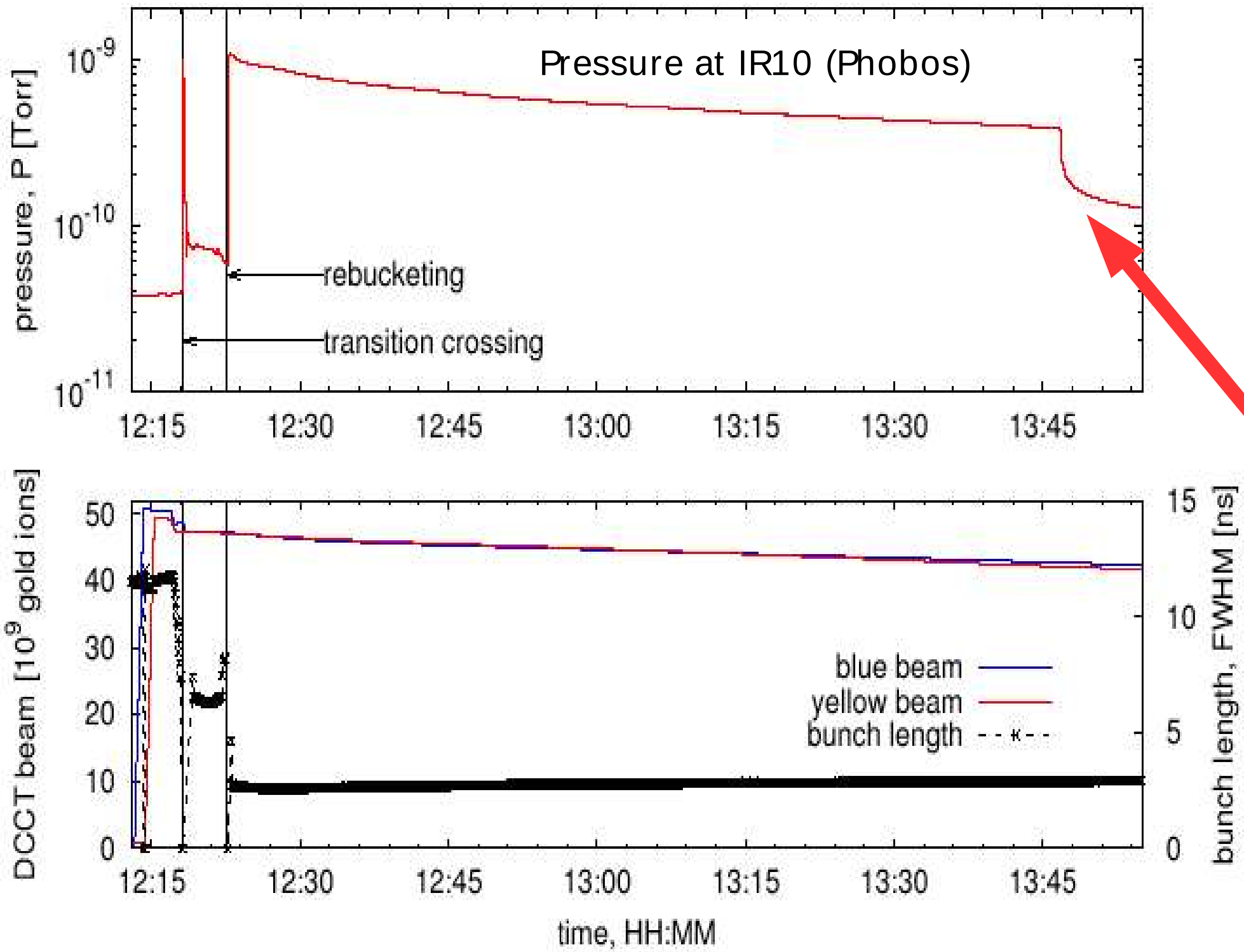


Intensity threshold decreases with more bunches (smaller bunch spacing)

Pressure rises at IRs are caused by electron clouds

Cloud evolution through a store

Abrupt behavior as population decays?

