ANALYSIS OF MEASURED TRANSVERSE BEAM ECHOES IN RHIC*

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Abstract

The transverse echo amplitudes observed in RHIC [1, 2] will be analyzed using particle tracking codes together with a kinetic intrabeam scattering model. We discuss the diffusion rates observed in heavy ion beams.

INTRODUCTION

The measurement of beam echoes is related to the determination of diffusion in beams. Longitudinal echoes were used in Ref. [3] to measure numerical diffusion in a 'computer beam' using different particle tracking schemes. In particle beams, diffusion leads to emittance growth and beam loss. In the present work we will concentrate on measurements of diffusion induced by intrabeam scattering (IBS). The IBS rates in RHIC and also in the proposed High Energy Storage Ring (HESR) [4] should be balanced by electron cooling at high energy in order to increase the luminosity. In RHIC typical IBS growth times are of the order of $1\ \mathrm{h}$. For such long time spans, the direct measurement of emittance growth is time consuming, especially under parameter variation, and so is within a direct calculation of the emittance growth. Here, the calculation and the measurement of beam echoes could provide an alternative way to determine diffusion times within a thousand revolutions only. In RHIC, echo measurements with Au^{79+} ions, Cu^{29+} ions, and protons were performed. The goal of this work is to simulate transverse beam echoes using realistic IBS rates within a particle tracking code and to compare the results with the experimental data. We restrict ourselves to the evaluation of the gold experiments.

MODELS

The echo effect

Transverse beam echoes can be generated in proton or heavy-ion storage rings using a one-turn dipole kick followed by a one-turn quadrupole kick. In addition octupoles may be required to provide a amplitude dependent tune shift. Here, we used 3 octupoles. The one-turn dipole kicker may kick the beam at a time $t = 0$ resulting in the betatron oscillation of the whole beam with the initial amplitude $\langle x \rangle_0$ (1st graphic of figure 2). The amplitude depending phase advance due to the octupoles, given by the phase advance μ measured for $x = \sigma$, leads to a detuning (2nd graphic of figure 2). As a consequence, the amplitude of the beam centroid oscillation disappears as shown in figure 3.

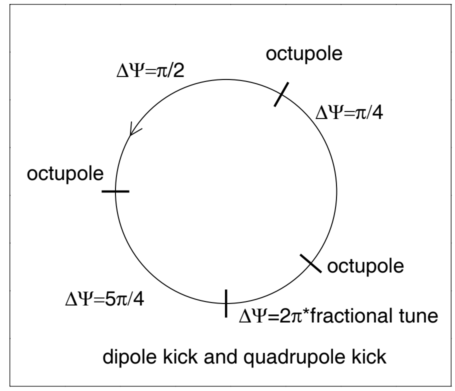

Figure 1: Scheme of RHIC taken from [5] and actually used in the measurements [1, 2] and the calculations for this work. The values of $\Delta\Psi$ denote the fractional part of the betatron phase advance of the focusing sections of RHIC between the kicks.

A time τ after the dipole kick the beam is kicked by a one turn quadrupole kick leading to a slight deformation of the beam in phase space (3rd graphic of figure 2). The particles become accumulated near $t = 2\tau$ (4th graphic of figure 2) resulting in a briefly appearing coherent betatron oscillation with finite amplitude. This is the transverse beam echo having its maximum at $t = 2\tau$, as figure 3 shows. The maximum amplitude of a beam echo decreases due to dif-

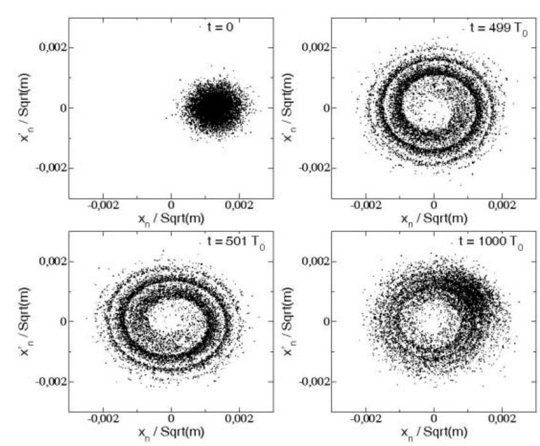

Figure 2: Phase space plots of a Gaussian beam 0, 499, 501, and 1000 turns after the initial dipole kick. The one turn quadrupole kick occurred 500 turns after the dipole kick. $x_n, x_n^{'}$ are normalized coordinates, see equation (2).

