Simulations of single bunch collective effects using HEADTAIL

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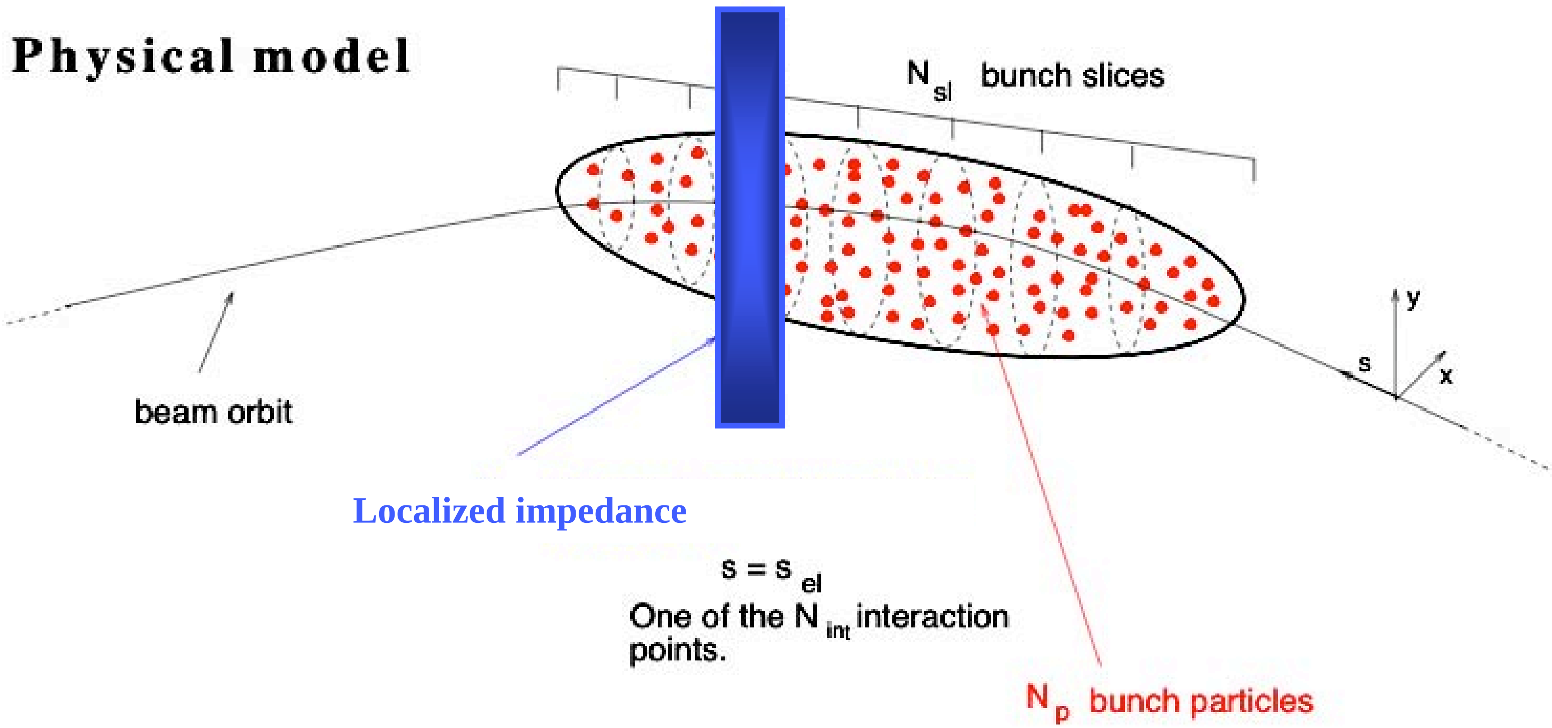
Overview

- Description of the HEADTAIL code
  - — what it does
  - — ingredients and features
- Examples of application (with benchmark against experiments/codes, where possible)
  - Electron cloud
  - Transverse Mode Coupling Instability in PS/SPS
  - Collimator induced tune shift
- Conclusions

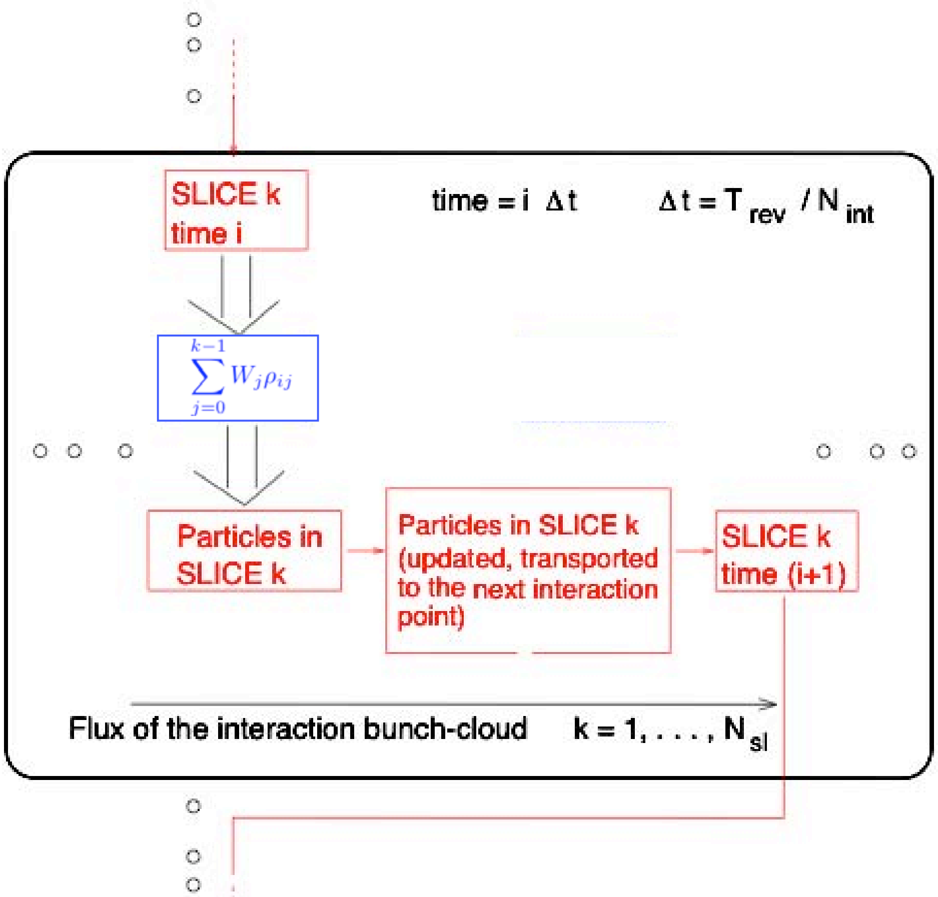
Code HEADTAIL

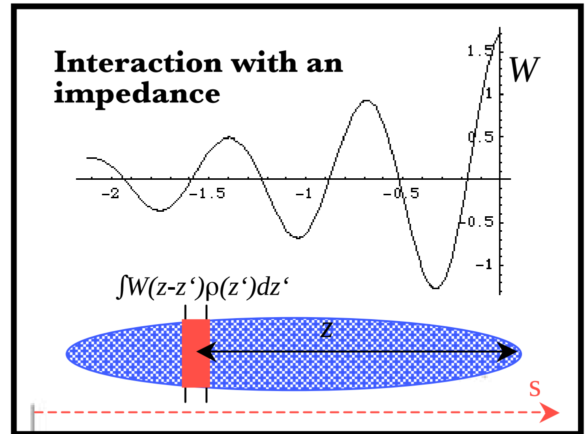
Physical model



Simulation model: electron cloud and bunch are both modelled as ensembles of macro–particles (typically 100000) and the bunch is divided into an adjustable number of slices (typically 50).

Numerical implementation





Time flux

$i = 0, \ldots, N_{turn} \times N_{int} - 1$

Features of HEADTAIL (I)

- Synchrotron motion included
- Single bunch can be Gaussian or uniform (barrier bucket). Longitudinal dynamics is solved in a linear, sinusoidal or no bucket ($\rightarrow$ debunching).
- Chromaticity and detuning with amplitude
- Dispersion at the kick section(s).
- Electron cloud kick(s):
  - Soft Gaussian approach (finite size electrons)
  - PIC module on a grid inside the beam pipe (using solvers with or without conducting boundary conditions)
  - Uniform or 1-2 stripes initial e-distributions
  - Kicks can be given at locations with different beta functions, which need to be input on a different file.