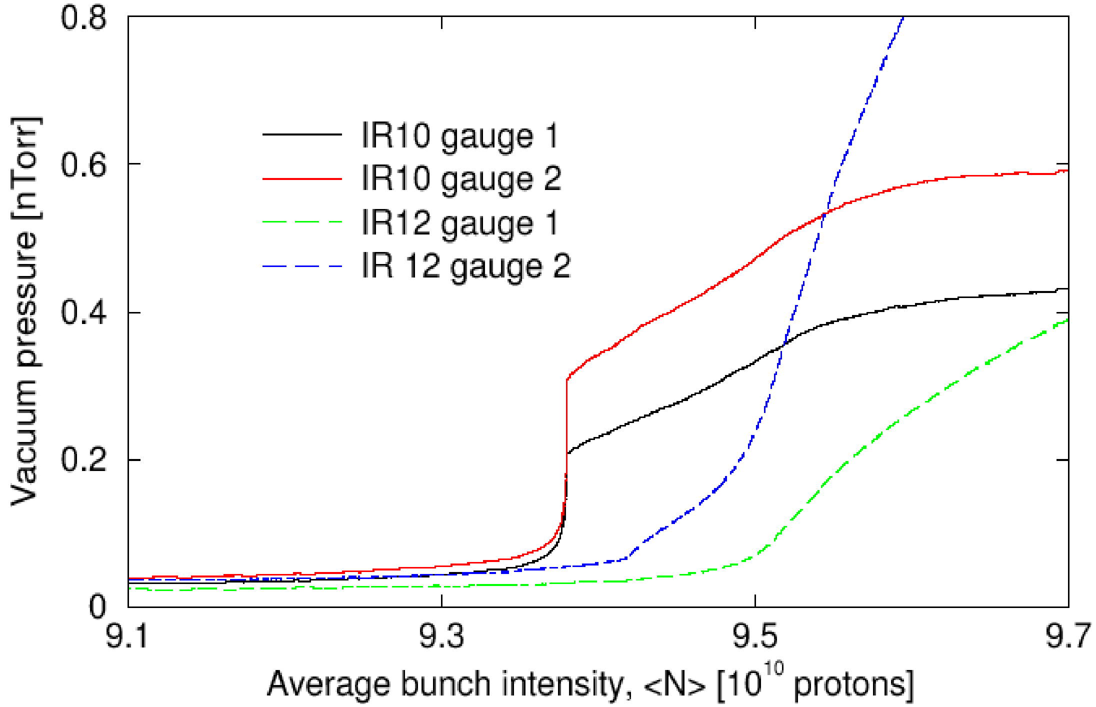


Pressure rises during "transition" & "rebucketing" are due to shortening

But what happened in IR10 at 13:45?