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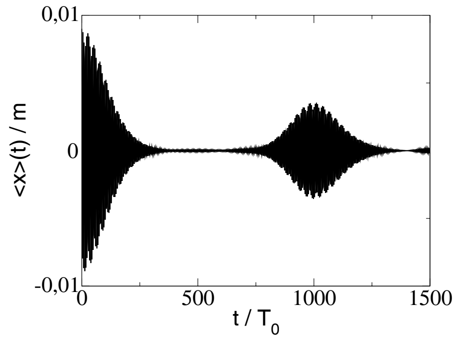

Figure 3: Amplitude of the coherent betatron oscillation as a function of time for condition as in [5]. It is $\langle x\rangle_0 = 9$ mm and $\tau = 500\ T_0$. Note that is a simulation.

fusion processes, which we used to determine diffusion in the beams of RHIC.

The tracking model

We used the simple tracking scheme

$$\begin{pmatrix} x_{n,i+1} \\ x'_{n,i+1} \end{pmatrix} = \begin{pmatrix} \cos\Psi & \sin\Psi \\ -\sin\Psi & \cos\Psi \end{pmatrix} \times \begin{pmatrix} x_{n,i} + \Delta x_n(D_0) \\ x'_{n,i} + \Delta x'_n(x_{n,i}, y_{n,i}, D_0) \end{pmatrix} \tag{1}$$

with the normalized coordinates

$$x_n = \frac{x}{\sqrt{\beta_x}} \quad \text{and} \quad x'_n = \sqrt{\beta_x}x' + \frac{\alpha_x}{\sqrt{\beta_x}}x. \tag{2}$$

In this equation, Ψ is the phase advance of a focusing part of lattice as given in figure 1. Δx_n and $\Delta x'_n$ denote the kicks due to the octupoles, the one turn quadrupole depending on the spatial coordinates x_n, y_n, and the random displacements and forces leading to diffusion depending on the diffusion coefficient D_0. The configuration of the kicks is shown in figure 1. The random kicks for the normalized coordinates x_n, x'_n are given by

$$\Delta x_n = \Delta x'_n = R_{\rm gauss}\sqrt{D_0 T_0}. \tag{3}$$

Here, $R_{\rm gauss}$ are random numbers satisfying a Gaussian distribution function with the variance $\sigma = 1$, T_0 is the revolution time. The diffusion coefficients were used in the form

$$D_0 = \frac{\varepsilon_{\rm rms}}{\tau_{\rm diff}}, \tag{4}$$

where the rms emittance $\varepsilon_{\rm rms}$ and the diffusion time $\tau_{\rm diff}$ were assumed to be constant in time and space, and they are calculated for a Gaussian beam.

Analytic formula

To validate our tracking code and for analytic estimates, we used the analytic formula for the relative echo amplitude from Ref. [6]

mass / charge number A/Z	179/79
relativistic γ	10.5
circumference/m	3834
revolution time T_0/μs	12.8
rms emittance, unnormormalized	
$\varepsilon_{\rm rms}/(\rm mm\ mrad)$	0.16
rms beam size, σ/mm	2.5
initial dipole displacement $\langle x\rangle_0$/mm	10
detuning, μ, for $x = \sigma$	0.0005 ... 0.0022
value most used	0.0014
normalized quadrupole strength q	0.025
quadrupole kick time τ/T_0	450
bunch intensity $N_b/10^9$	0.1 ... 1.0

Table 1: Parameters used in the experiments of [1, 2] with gold and in the calculations.

$$A_{\rm echo,rel} = \frac{\langle x\rangle_{\rm echo,max}}{\langle x\rangle_0} = \frac{q}{\alpha^3}\frac{\tau}{\tau_{\rm dec}} \tag{5}$$

with

$$\alpha = \frac{2}{3}\frac{\tau}{\tau_{\rm diff}}\left(\frac{\tau}{\tau_{\rm dec}}\right)^2 + 1. \tag{6}$$