Features of HEADTAIL (II)

- Short range wake field due to a broad band impedance

$$Z_{1\perp}(\omega) = \frac{\omega_R}{\omega} \frac{Z_\perp}{1 + iQ_\perp\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)}$$

  or to resistive wall.

  x and y components of the wakes can be weighed by the Yokoya coefficients to include the effect of flat chamber/structure.

- Space charge: each bunch particle receives a transverse kick proportional to the local bunch density around the local centroid. Longitudinal space charge is optional.
- Linear coupling between transverse planes

$$\begin{pmatrix} x_{n+1} \\ x'_{n+1} \end{pmatrix} = M_1(\delta p) M_2(I_x, I_y) \left[M_{sc}(z) \begin{pmatrix} x_n - \hat{x}(z) \\ x'_n + \Delta x'_{EC,Z_{1\perp}} - \hat{x}'(z) \end{pmatrix} + \begin{pmatrix} \hat{x}(z) \\ \hat{x}'(z) \end{pmatrix} \right]$$

Features of HEADTAIL (III)

- Number and charge of macro-electrons fixed.
- Electron dynamics is resolved transversely in the nonlinear beam field and in optional configurations of magnetic field (field-free, dipole, solenoid, combined function)

$$\ddot{\bar{r}}_e - \frac{e}{m_e}\left(\bar{E}(\bar{r}_e) + \dot{\bar{r}}_e \times \bar{B}_{ext}\right) = 0$$

- Electrons elastically reflected at the walls.
- But for **coasting beams**:
  - electrons are generated turn by turn according to the ionization or proton/ion loss rate
  - they can generate secondaries by impact with the pipe wall (charge of the macro-electrons is changed upon impact)

Features of HEADTAIL (IV)

- The **output files** of HEADTAIL give:
  - Bunch centroid positions, rms-sizes and emittances (horizontal, vertical and longitudinal) as a function of time
  - Slice by slice centroid positions and rms-sizes. Coherent intra-bunch patterns can be resolved using this information.
  - Transverse and longitudinal phase space of the bunch
- Off line analysis of the HEADTAIL output allows evaluating tune shifts, growth rates, mode spectra.
- Instability thresholds can be determined through massive simulation campaigns with different bunch intensities or lengths or emittances.

Features of HEADTAIL (V)

- In 2006 a number of new features have been added to **HEADTAIL**...
  - Interaction of the bunch with **several resonators** placed at locations with different beta functions (list needs to be input on a separated file)
  - The resistive wall impedance has been extended to include the **inductive by-pass effect** (benchmark of **HEADTAIL** tracking with the SPS collimator experiment)
  - The **initial distribution of electrons** can be self-consistently loaded from a build-up code (**ECLOUD)** run → see next slide

Headtail upgraded

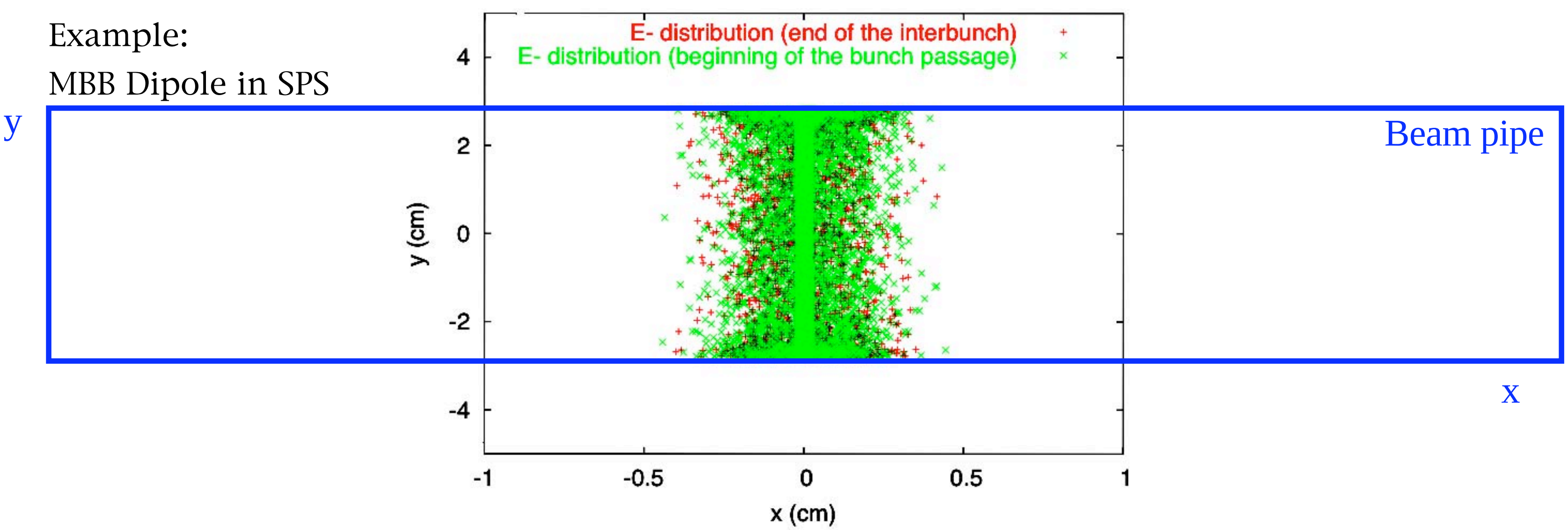
The electron distribution used in HEADTAIL was uniform in the beam pipe or with a single- or two-stripes to better fit the real distribution in a dipole field region...

→ Why not improve the model by using as an input the real distribution of electrons as it comes out of the build up ECLOUD code??

→ The electron distribution at the very beginning of a bunch passage is stored in a file from an ECLOUD run and subsequently fed into HEADTAIL. **This model is closer to self-consistent!**

Example:

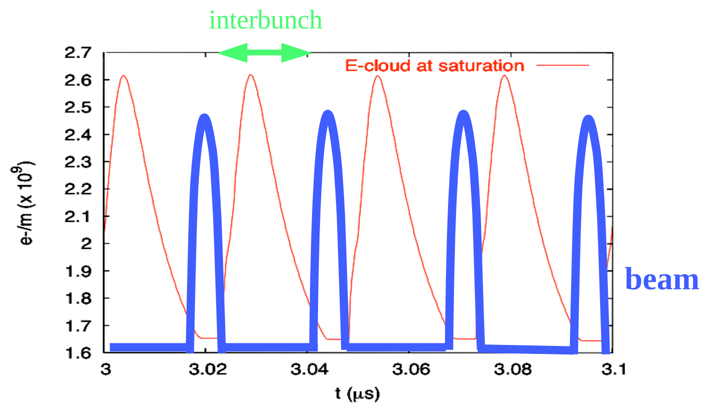
MBB Dipole in SPS

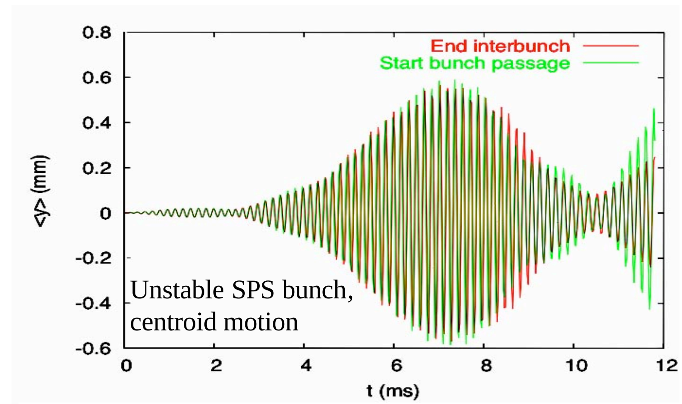


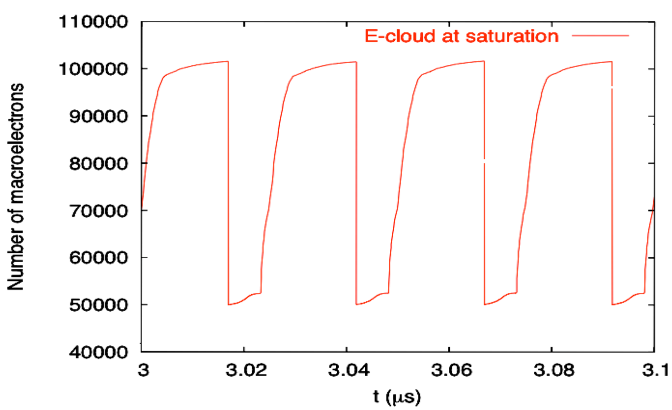
Headtail upgraded (II)

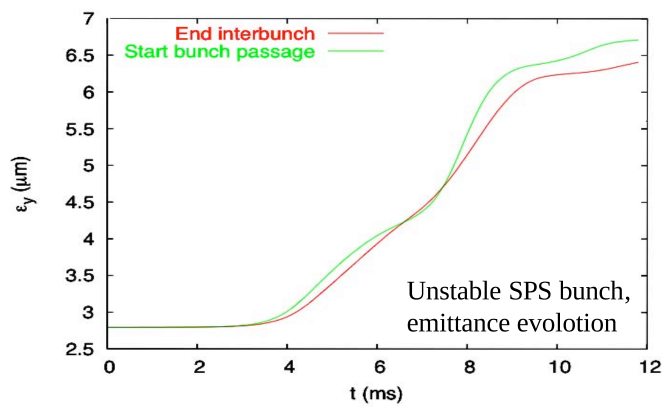
Between interbunch gap and bunch passage ECLOUD runs a `clean` routine to remove all the macro-electrons with very low charge.

