NUMERICAL COMPUTATION OF KICKER IMPEDANCES : TOWARDS A COMPLETE DATABASE FOR THE GSI SIS100/300 KICKERS*

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Abstract

Fast kicker modules represent a potential source of beam instabilities in the planned Facility for Antiproton and Ion Research (FAIR) at the Gesellschaft fr Schwerionenforschung (GSI), Darmstadt. Containing approximately six tons of lossy ferrite material, the more than forty kicker modules to be installed in the SIS-100 and SIS-300 synchrotrons are expected to have a considerable parasitic influence on the high-current beam dynamics. In order to be able to take these effects into account in the kicker design, a dedicated electromagnetic field software for the calculation of coupling impedances has been developed. Here we present our numerical results on the longitudinal and transverse kicker coupling impedances for the planned components. Besides the inductive coupling of the beam to the external network -relevant below 250 MHz- particular attention is paid to the impact of ferrite losses up to the beampipe cutoff frequency.

INTRODUCTION

Ensuring the stability of high-intensity ion beams is one of the most important research and development goals within the design work of the planned Facility for Antiproton and Ion Research (FAIR) at the Gesellschaft für Schwerionenforschung (GSI), Darmstadt. Currently, detailed beam dynamics studies are aiming at optimizing critical parts of the existing and the planned parts of the accelerator.

The goal of the work presented here is to characterize the various planned kicker devices of the SIS-100/SIS-300 synchrotrons (injection, extraction/emergency, transfer, and Q kickers) in terms of their coupling impedances, which in turn can be used as input data for beam stability investigations. In this way, the parasitic influence of the kickers on the ion beam can be quantified during the design phase, and, if needed, reduced by the optimization of the devices. As we have demonstrated in previous publications [1, 2, 3], numerical field calculations offer a convenient way of providing coupling impedance data without the need of protoypes. Moreover, comparisons with measurements have shown the reliability of computer simulations in this respect (see Arevalo et *al.*, this conference).

Here, we review the essential numerical aspects of our dedicated impedance code, before discussing in detail the simulation results for one of the SIS-100 kickers.

Definitions

We follow the notation of [4]. The vertical coupling impedance is defined by

$$Z_y(\omega) \equiv \frac{\mathrm{i}}{qd} \int_{-\infty}^{\infty} \mathrm{d}z \left(E_y + \beta c B_x \right) |_{(0,0,z;\omega)} e^{\mathrm{i}kz}, \quad (1)$$

where βc is the velocity of the beam and $k = \omega/\beta c$. The electro-magnetic field ($E(x, y, z; \omega)$), $B(x, y, z; \omega)$) is due to the source current, $j(x, y, z; \omega)$, given by $j_x = j_y = 0$, and

$$j_z(x, z, y; \omega) = q\delta(x)\delta(y - d)e^{-ikz}, \qquad (2)$$

which is the frequency-domain description of a point charge q travelling along the z-axis with velocity βc and vertical offset y = d, i.e.

$$j_z(x, z, y; t) = q\beta c \,\delta(x)\delta(y - d)\delta(z - \beta ct).$$

We remark that the offset d is essential to the definition of Z_y , as well as the fact that fields in Eq. 1 are evaluated at x = y = 0. Analogous expressions can be written down for the horizontal coupling impedance.

Please note that, after our convention, the fields E and B carry the units of Vs/m and Ts, respectively, as they constitute spectral densities. Similarly, $[j] = As/m^2$, so that Z_y has the unit of Ω/m . The reason for the ocurrence of 1/m is that Z_y represents an impedance *per transverse displacement*, d: For small enough d, Z_y is expected to become independent of d, which is why this parameter has been suppressed in $Z_y(\omega)$. Note that in the center of a symmetrical device, there cannot be any transverse force, i.e. $\lim_{d\to 0} Z_y d = 0$, whereas in general $Z_y \neq 0$.

The longitudinal coupling impedance is defined by

$$Z_{||}(\omega) \equiv -\frac{1}{q^2} \int \mathrm{d}V \boldsymbol{j}^* \cdot \boldsymbol{E},\tag{3}$$

with the current of Eq. 2 at d = 0.