Here, $q = \beta/f$ is the normalized quadrupole strength of the one turn quadrupole kick with the beta function β at this position and the focal length of the quadrupole, τ is the time between the initial dipole kick and the one turn quadrupole kick, and

$$\tau_{\rm dec} = T_0/(4\pi\mu) \tag{7}$$

[7] is the decoherence time, the time that characterizes the decrease of the coherent betatron oscillation amplitude of the beam centroid after the dipole kick due to the octupole. Here, μ is the detuning due to the octupoles measured at $x = \sigma$, T_0 is the revolution time, and $\tau_{\rm diff}$ is the diffusion time.

Equation (5) for the echo amplitude was derived considering only the one dimensional description of beam motion,

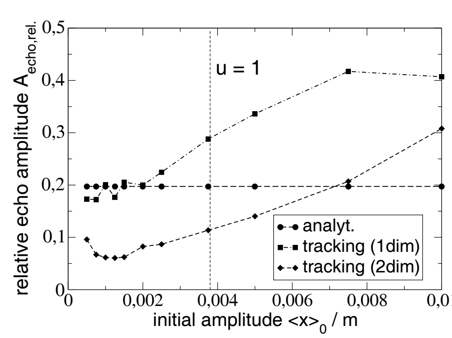

Figure 4: Relative maximum echo amplitude $A_{\rm echo,rel} = \langle x\rangle_{\rm echo}/\langle x\rangle_0$ calculated without diffusion as a function of the initial betatron amplitude $\langle x\rangle_0$. Parameters are those in table 1.

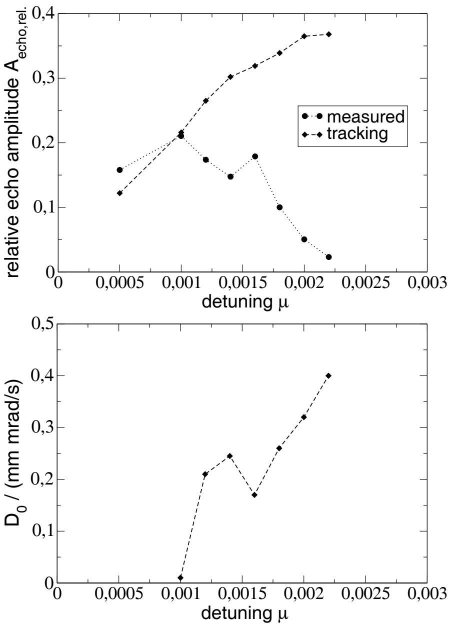

Figure 5: Up: Maximum echo amplitude $\langle x\rangle_{\rm echo}$ from experiment (figure 4 in [1]) and calculated without diffusion as a function of the detuning μ using the tracking code. Down: Diffusion coefficients D_0 needed to reproduce the experimental results in the upper graphic.

i.e. without coupling of vertical and horizontal motion due to the octupoles. Furthermore, the condition

$$u = \frac{\langle x\rangle_0^2}{2\beta_x\varepsilon_{\rm rms}} \ll 1. \tag{8}$$

has been assumed in the derivation. Consequently, the maximum echo amplitudes calculated with equation (5) coincide with those yielded from tracking only under these conditions, as one can see in figure 4. The inclusion of coupling for both perpendicular directions, which is referred to as 2-dim in this figure, leads to a decrease of the maximum echo amplitude.

In our tracking calculations we used up to 50000 particles in the case of small $\langle x\rangle_0$ in order to obtain results with low fluctuations.

RESULTS

To obtain diffusion times and coefficients from the experimental data, we evaluated two sets of data from [1] using the parameters given in table 1.