Results of HEADTAIL simulations stay unchanged. The CPU time is about halved.









Applications 1

E-cloud instability in the SPS, energy dependence of threshold

Using the upgraded HEADTAIL code, the dependence of the e-cloud instability threshold on energy has been studied (for constant bunch length and longitudinal emittance)

Table 1: SPS parameters used in the simulation

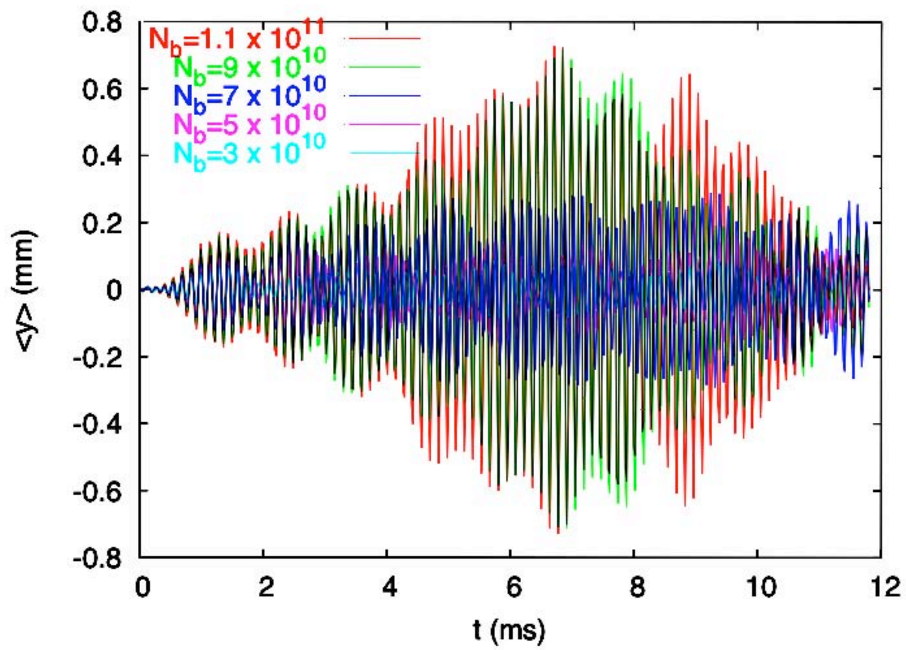
Parameter	Symbol (unit)	Value
Momentum	p_0 (GeV/c)	scanned between 14 and 270
Bunch intensity	$N_b(\times 10^{11})$	scanned between 0.3 and 1.1
Longitudinal emittance (2σ)	ϵ_z (eVs)	0.35
Bunch length ($1{\cdot}\sigma$)	σ_z (m)	0.3
Mom. compaction	α	1.92×10^{-3}
Norm. r.m.s. emittances	$\epsilon_{x,y}$ (μm)	2.8/2.8
Tunes	$Q_{x,y}$	26.185/26.13
Chromaticities	$\xi_{x,y}$	corrected, corrected
E-cloud density (average)	ρ_e (m^{-3})	$0.3 - 1 \times 10^{12}$

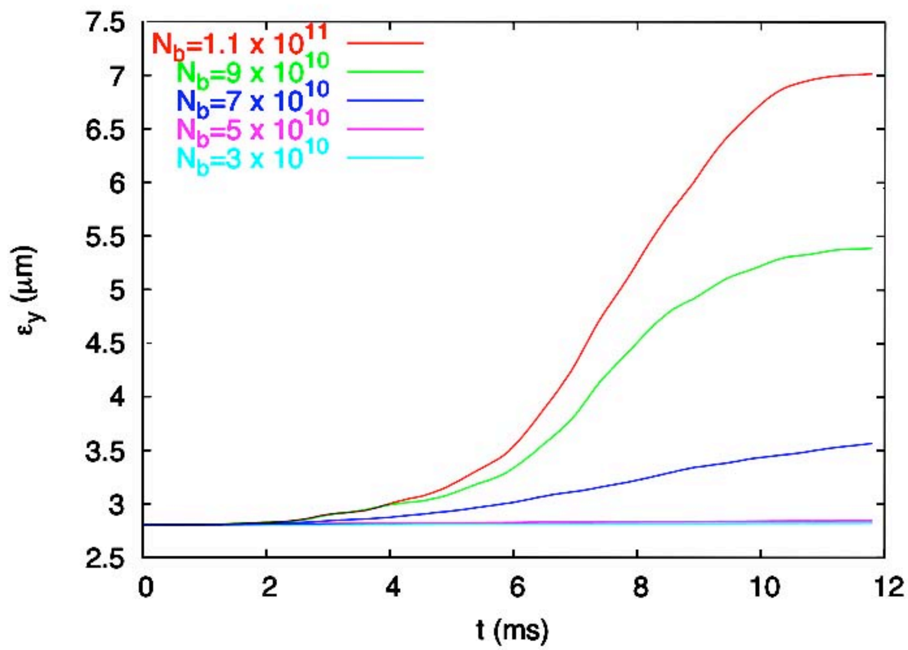
E-cloud instability in the SPS, energy dependence of threshold (II)

- Looking for the dependence of the e-cloud instability threshold on energy with **HEADTAIL** means:
  - → „Coarse“ intensity scan → at least 10 bunch intensity values scanned for each energy value (**10 x 10 runs**)
  - → As many (100) ECLOUD runs needed beforehand to get the electron distributions that have to be used in HEADTAIL
  - → N_{el} comes from ECLOUD and ranges usually from **5×10^4 to 10^5**.
  - → N_{pr} and N_{bin} need to be chosen as a balance between:
    - Bunch slicing still assures a good resolution of the electron motion: $N_{bin} >> n_{e,osc}$
    - All slices are enough populated (>10^3) , even those in the tails.

    Typical numbers are **$N_{pr}=3 \times 10^5$ and $N_{bin}=80$**
  - → CPU times amount to **~10h per run (512 turns)**

E-cloud instability in the SPS, energy dependence of threshold (III)