First & second order phase transitions

IR10 consistently showed abrupt e-cloud collapse

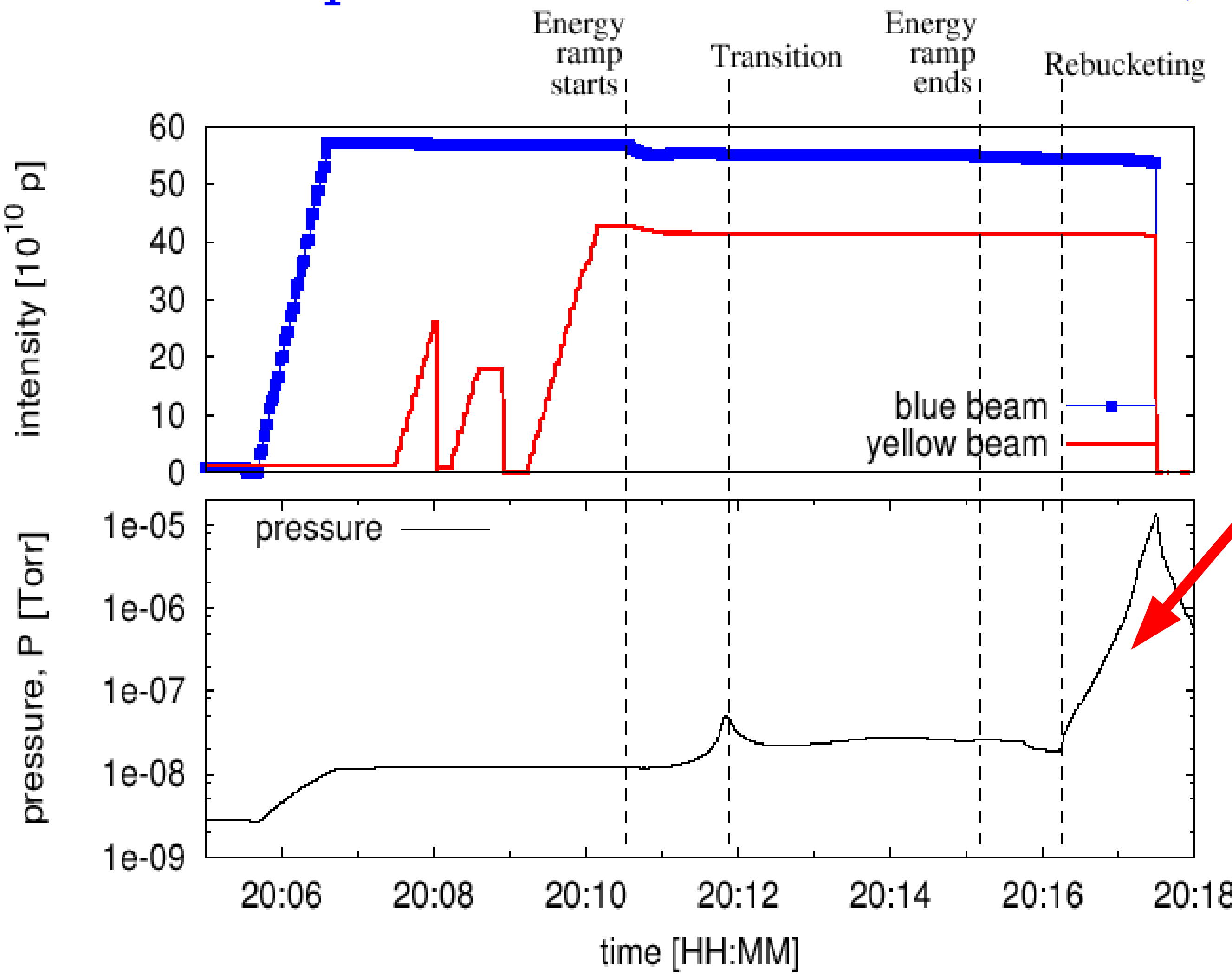


Contemporary simulation codes only reproduce a smooth transition from "off" to "on" (Iriso & Peggs, ECLOUD 04)

How can both first and second order phase transitions occur in e-clouds?

Slow vacuum instability - driven by e-clouds?

Too complex for current codes CSEC, ECLOUD, etc



e-clouds create partial ion-clouds create e-clouds, slowly running away?(Fischer et al, CARE-HHH 04)

Ion timescales ~3-6 orders of magnitude longer than for electrons

CPU times?

(See Wednesday talk by J-L Vay: POSINST & WARP)

Coupled cloud maps

Fixed points

Ion clouds couple to electron cloud via bunch-to-bunch maps:

$$\begin{aligned} \rho_{m+1} &= f(\rho_m, R_m) \\ R_{m+1} &= g(\rho_m, R_m) \end{aligned}$$