First, we considered the relative echo amplitudes measured for different detuning values. In a first step, we compared them to echo amplitudes calculated without diffusion using our tracking code, shown in the upper graphic of figure 5. As one can see, the so calculated echo amplitudes are larger than the measured except for very small detuning values. Therefore it was possible to adjust diffusion coefficients to bring the calculated echo amplitudes in coincidence with the measured ones. Except for small μ, we found diffusion coefficients within $D_0 \sim (0.1 \;...\; 1)$ mm mrad/s corresponding to diffusion times $\tau_{\rm diff} \sim (1.6 \;...\; 0.16)$ s. The diffusion coefficients found seem to depend systematically on the detuning. This is unexpected, because the beam parameters were the same for all points in figure 5.

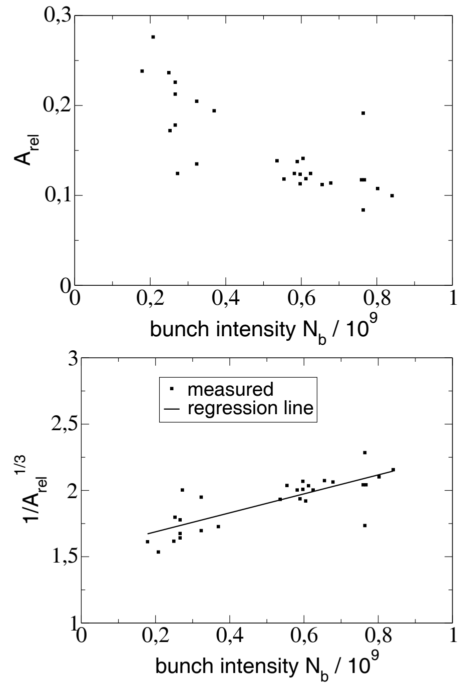

Figure 6: Up: Measured relative maximum echo amplitude $A_{\rm rel,echo} = \langle x\rangle_{\rm echo}/\langle x\rangle_0$ depending on the bunch intensity N_b (from figure 6 in [1]). Down: $A_{\rm echo,rel}^{-1/3}$ according to experimental data points and straight regression line $A_{\rm echo,rel}^{-1/3} = a + bN_b$.

As a second example, we evaluated the echo amplitudes measured as a function of the bunch intensity shown in the upper graphic of figure 6. Here, we applied

$$\begin{aligned} A_{\rm echo,rel}^{-\frac{1}{3}} &= \left(\frac{\tau_{\rm dec}}{q\tau}\right)^{\frac{1}{3}}\left[1+\frac{2}{3}\frac{\tau}{\tau_{\rm diff}}\left(\frac{\tau}{\tau_{\rm dec}}\right)^2\right] \\ &= a + bN_b \end{aligned} \tag{9}$$

yielded from equations (5) and (6), and assuming $\tau_{\rm diff} \propto N_b^{-1}$. Now, a and b may be determined by linear regression. In doing so, we got $\tau_{\rm dec} = 41\ T_0$, which is near $\tau_{\rm dec} = 57\ T_0$ yielded from equation (7). This results in $\tau_{\rm diff} = 5.5\cdot 10^{13}\ T_0/N_b = 7.0\ \cdot 10^8\ {\rm s}/N_b$. Hence, it is $\tau_{\rm diff} = 7$ s for $N_b = 10^8$.

In general, we found, in agreement with [1], diffusion coefficients corresponding to diffusion times $\tau_{\rm diff} \sim (0.1 \;...\; 10)$ s from the evaluation of the experimental data. Such diffusion times are unrealistic. Applying the IBS model of Bjorken-Mtingwa being a part of MAD-X, we found diffusion times $\tau_{\rm diff} = (2500 \;...\; 25000)\ {\rm s} = (2\,\cdot$

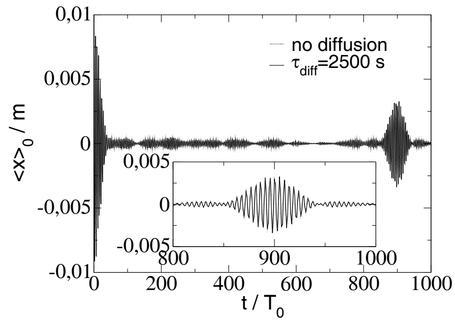

Figure 7: Coherent betatron oscillation as a function of time calculated with parameters given in table 1 without diffusion and with a diffusion coefficient corresponding to a diffusion time $\tau_{\rm diff} = 2500$ s calculated for $N_b = 10^9$.