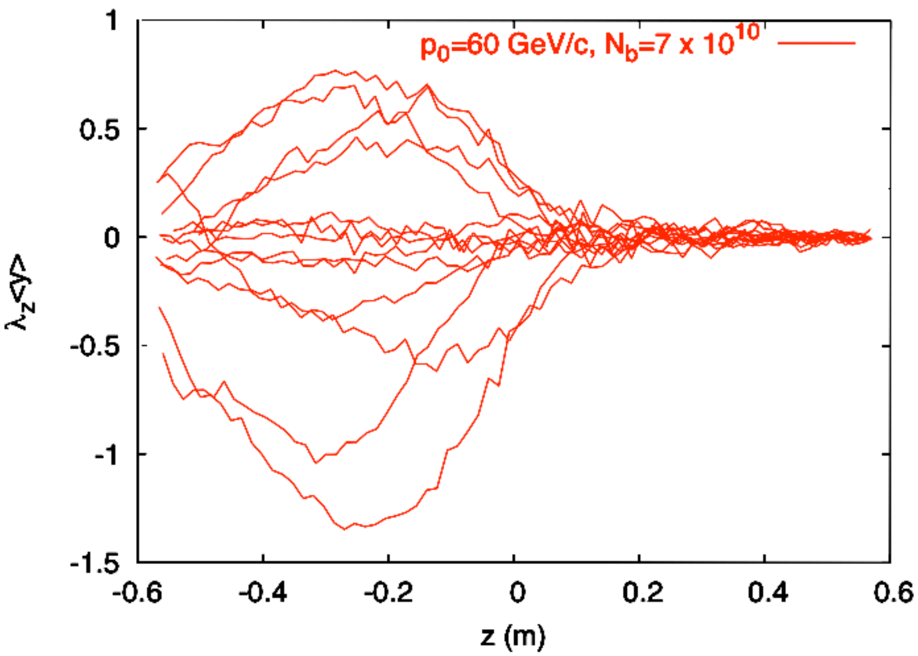


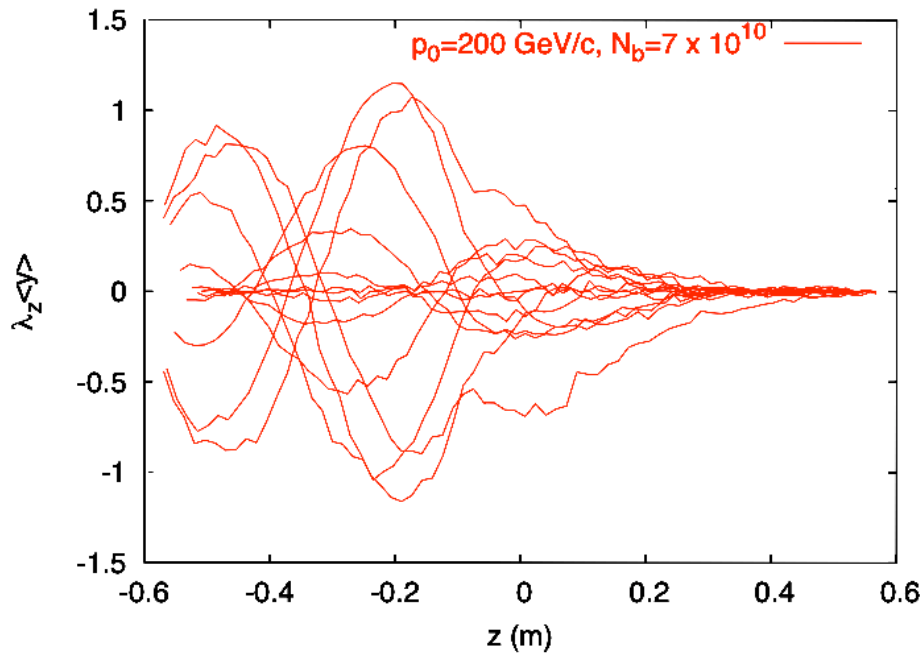
Example at 40 GeV/c with centroid motion and emittance evolution at different bunch intensities:

→ There is a coherent motion of the bunch with threshold between 5 and 7 x 10^{10}

→ simulations are in dipole field regions, the instability appears in the vertical plane.

E-cloud instability in the SPS, energy dependence of threshold (IV)

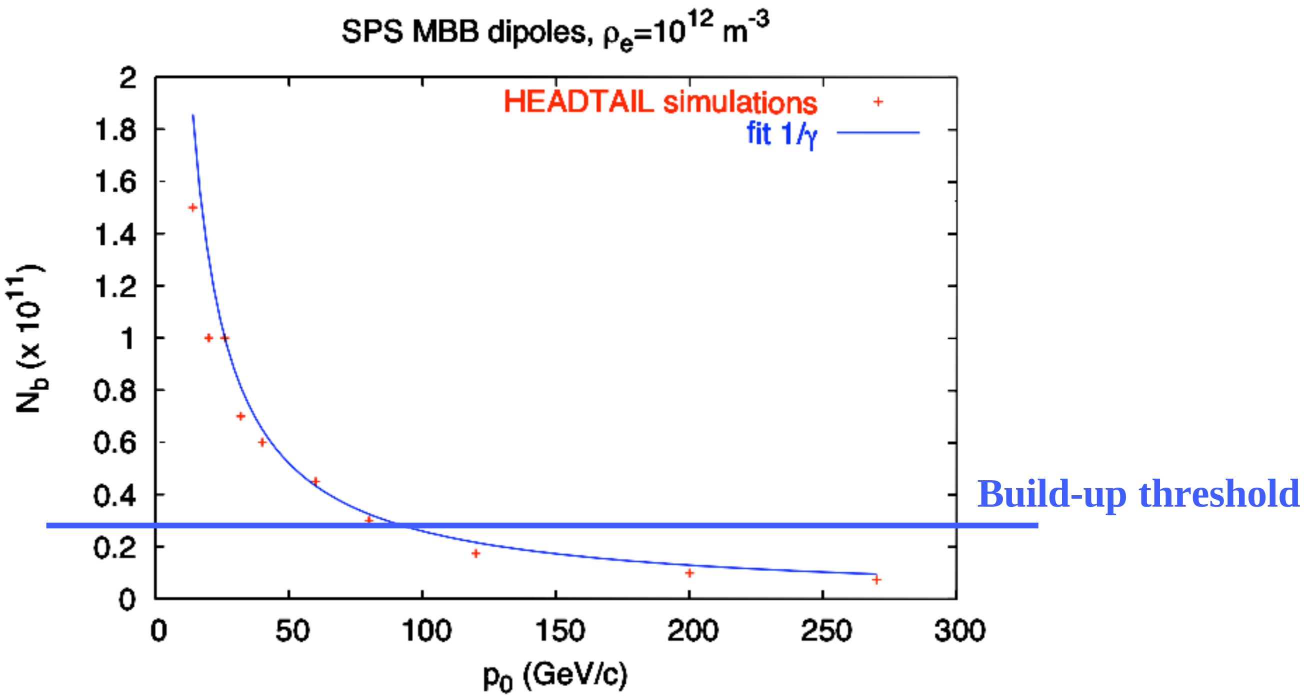




The coherent motion appears along the bunch with a typical TMCI pattern.

Example ↪ The figures above are superimposed snapshots of the centroid motion along the bunch at different times for the 60 and 200 GeV/c cases.

E-cloud instability in the SPS, energy dependence of threshold (V)



Full energy scan shows:

• Bunch intensity threshold **decreases with energy!**

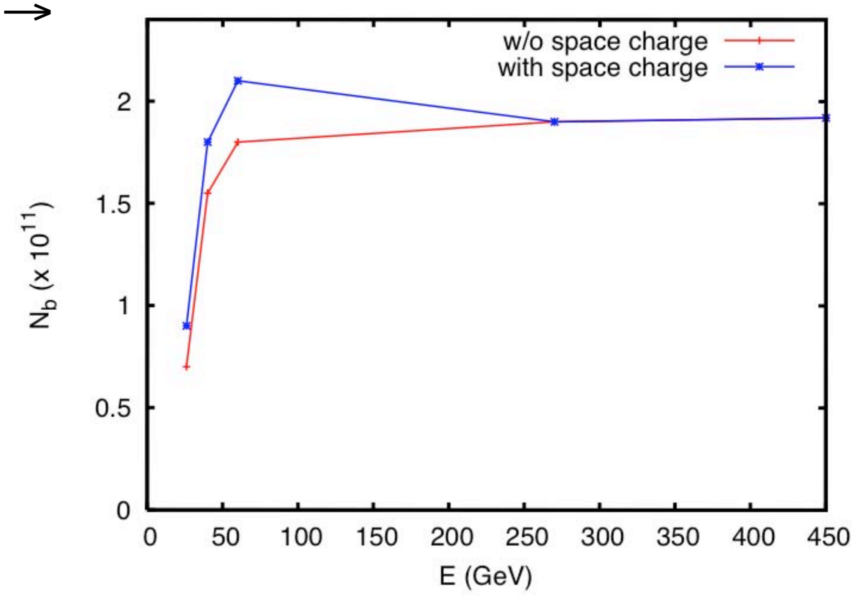
• Above ~80 GeV/c the instability threshold becomes lower than the build-up threshold, which means that the instability threshold would actually level off at the build up threshold value.

From E-cloud instability in the SPS to the Transverse Mode Coupling…

→ The e-cloud instability exhibits a different behaviour than the TMCI

„*Simulation Study on the Energy Dependence of the TMCI Threshold in the CERN-SPS*", G. Rumolo, E. Métral, E. Shaposhnikova, EPAC'06, Edinburgh

→

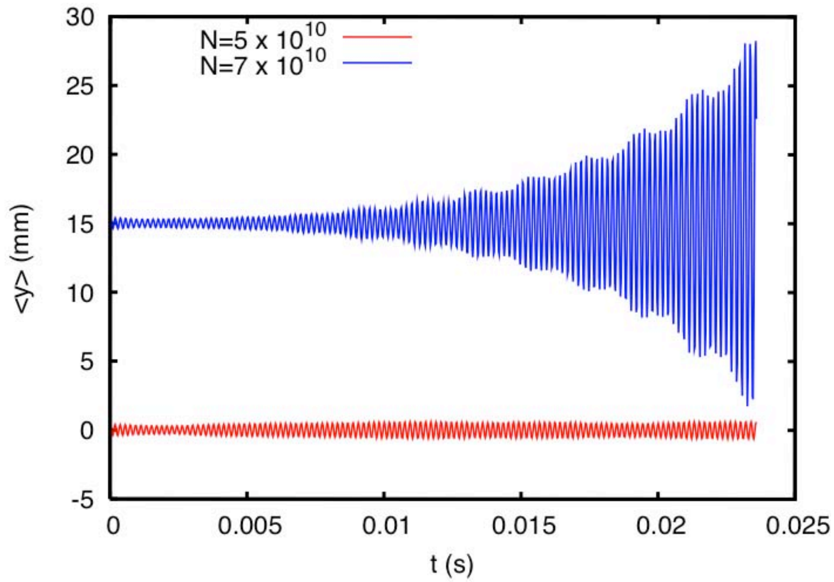


Based on HEADTAIL simulations, shows a scaling law $\propto|\eta|$
This is in agreement with the analytically derived formula by B. Zotter (1981).