→ for electron density

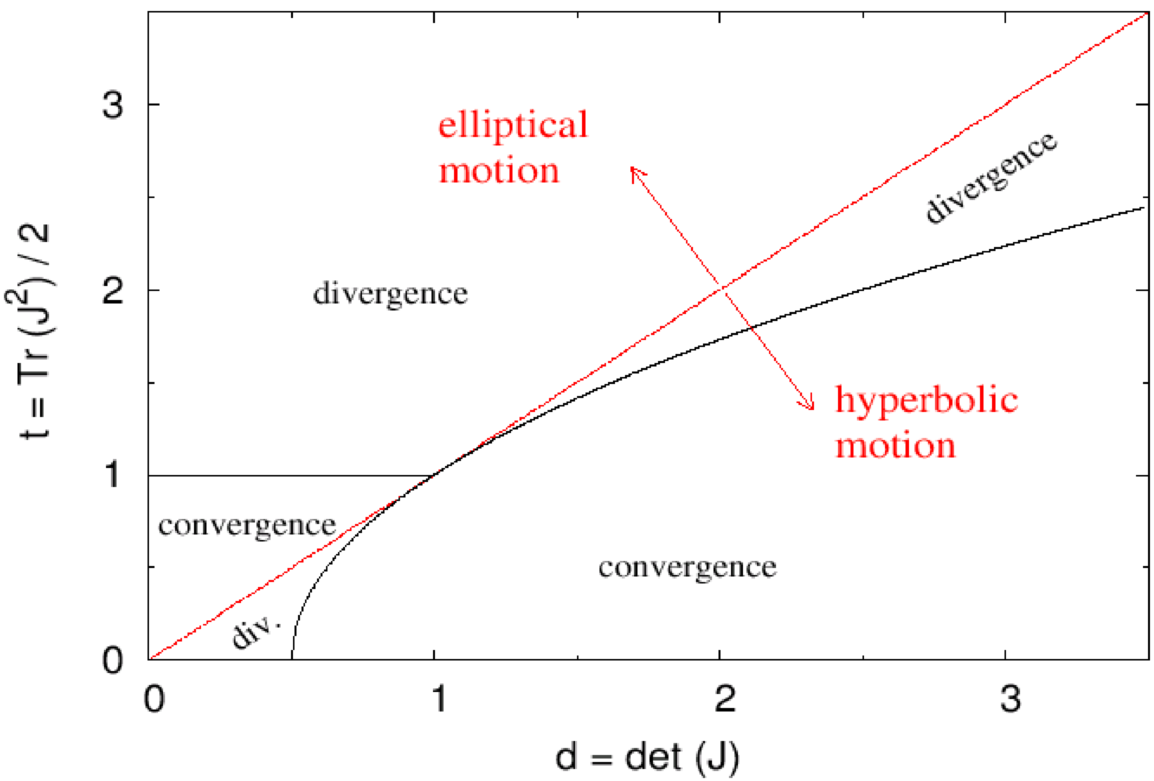
→ for ion density

Writing $\vec{r}_m = \begin{pmatrix} \rho_m \\ R_m \end{pmatrix}$ then the fixed point solution occurs when

$$\vec{r}_{\mathrm{m+1}} = \vec{r}_m \equiv \vec{r*}$$

Fixed point stability depends on the Jacobian matrix:

$$J = \begin{pmatrix} \frac{\partial f}{\partial \rho_m} & \frac{\partial f}{\partial R_m} \\ \frac{\partial g}{\partial \rho_m} & \frac{\partial g}{\partial R_m} \end{pmatrix}_{\vec{r}^*}$$