Two-wire Approximation

Expression Eq. 1 for the vertical impedance can be recast into a form that is more convenient for computations by using Faraday's law [4],

$$Z_y(\omega) \approx \frac{-1}{kq^2(2d)^2} \int dV \boldsymbol{E}^{(2)} \cdot \boldsymbol{j}^{(2)}.$$
 (4)

The superscript indicates that the excitation current consists of two anti-parallel currents with separation 2d, i.e.

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$$j_x^{(2)} = j_y^{(2)} = 0,$$

$$j_z^{(2)} = q\delta(x) \left(\delta(y-d) - \delta(y+d)\right) e^{-ikz}.$$
 (5)

In the derivation of Eq. 4, a partial derivative $\partial_y E_z$ is replaced by a difference quotient, meaning that it is an approximation of Eq. 1 which becomes exact for $d \rightarrow 0$. Eqs. 4 and 5 are the '2-wire' representation of the transverse, vertical coupling impedance. The horizontal impedance, Z_x , can be expressed in a similar way.

Eqs. 3 and 4 show why calling Z_x , Z_y , and $Z_{||}$ impedances is plausible, as they are given by integrals over power densities, devided by the current squared (leaving alone the factor $1/k\Delta^2$ for the transverse case). The minus sign ensures that their real parts are positive or zero because the integral over a power density must always be negative in a passive device - the beam can only lose energy.

COMPUTATIONAL APPROACH

Discretization of the Field Problem

We solve the wave equation

$$\partial \times \mu^{-1} \partial \times \boldsymbol{E} - \omega^2 \epsilon \boldsymbol{E} = -\mathrm{i}\omega \boldsymbol{j}$$
 (6)

within the framework of the Finite Integration Technique (FIT), first proposed in [5], see also [6]. This scheme allows the approximation of Eq. 6 by the system of linear, algebraic equations,

$$\boldsymbol{C}^{T}\boldsymbol{M}_{\mu}^{-1}\boldsymbol{C}\widehat{\boldsymbol{e}} - \omega^{2}\boldsymbol{M}_{\epsilon}\widehat{\boldsymbol{e}} = -\mathrm{i}\omega\widehat{\boldsymbol{j}} \qquad (7)$$

the unknowns of which ($\widehat{e} \equiv \{e_i\}$) are the line integrals of the electric field along the edges of a structured, hexagonal grid. The righthand-side excitation \widehat{j} is a vector of currents through dual x - y grid facets [7]. Alternatively, \widehat{j} can be conceived as a filament current along the primary grid edges (see next section). The matrix C may be interpreted as a discretized version of the continous curl operator, whereas the matrices M_{ϵ} and M_{μ} reflect the permittivity and (complex) permeability, respectively, plus mesh metrics [8].

Discretization of the Beam Current

The beam currents given by Eq. 2 (d = 0, computation of $Z_{||}$) and Eq. 5 (Z_y calculation) are approximated by one or two filament currents on the primary FIT grid along the z-axis, respectively. Each filament segment carries the phase $\exp(-ikz)$ corresponding to the center of the respective grid edge.

As in the discrete setting the delta functions are approximated by Kronecker deltas w.r.t. grid indices, one may wonder which effective thickness the single excitation filaments have. By comparing simulation results with analytical expressions for simple 2D models, we found that the effective radius of a filament is approximately *one half* of the transverse spacing of grid lines [9]. Comparing computations with different mesh resolutions, we found that kicker coupling impedances are rather insensitive to the thickness of the filaments (data not shown here).

Boundary Conditions

In our simulations, the walls of the vacuum vessel containing the kicker module are assumed to be perfectly conducting (i.e. tangential electrical fields vanish on the wall). At the beam-entry and exit planes z_{\min} and z_{\max} , however, special boundary conditions are needed, which we discuss in the following. Generally, the transitions from the module to the adjacent beam pipe have to be included into the computation, since jumps in pipe cross sections are known to contribute to the coupling impedance. For frequencies below beam-pipe cutoff, the additional field excited within the module decays exponentially into the beam pipe. Thus, at some distance along the beam pipe, the perturbation resulting from the module can be neglected and fields can be considered stationary in the sense that $(\boldsymbol{E}, \boldsymbol{B})_{\text{pipe}} \propto \exp(-ikz)$, where $k = \omega/\beta c$. Using this property, we set up boundary conditions for the 3D problem by solving for $(E, B)_{\text{pipe}}$ in a 2D cross section of the beam pipe. For a complete discussion of beam-adapted boundary conditions, see [10].