Applications 2

TMCI simulation and thresholds

→ Like the e-cloud instability threshold, the TMCI threshold is inferred by HEADTAIL tracking when unstable coherent motion of the bunch centroid with exponential growth suddenly appears for a tiny change of bunch current.



Advantages of HEADTAIL:

- It allows for full simulations of flat geometries by using dipole and quadrupole wakes appropriately scaled by the Yokoya factors.
- It allows for simulation of longitudinally unmatched bunches
- It allows for combined impedance-space charge simulations.
- It gives as an output the full bunch dynamics in the unstable regime.

TMCI simulation and thresholds (II)

- **HEADTAIL** tracking to investigate on a TMCI threshold requires a bunch intensity scan with a **fixed impedance model**.

→ The range where the threshold lies is usually roughly estimated with an analytical formula. Typically then about 10 intensity values in this range are scanned. To find the threshold might require more iterations.

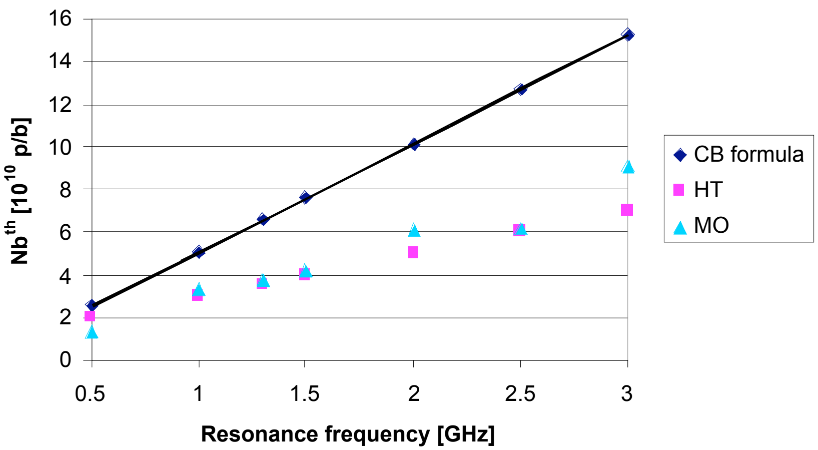
→ The numerical parameters depend on the specific problem:

- Bunch length
- Resonance frequency of the resonator(s) interacting with the beam

The criterion is to have enough bunch slices to resolve the oscillation of the wake field over the bunch longitudinal extension. Some times N_{pr}=**3 x 10^6** and N_{bin}=**500** might be required.

→ CPU times per run range **between the 10 and the 30 hours**.

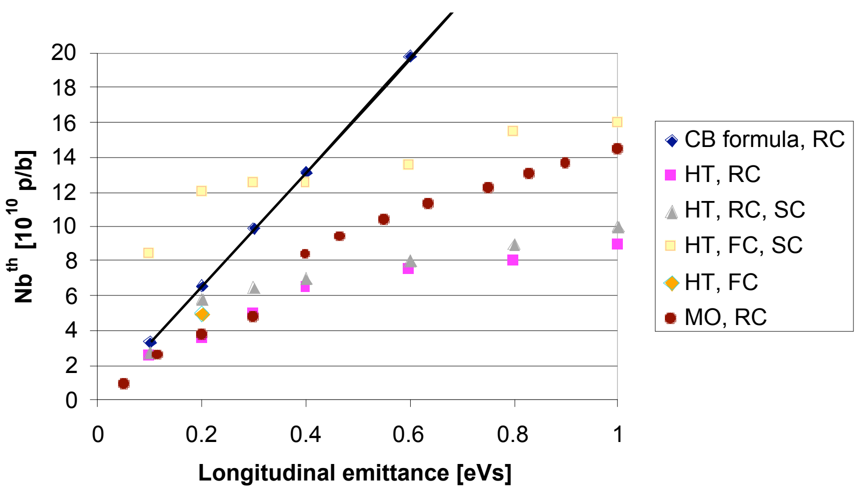
Benchmarking MOSES and HEADTAIL



E. Métral et al.

„Transverse Mode Coupling Instability in the CERN-SPS"

ICFA-HB2004, Bensheim, Germany, 18-22/10/2004



- The agreement between HEADTAIL and MOSES is excellent for low longitudinal emittances.
- The agreement is worse but still keeps within a factor less than 2 for higher longitudinal emittances (MOSES uses the linearized synchrotron motion)