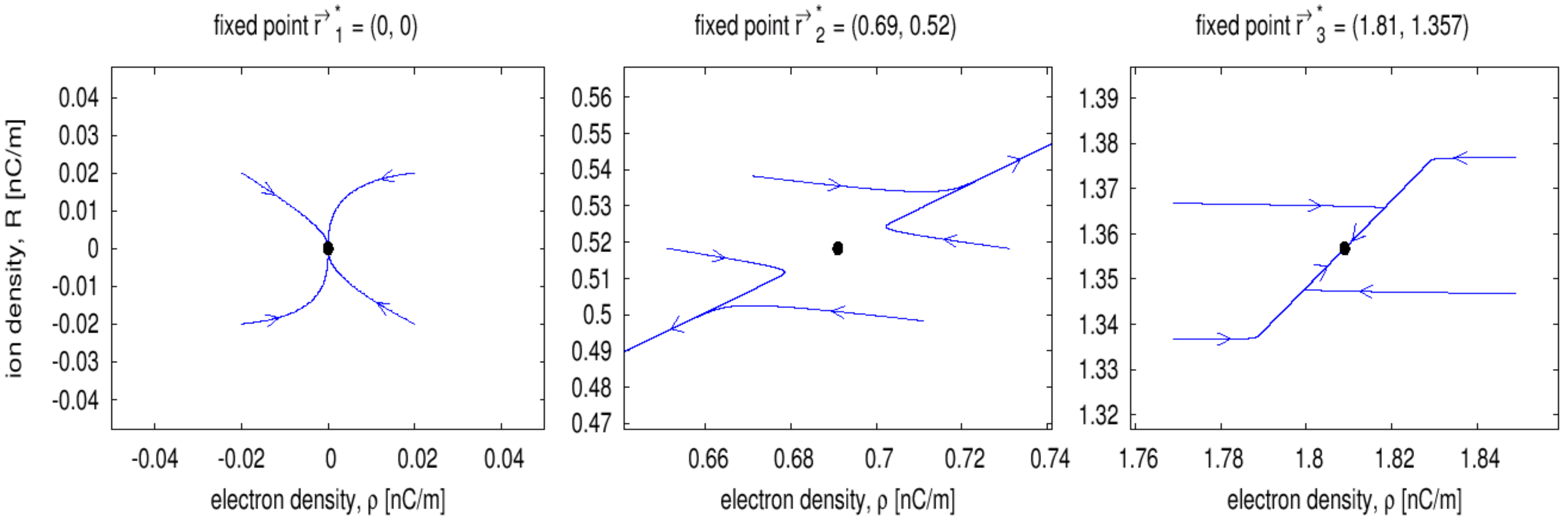
Extending the cubic e-cloud map

$$\rho_{m+1} = (a + b\rho_m + yR_m)\rho_m + c\rho_m^3$$
$$R_{m+1} = AR_m + Y\rho_m$$

Physical meaning can be attached to the new coeefficients y, A & Y (Iriso's thesis, & PRST 9, 071002)

For a given set of constant coefficients (except that a is linear in N) there are 3 fixed point solutions for