Solution of the Linear Equations

The solution of the complex-symmetric, non-hermitian linear system, Eq. 7, is carried out as follows. We split the solution into two contributions, $\hat{e} = \hat{e}_0 + \hat{e}_{dyn}$, which satisfy $C\hat{e}_0 = 0$ and $G^T M_{\epsilon} \hat{e}_{dyn} = 0$. Here, G denotes the discrete gradient operator (a grid incidence matrix as is C). Non-trivial (multiple-connected) model topologies do *not* lead to an additional term in the decomposition, because all boundaries are perfectly conducting. First, the 'electrostatic' contribution is determined from the system,

$$\boldsymbol{G}^{T}\boldsymbol{M}_{\epsilon}\widehat{\boldsymbol{e}}_{0}=-rac{1}{\mathrm{i}\omega}\boldsymbol{G}^{T}\widehat{\boldsymbol{j}},$$
 (8)

the righthand side of which represents the vector of discrete charges. Knowing the solution \hat{e}_0 enables us to rewrite Eq. 7,

$$\boldsymbol{C}^{T}\boldsymbol{M}_{\mu}^{-1}\boldsymbol{C}\widehat{\boldsymbol{e}}_{dyn} - \omega^{2}\boldsymbol{M}_{\epsilon}\widehat{\boldsymbol{e}}_{dyn} = -\mathrm{i}\omega\widehat{\boldsymbol{j}} + \omega^{2}\boldsymbol{M}_{\epsilon}\widehat{\boldsymbol{e}}_{0}.$$
 (9)

The righthand side, **b**, of Eq. 9 (abbreviated by $A\widehat{e}_{dyn} = \mathbf{b}$) is discretely divergence free, i.e. $\mathbf{G}^T \mathbf{b} = 0$, which is favorable for the solution of Eq. 9 by a Krylov subspace solver, as we will see in the following. As a next step, Eq. 9 is rescaled by the inverse square root of the diagonal matrix M_{ϵ} :

$$\boldsymbol{M}_{\epsilon}^{-1/2}\boldsymbol{C}^{T}\boldsymbol{M}_{\mu}^{-1}\boldsymbol{C}\boldsymbol{M}_{\epsilon}^{-1/2}\boldsymbol{\widehat{e}}_{dyn}^{\prime} - \omega^{2}\boldsymbol{\widehat{e}}_{dyn}^{\prime} = \boldsymbol{M}_{\epsilon}^{-1/2}\boldsymbol{b},$$
(10)

(abbreviated by $A' \hat{e}'_{dyn} = b'$) with the rescaled unknown vector $\hat{e}'_{dyn} \equiv M_{\epsilon}^{1/2} \hat{e}_{dyn}$. The advantage of this form is that the subspaces $K_n = \{b', A'b', A'^2b', ..., A'^nb'\}$, produced by a Krylov iteration scheme, always fulfill $K_n \subset$

 $\left\{ \widehat{\boldsymbol{e}}' | \boldsymbol{G}^T \boldsymbol{M}_{\epsilon}^{1/2} \widehat{\boldsymbol{e}}' = 0 \right\}$, so that the convergence of the linear solver is not impaired by the null-space of the curl-curl operator.

The iterative solution of Eq. 10 constitutes the bottleneck in the computation of \hat{e} . The reason is that the large permeability difference between vaccum and ferrite parts results in a large condition number of the system matrix, A', leading to a slow convergence of the Krylov subspace solver. In our code, we have used the Trilinos linear algebra library, which offers various types of iterative solvers and powerful preconditioners [11]. After our experience, however, the most efficient and robust way to solve the complexsymmetric Eq. 10 is to use the conjugate-orthogonal conjugate gradient algorithm (COCG) [12], *without* any further preconditioning. Thus, the rescaling of Eq. 9 leading to Eq. 10 already constitutes a near-optimum preconditioning. This fact has also been reported in [13].