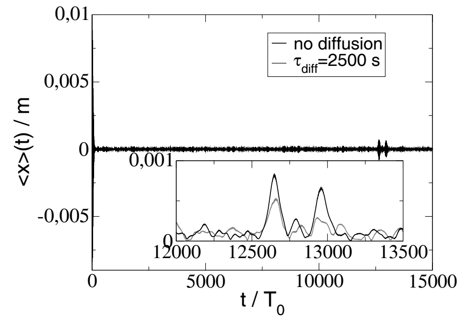

Figure 8: The same as in figure 7 but with an extended time $\tau = 6400\ T_0$. The curve in the small window is the envelope of the amplitude of the betatron oscillation.

10^8 ... $2 \cdot 10^9$) T_0 for a Gaussian beam having a rms bunch length $l_b = 1$ m, a momentum spread $\Delta p/p = 10^{-3}$, and other parameters according to table 1. To investigate the influence of such small diffusion rates, we performed a further tracking calculation with a diffusion coefficient $D_0 = 1.3 \cdot 10^{-10}$ mm mrad/s corresponding to $\tau = 2500$ s. The results are shown in figure 7. As one can see, the differences are too small to see them in this figure and, in fact, the curves are almost identical. This agrees with equation (5), where the relative reduction of the echo amplitude due to diffusion is given by $1/\alpha^3$. For $\tau_{\rm diff} = 2 \cdot 10^8\ T_0$, $\tau = 450\ T_0$, and $\tau_{\rm dec} = 57\ T_0$, one finds $\alpha = 1.0000935$ and, thus, it is $1/\alpha^3 = 0.99972$. To reach a significant amplitude reduction, let us say 50 %, the condition $\tau^3 \approx 0.4\ \tau_{\rm dec}\tau_{\rm diff}$ must be satisfied. This might be reached by increasing the quadrupole kick time to $\tau \approx 6400\ T_0$. The resulting beam echoes, shown in figure 8, at least appear at the correct time $t = 2\tau = 12800\ T_0$, and the amplitude is reduced even by about 50 %. This shows a good coincidence of our tracking code to the analytic formula (5) in respect to the reduction of the echo amplitude due to diffusion. Nevertheless, it remains, that in [1, 2] echoes were experimentally found only for $400\ T_0 < \tau < 550\ T_0$, and that the experimental data can be explained only with very large diffusion rates.

CONCLUSIONS

Transverse beam echoes were calculated in order to evaluate echo amplitudes measured for Au^{79+} beams in RHIC [1, 2]. The goal was to estimate the diffusion times in the beams. For this purpose we used a simple particle tracking code and an analytic formula taken from [6]. ¿From both we calculated echo amplitudes as a function of the detuning due to the octupoles and as a function of the bunch intensity. We found that diffusion times $\tau < 10$ s are necessary to explain the measured echo amplitudes in agreement with [1]. In contrast, calculations applying the IBS model of Bjorken and Mtingwa yielded diffusion times $\tau_{\rm diff} \sim (2500\ ...\ 25000)$ s for $N_b = 10^8$ ... 10^9. The influence of such small diffusion rates cannot be detected with quadrupole kick delay times $\tau = 450\ T_0$ used in the experiments. Instead a quadrupole kick delay of some thousand revolution times would be needed. The diffusion rates found to be necessary to explain the measured echo amplitudes are 2 orders of magnitude larger than those obtained from the IBS model. This could be a hint for other intensity dependent mechanisms besides IBS, that reduce echo amplitudes. Furthermore, with our tracking method we found an increase in the diffusion coefficient needed to reproduce measured echo amplitudes, when the detuning is enhanced. A possible reason for this could be a significant enhancement of transverse diffusion in the measurement when nonlinearities are increased. The IBS diffusion coefficients used were calculated using strong approximations and simplifications, e.g. assuming a constant coefficient and ignoring the betatron amplitude dependence of the IBS diffusion. The influence of these approximations and simplifications should be further investigated.

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