N=5 x 10^{10} protons/bunch

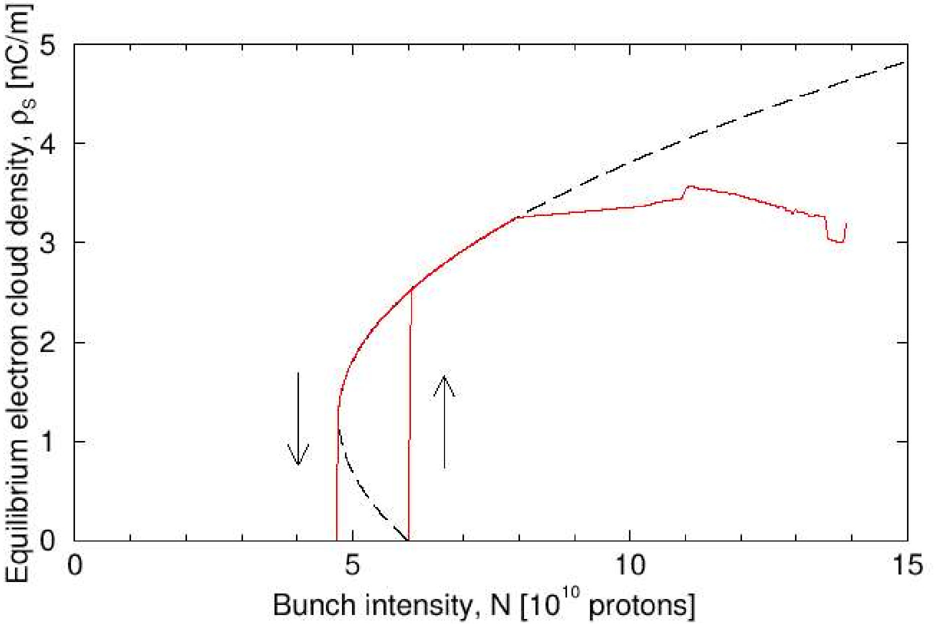


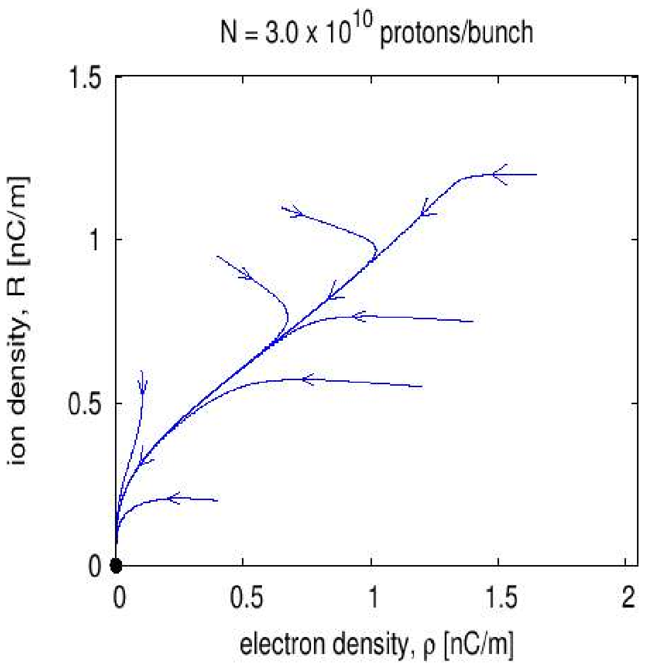
Dynamics – growing & collapsing clouds

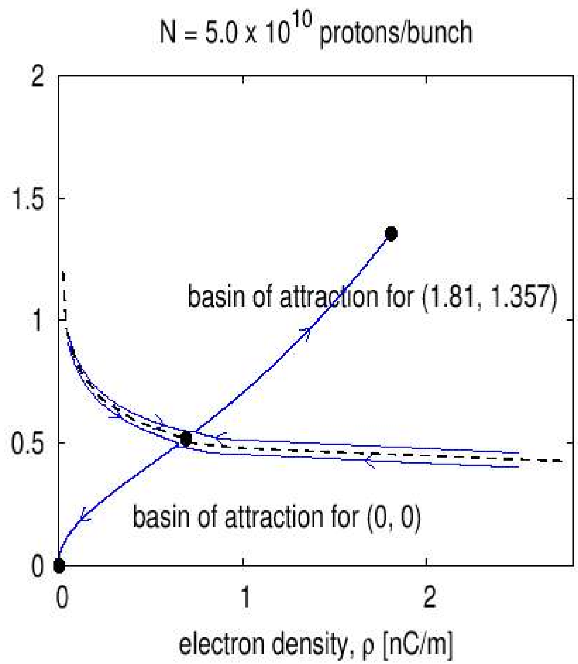
Simulated behavior as **N** is slowly increased, then slowly decreased

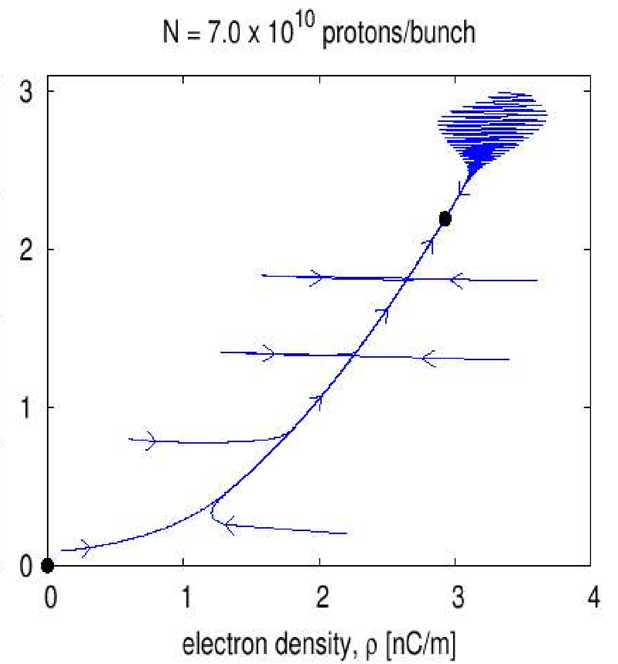
Hysteresis – ion & electron clouds grow spontaneously or collapse.

First order phase transitions!