Inclusion of the Pulse-Forming Network (PFN)

It is well known that in a kicker device, a charged particle beam, displaced from the axis into the kick direction, creates an oscillating magnetic flux in the perpendicular transverse direction. This flux couples to the kicker-magnet coil, inducing an oscillating voltage at the plugs of the external, pulse-forming network (PFN), and thus a current through the PFN. As the PFN acts back on the beam through the magnet coil, it is important to include this effect into the EM field simulations. The PFN is not taken into account explicitly in our field computations, instead we use the equivalent lumped impedance, $Z_q(\omega)$ as its representation. At a given frequency, $Z_q(\omega)$ is introduced into the simulation as follows. One first localizes one of the FIT grid edges that join the two ends of the magnet winding (the ends are supposed to be one grid line apart). One then modifies the material matrix M_{ϵ} at the respective component j,

$$(\boldsymbol{M}_{\epsilon})_{jj} \to (\boldsymbol{M}_{\epsilon})_{jj} + \frac{1}{\mathrm{i}\omega Z_g(\omega)}.$$
 (11)

SOFTWARE

Constructing the computational models, meshing, and visualization of calculated fields has been done with the commercial program CST MICROWAVE STUDIO^(R) [14]. Since this tool is partly based on the Finite Integration Technique, it is able to produce the mesh and material data needed.

The implementation of the impedance code, i.e. the FIT electromagnetic field solver, pre- and postprocessing has been carried out in a hybrid C++/Python framework. Most of the code specific to our application has been implemented in Python [15], while the performance-critical linear-algebra computations are carried out within C++ subroutines (most notably of the Trilinos linear algebra package [11]).

Making the coupling impedance data of the SIS-100/300 kickers available to beam-dynamics simulations is the pri-



Figure 1: CAD model of one of the six modules of the SIS-100 extraction/emergency kicker. Ferrite (8C11) length: 90cm, ferrite thickness: 4cm, window width: 15.5cm, window height: 13cm (minus 2x coil thickness), coil thickness: 0.5cm. The ceramic is specified with $\epsilon_r = 9.6$, $\mu_r = 1$. Discretization is performed on a cartesian FIT grid with 180090 grid points. The number of unknowns (electric grid voltages) is 264827. (a) total view of the model. (b) details of the coil (two windings).

mary goal of the present work. In order to keep track of the many data sets gained for different parameters we have chosen to implement a Python-based database management which comprises containers for raw impedance data as well as methods for its further processing. Automatic interpolation between simulated frequency points and values of β , as well as more complicated tasks as implicitly processing raw impedance data within a PFN model (see next section) are required. The advantage of using the programming language Python for these tasks is that creating, storing, and interactive manipulation of 'intelligent' datasets (i.e. classes) is a built-in feature. Moreover, the code and data are largely platform independent.

EXAMPLE: THE SIS-100 EXTRACTION/EMERGENCY KICKER

For reasons of brevity, we consider only one of the SIS-100 kickers in the following. Figure 1 depicts one of the six extraction/emergency kicker modules. The bipolar magnet contains two windings and 4cm-thick ferrite plates of length 90cm. The beam-induced heating of the ferrite has been reduced by metal ('eddy-current') strips. A ceramic pipe leads through the magnet so that ferrite parts are placed outside of the UHV.

PFN-Dominated Regime

As described above, the beam current used in transverse impedance calculations is modelled as a twin-wire excitation. From the setup of the SIS-100 extraction/emergency kicker (Fig. 1), we see that the kick direction is vertical (y), since the winding creates a magnetic field pointing into the x-direction. Very much like in an ordinary transformer, the magnetic flux created by the two-wire excitation Eq. 5 used in the calculation of Z_y couples to the magnet winding. As described above, the consequent footprint of the PFN can be obtained by including the equivalent lumped impedance of the PFN, $Z_g(\omega)$, into the EM field simulation. At the time of this writing, however, the SIS-100 PFNs are still in the design phase, so that only preliminary impedances $Z_q(\omega)$ are available.