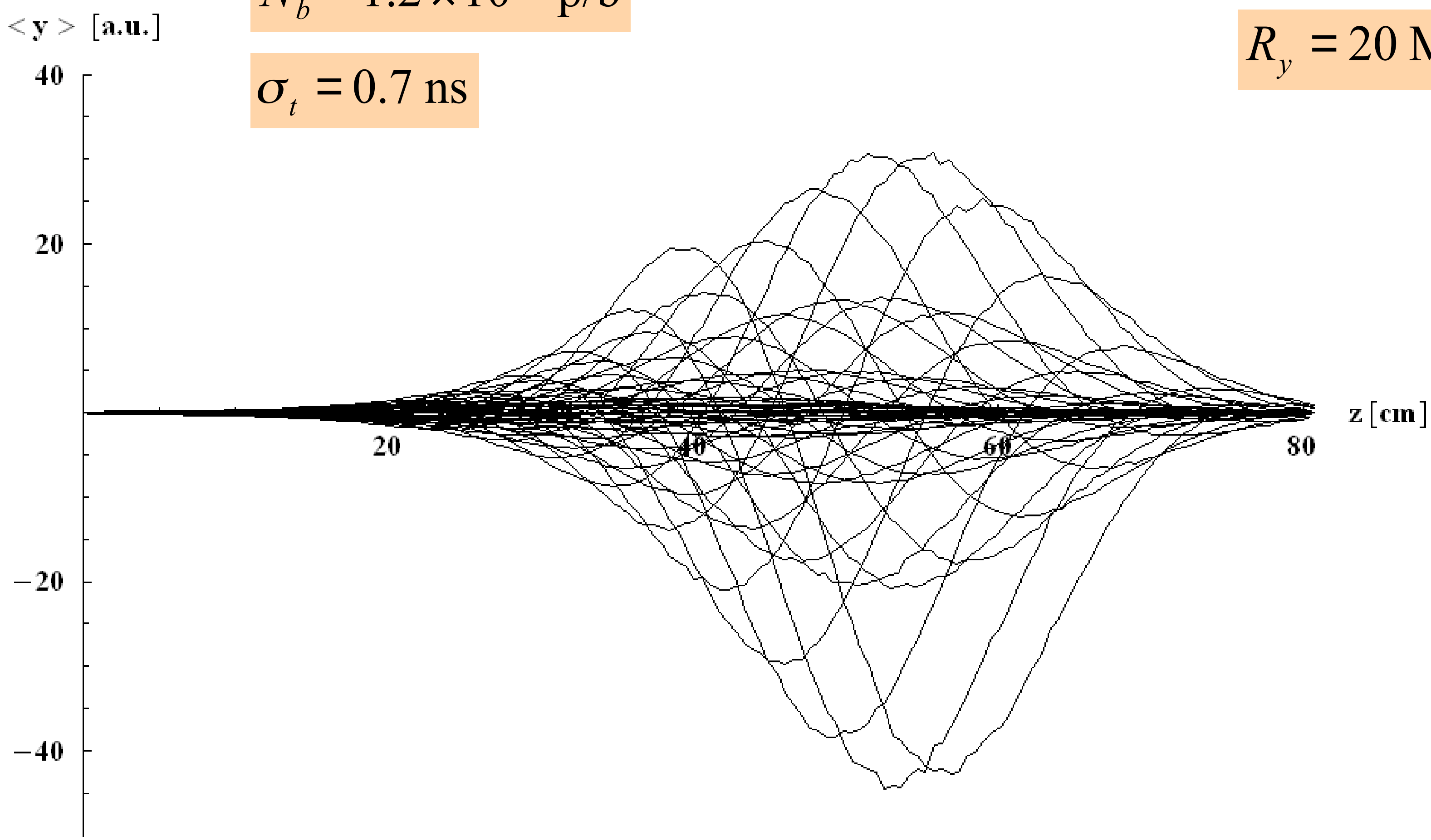
$N_b = 1.2 \times 10^{11}$ p/b

$\sigma_t = 0.7$ ns

$f_r = 1$ GHz

$R_y = 20$ MΩ/m

$Q = 1$



SPS TMCI simulated with HEADTAIL using a BB-impedance

The case of the PS instability with HEADTAIL - I

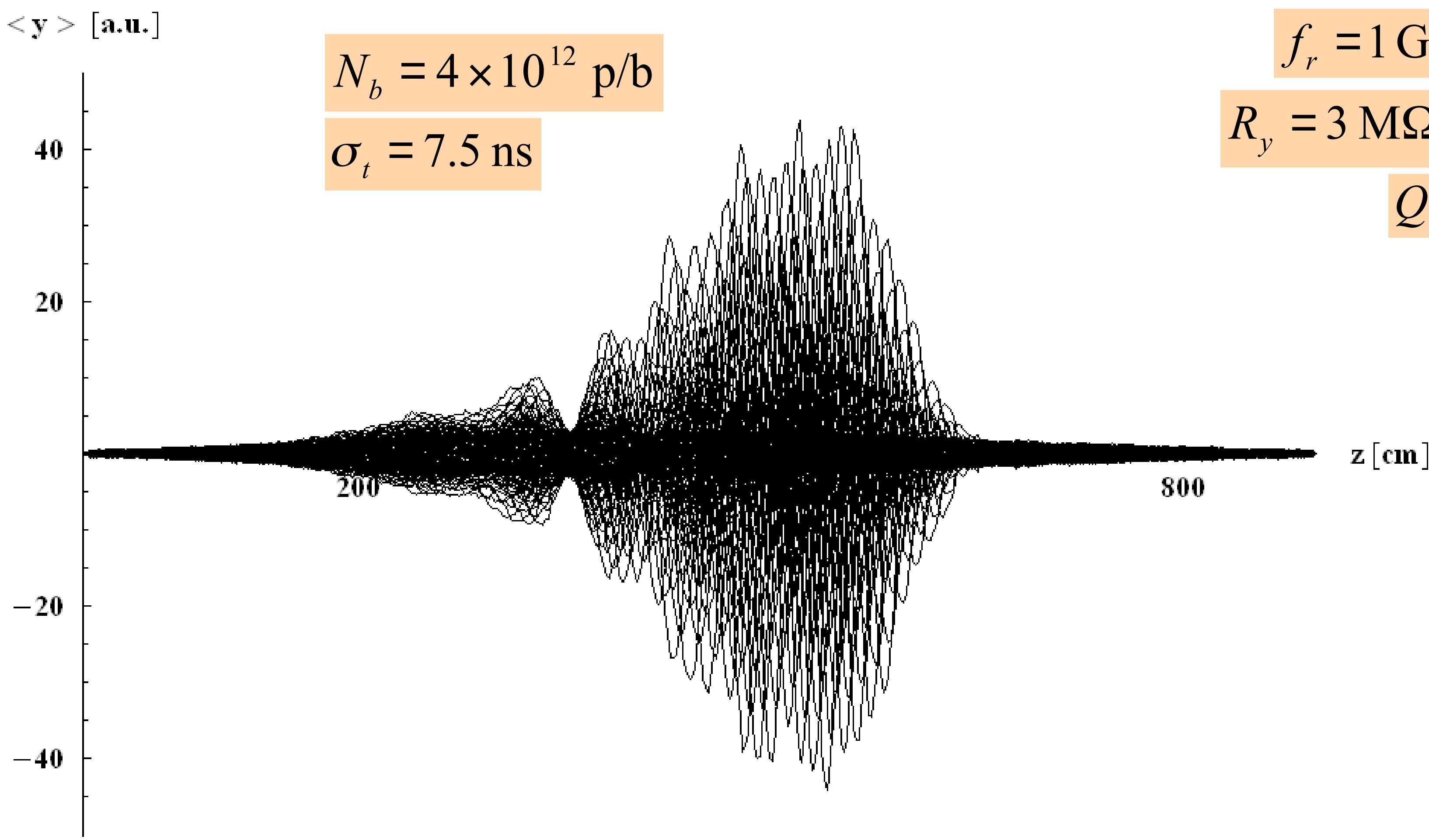
$N_b = 4 \times 10^{12}$ p/b

$\sigma_t = 7.5$ ns

$f_r = 1$ GHz

$R_y = 3$ MΩ/m

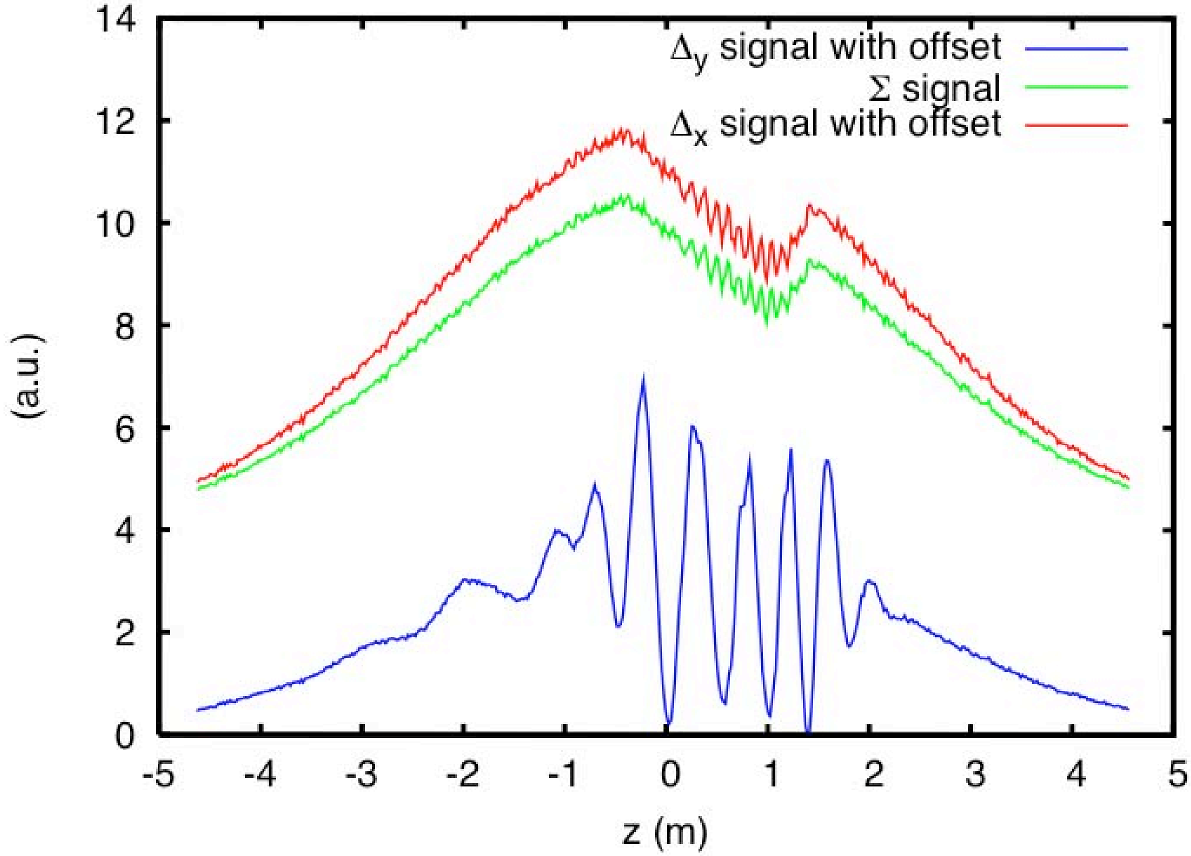
$Q = 1$

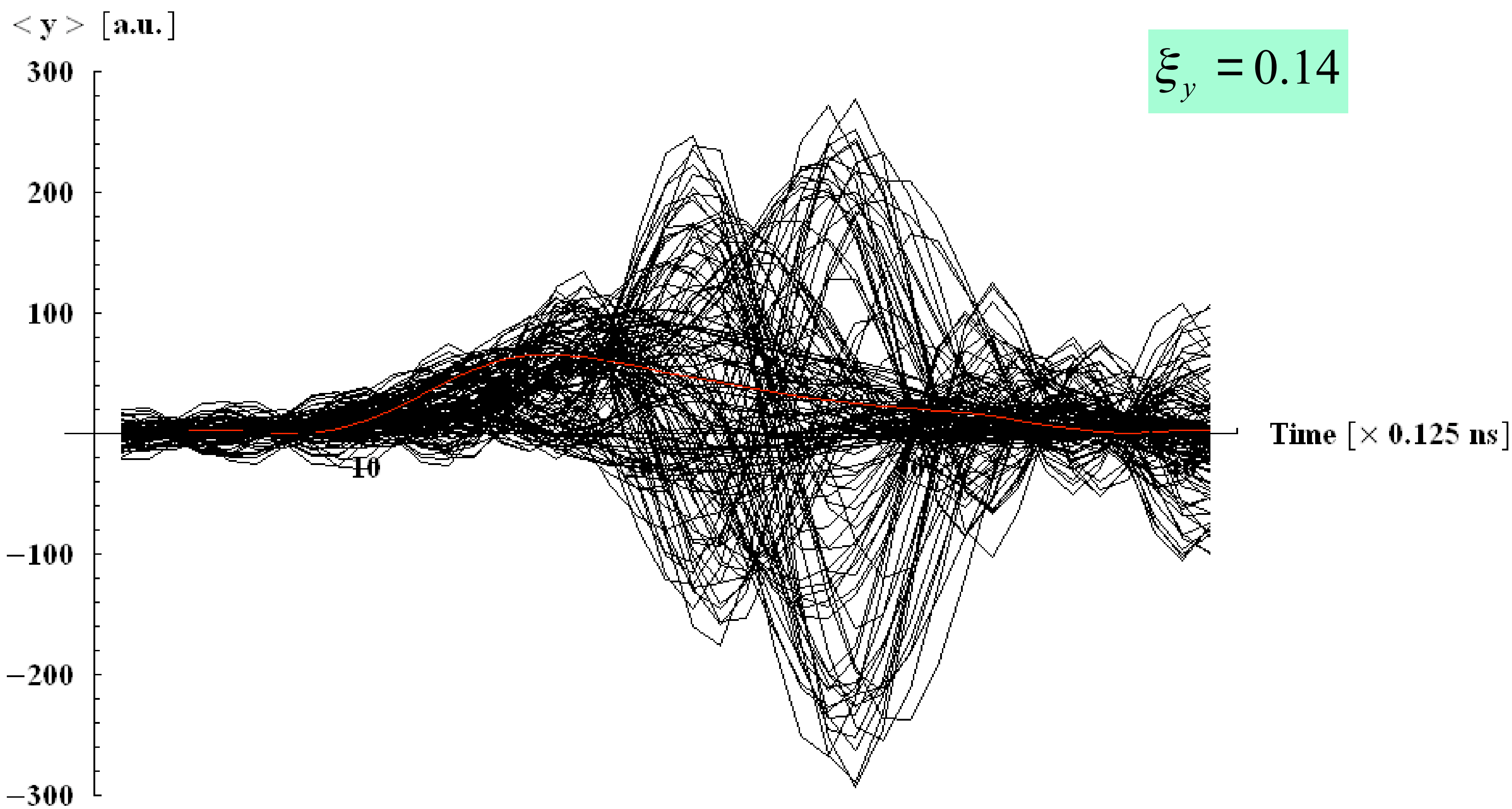