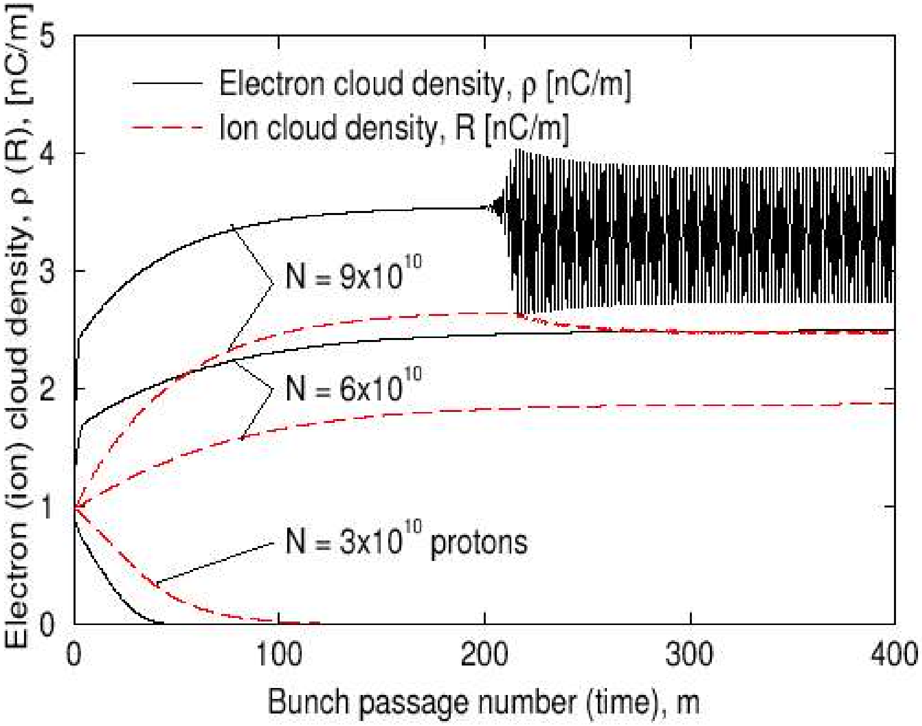


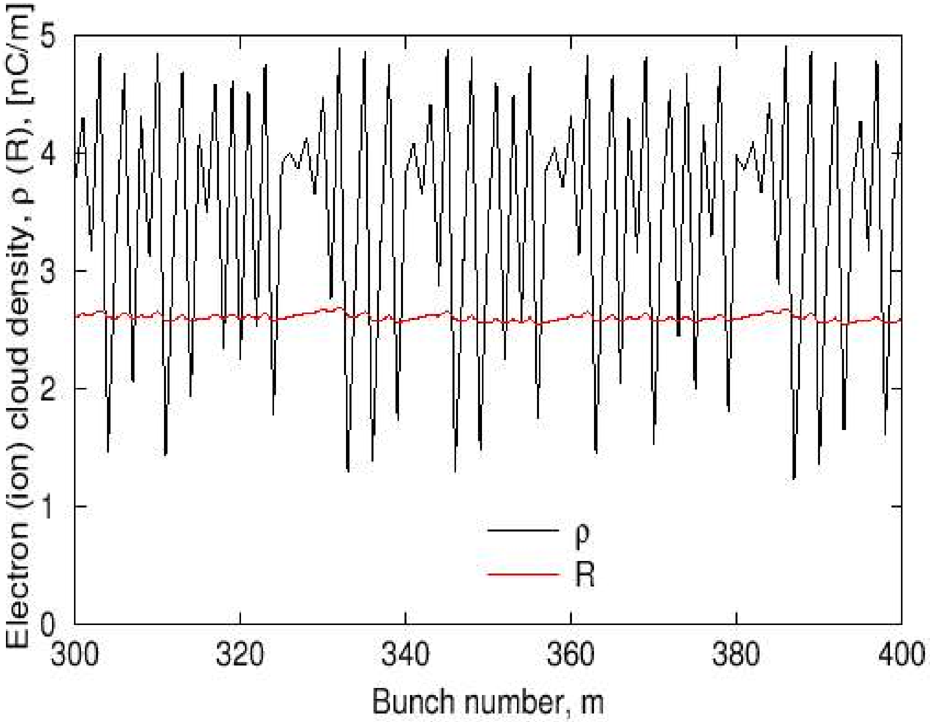




Additional dynamical phases

Period doubling, and even chaos .....





It is NOT clear that these are present in the “real” world!

Summary

Generic simple maps

Transient violence: maps and transients go together - jellyfish breeding, RF cavities, thin sextupoles, bunch passage, ...

E-cloud and beam-beam: simulations will go on for ever, never solved, always useful

Parametric behavior counts: not dynamic effects. Eg beam-beam tune plane, EC threshold vs bunch length, ...

Maps are shorthand: for complex physics. Eg one-turn maps, EC, ...

Uncoupled EC maps work: for RHIC (just), and LHC

Coupled e & i-maps

Reproduce unexpected observations: RHIC – 1st order phase transitions, hysteresis

Summarize simulations: parametric dependence

Enhance comprehension: coefficients have meaning, connect to semi-analytic theory

New dynamics: Period doubling and chaos may be observed?

Are fast: hours become milliseconds – 6 or 7 order speed-up