It is therefore of interest to parameterize the transverse kicker impedance in terms of an arbitrary PFN impedance $Z_g(\omega)$. An approach to achieve this has been proposed by Nassibian and Sacherer [16]. Interpreting the beam current and the kicker-magnet coil as the primary and secondary winding of a transformer, respectively, they developed a simple model for the inductive coupling of the beam to the PFN. Following a very similar route, slightly generalizing their argument, we have proposed the following parametization [2]:

$$Z_y(\omega) = a(\omega) - \frac{b(\omega)}{c(\omega) + Z_g(\omega)}.$$
 (12)

The value of Eq. 12 is that, knowing a, b, and c, one is able to specify Z_y in terms of a given, arbitrary PFN impedance Z_g . The only assumptions used in the derivation of Eq. 12 are the linearity of Maxwell's equations and the absence of wave effects in the winding. The parameters $a(\omega), b(\omega)$, and $c(\omega)$ can be obtained from numerical field simulation by computing three frequency sweeps of Z_y for different Z_g 's, e.g. $Z_g \in \{0, \infty, 50 \Omega\}$.

The corresponding data are shown in Fig. 2 for f < 50MHz. In the case $Z_g \equiv 0$ no net magnetic flux is able to reach the ferrite parts through the left (-x) or right side (+x) of the kicker window (after Faraday's law). Hence, the observed losses are comparably small for the shorted termination. For $Z_g = \infty$ (open), we observe a resonance at 5.5MHz, which is due to the interplay between the capacitance and the self-inductance of the magnet winding.

The vertical coupling impedance expected with real PFN is depicted in Fig. 2, bottom. The $Z_g(\omega)$ data used to produce this plot in connection with Eq. 12 has been computed by a SPICE simulation [17]. The oscillations in Z_y are due to resonances in the pulse transmission cables.

Considering higher frequencies, one expects that the inductive coupling to the PFN becomes less pronounced, because the phase factor inherent to the beam excitation current, $\exp(-ikz)$, causes sign changes of the current within the magnet, implying partial cancelation of magnetic flux through the windings. As the magnet has the length 0.9m, a phase change of 2π within the magnet corresponds to



Figure 2: Vertical coupling impedance ($\beta = 1$) for the SIS-100 extraction/emergency kicker in the PFN-dominated frequency regime, for different termination resistences (top: short, 50 Ω , middle: open, bottom: the planned bipolar PFN).

 $f \approx 333$ MHz ($\beta = 1$). Figure 3 supports this reasoning, because the difference between different magnet termina-



Figure 3: Vertical coupling impedance for the same three kicker terminations as in Fig. 2, $\beta = 1$. For frequencies above 250MHz, the PFN coupling is neglected, since curves coincide to a good approximation.

tions becomes small above 250MHz. We therefore neglect the PFN coupling of the extraction/emergency kicker for frequencies above 250MHz.

In contrast to Z_y , the horizontal and longitudinal impedances $(Z_x, Z_{||})$, are largely insensistive to the PFN impedance Z_g ; see [2] for a demonstration of this fact.

Ferrite-Dominated Regime

In the frequency regime above 250MHz, losses due to the motion of Weiss domain boundaries in the ferrite [18] dominate $\text{Re}Z_y$. Since the manufacturer's data sheet (www.ferroxcube.com) for the permeability of the ferrite 8C11 only covers frequencies up to 100MHz, we have continued $\mu(\omega)$ up to the cutoff frequency of the beam pipe (1.325GHz) by the scaling laws given in [18]. Figure 4 shows the corresponding simulation results for Z_y , Z_x and $Z_{||}$.

As for the low-frequency, PFN-dominated regime, one would like to have a physical model for the present highfrequency range. The successful application of the analytical transverse impedance formula proposed by Zotter [19] to the CERN MKE kicker [20], i.e. the agreement with measurements and own simulations motivated us to apply the formula to the SIS-100 extraction/emergency-kicker. However, as can be seen from Fig. 4 (middle), Zotter's prediction of Z_x is in disagreement with the observations from respective simulations. In particular, the resonance peak of Re Z_x , observed at approximately 740MHz, is much sharper than that predicted analytically. It is a matter of our current research to explain this discrepancy.

CONCLUSIONS

We have reported on our ongoing effort to tabulate the coupling impedances of the SIS-100/300 kickers using



Figure 4: View of the simulation results for vertical (top), horizontal (middle), and longitudinal (bottom) coupling impedances for the SIS-100 extraction/emergency kicker, $\beta = 1$.

electromagnetic field simulations. After reporting the computational features of our dedicated impedance software, results for one SIS-100 kicker type have been discussed in detail. Although impedance calculations are still time consuming (e.g. one week on a two-processor, 3GHz machine for the results presented here), computer simulations have proven as a convenient method to support the kicker design.

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