PS TMCI simulated with HEADTAIL using a BB-impedance

The case of the PS instability with HEADTAIL - II

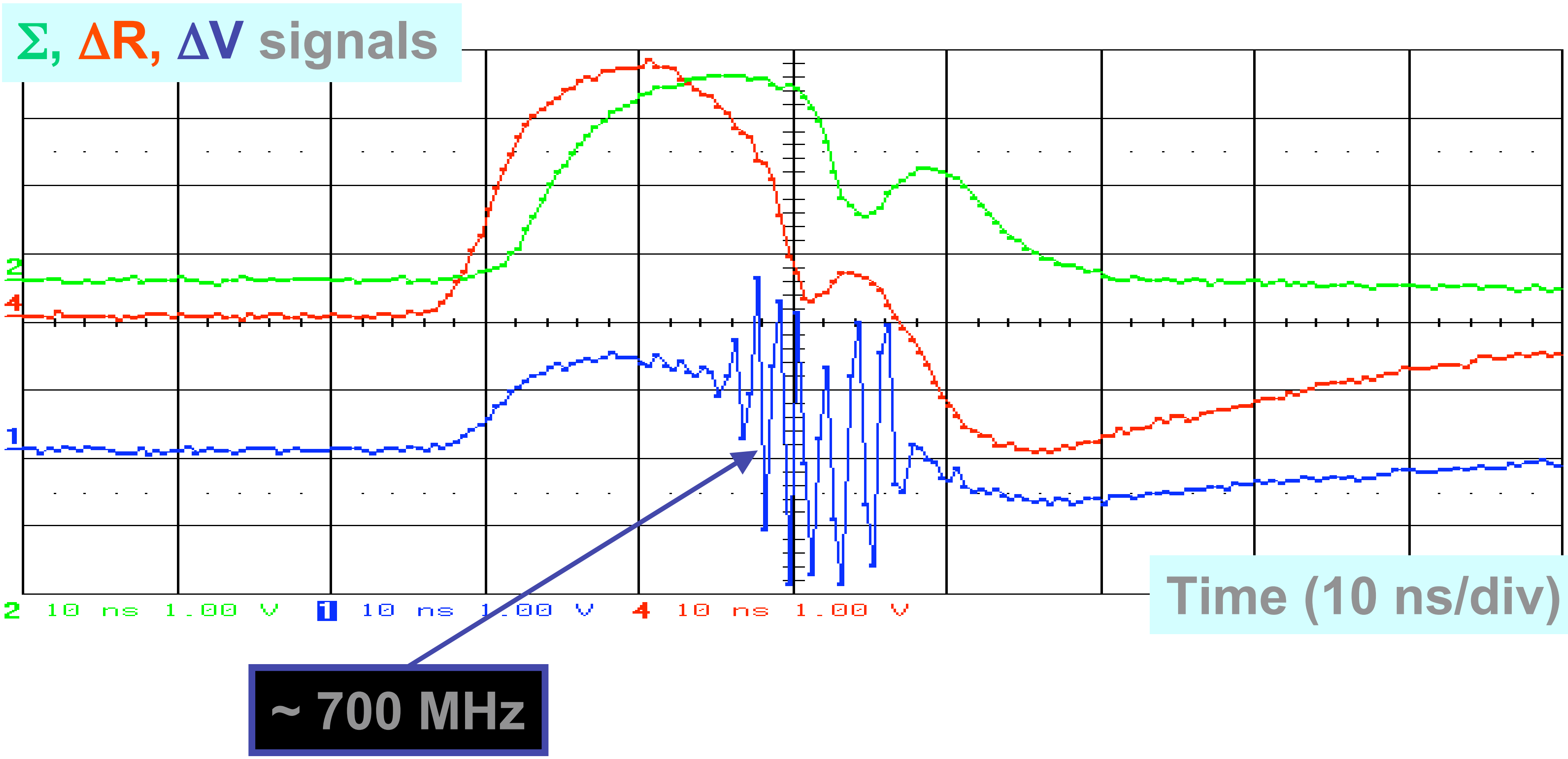
HEADTAIL simulation - turn # 149





Experimental data from the SPS: observed TMCI

Observation of a fast vertical single-bunch instability near transition (~ 6 GeV) in the CERN-PS



Applications 3

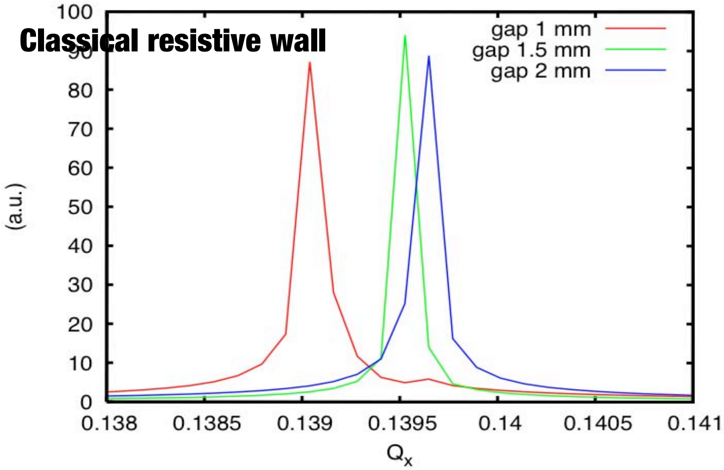
The collimator impedance (resistive wall with inductive by-pass)

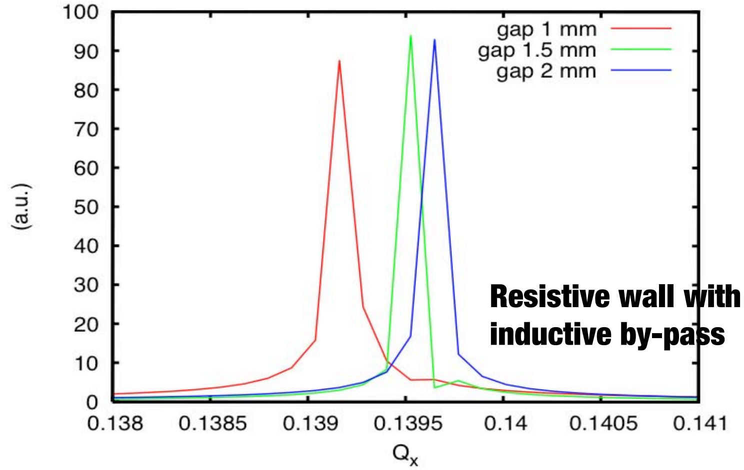
- In 2004 measurements were done at the SPS with a prototype of an LHC collimator:
  - The collimator jaws were moved in and out to **different gap aperture values**.
  - The **tune shift** was measured for a few gap values.
- The resistive wall model in HEADTAIL was improved to include both the **inductive by-pass effect** (impedance formula by L. Vos and Burov-Lebedev, wake field by A. Koschik) and the **nonlinear coefficients** for the wake at large (near-wall) amplitudes (Piwinski)
- Tracking results could be fully benchmarked against experimental data.

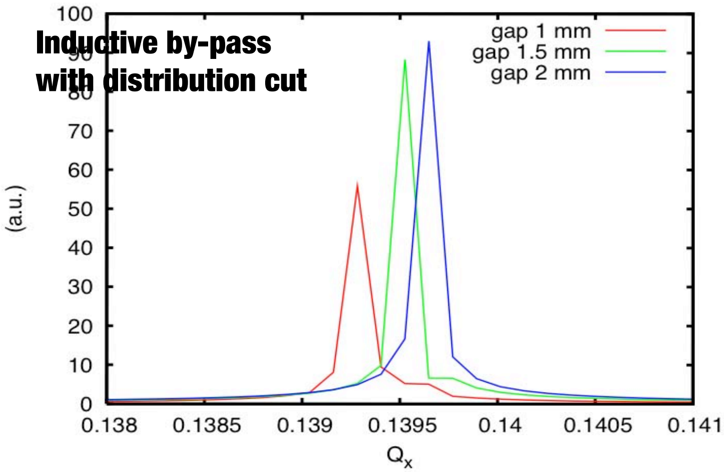
The collimator impedance (resistive wall with inductive by-pass) (II)

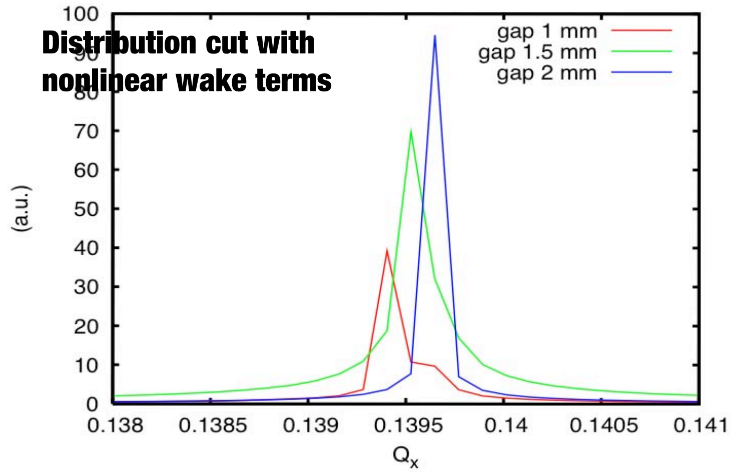
- **HEADTAIL** tracking using resistive wall with inductive by-pass wake field and nonlinear wakes requires compromises between accuracy and computational speed.
  - → The use of external libraries for the special functions can significantly slow down the execution of the program:
    - A „frozen“ wake field option has been implemented, which computes the slice-to-slice wakes at the beginning of the execution, stores the values and then uses them turn by turn.
    - Applicable only for matched bunches!
  - → A too frequent call of sine and cosine functions for the nonlinear wakes (6 x slice-particle pair) needs to be avoided:
    - The non-oscillatory character of the wake allows for a coarser slicing of the bunch with respect to resonator runs (N_{pr}=10^5 and N_{bin}=50)
  - → CPU times per run are ~8h.

The collimator impedance (resistive wall with inductive by-pass) (III)

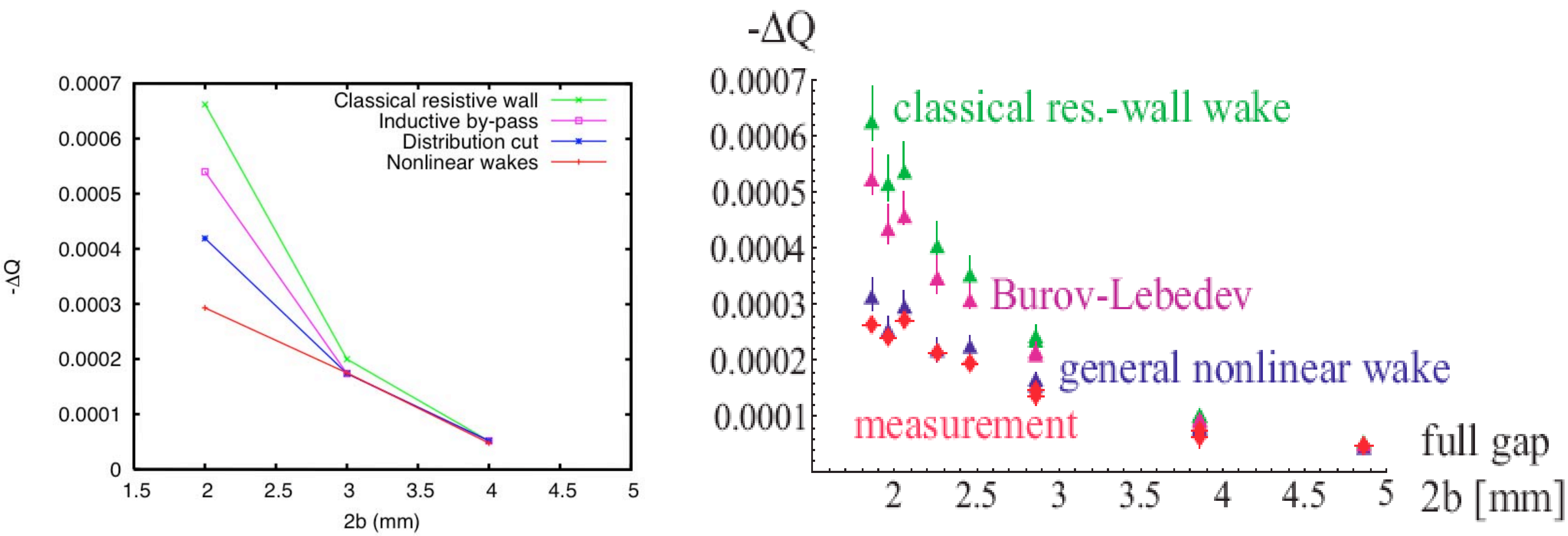








The collimator impedance (resistive wall with inductive by-pass) (IV)



Comparing the tune shifts extrapolated from **HEADTAIL simulations** (left plot) with the experimental ones (right plot, **red** points) and those from analytical theory (right plot, **green**, **magenta**, **blue** points), the agreement is excellent.

Conclusions

- HEADTAIL is a versatile tool that can be used to do particle tracking with a variety of collective interactions (electron cloud, broad-band impedances, resistive wall, space charge)
- Upgrade of HEADTAIL over the last year includes
  - Use of **self-consistent electron distribution** for the e-cloud simulations imported from build-up code ECLOUD
  - Use of **arbitrary number of resonators** interacting with the beam
  - Improved model of **resistive wall and nonlinear wakes**
- HEADTAIL performances are satisfactory in terms of **computational speed** with an appropriate choice of modeling and numeric parameters!! (CPU times never exceed ~1 day/run)
- Benchmark of HEADTAIL against other codes and against experiments (where possible) is successful.