EIGENMODE EXPANSION METHOD IN THE INDIRECT CALCULATION OF WAKE POTENTIAL IN 3D STRUCTURES*

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Abstract

The eigenmode expansion method was used in the early 1980's in calculating wake potential for 2D rotational symmetric structures. In this paper, it is extended to general 3D cases. The idea is to calculate the indirect wake contributed by the infinitely long outgoing pipe analytically based on the field dependence of $e^{jk_z z}$. Numerical results show the accuracy and efficiency of the method.

INTRODUCTION

The determination of wake potentials and impedances is important in the design of accelerators. It is difficult to calculate the wake potential directly by an integration of electric field along a very long outgoing beam pipe, especially, when the bunch length is short. An indirect integration method was proposed for 2D rotational symmetric structures, in which the integral path was shifted to the boundary of beam pipe [1, 2]. Later this idea was generalized [3]. For 3D cavity-like structures (provided that the minimum aperture of the structure is larger than the aperture of the beam pipe), the wake potentials at beam positions could be found by solving a Poisson's equation with known boundary wake potentials [4]. This method has been implemented in the code MAFIA. Another approach was given in [5] for 3D structures ending with round beam pipe.

There was no general method available for arbitrary 3D structures when we submitted the abstract of this paper. It was very recently that we found a submitted paper mentioning also the eigenmode expansion method [6]. The ideas of our work and ref. [6] (see section IV. D) are the same, despite slight differences in the implementation. Meanwhile, we noted a further generalization of the indirect method into 3D presented at EPAC'06 [7]. In this approach, a TM-/TEM- potential is defined in the transverse plane such that the indirect wake is the value of the potential at bunch position. This potential can be extracted by solving the Poisson's equation in a transverse plane of the structure.

In this paper, we describe the eigenmode expansion method in calculating the longitudinal wake potential for general 3D structures. The transverse one could be obtained by applying the Panofsky-Wenzel theorem [8]. The wake potential is computed as a sum of two parts, direct and indirect ones. The direct wake potential is obtained by a full wave discrete solution, which stops after covering all the structure discontinuities. The indirect wake potential is then calculated analytically through the eigenmode expansion of the field values recorded at the truncation aperture. Numerical results are compared to the generalized 3D method given in [7].

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EIGENMODE EXPANSION METHOD

We use the Finite Integration Technique (FIT) [9-11] with moving mesh window [12] to calculate the direct wake potential. As shown in Fig. 1, the full wave simulation stops after the bunch passing all the discontinuity. The E_z fields are recorded at the truncation boundary ($z=0$) as a function of time.

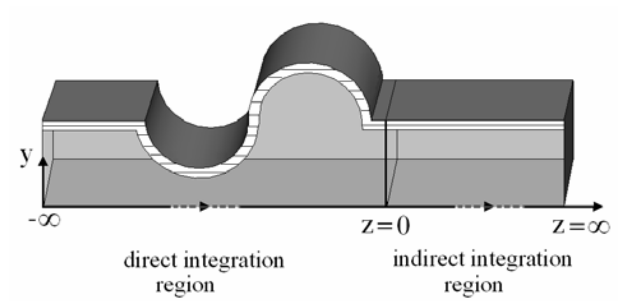

Figure 1: Sketch of the wake field integration regions.

The indirect wake potential is given by

$$W_{//,in}(s) = -\frac{1}{Q}\int_0^\infty E_z\left(x,y,z,t=(z+s)/c\right)dz \qquad (1)$$

It is known that the TM fields inside the waveguide satisfy [8, 2]

$$\left(\nabla_\perp^2 + \omega^2\mu\varepsilon - k_z^2\right)E_z = 0 \qquad (2)$$

By solving the eigenvalue problem

$$\left(\nabla_\perp^2 + k_\perp^2\right)E_z = 0 \qquad (3)$$

the electric field can be written as a combination of discrete eigenmodes

$$E_z(x,y) = \sum_n c_n E_n(x,y) \qquad (4)$$

where $E_n(x,y)$ and c_n are the eigenvector and eigenmode coefficient, respectively. As obtained from the FIT solution the electric field E_z at $z=0$ plane as a function of time, we have

$$E_z(t,x,y,z=0) = \sum_n c_n(t)E_n(x,y) \qquad (5)$$

where

$$c_n(t) = \frac{1}{|A_n|}\int E_z(x,y,z=0,t)E_n(x,y)dxdy \qquad (6)$$

is the time domain eigenmode expansion coefficient. The electric field E_z at a propagation distance z is given by

$$E_z(\omega,x,y,z) = \sum_n C_n(\omega)E_n(x,y)e^{jk_{z,n}(\omega)z} \qquad (7)$$

In (7), E_z is written in frequency domain since the propagation constant k_z is frequency dependent. Therefore, the time domain eigenmode coefficient $c_n(t)$ must be transferred into frequency domain to get $C_n(\omega)$. The propagation constant is given by

$$k_{z,n}(\omega) = \sqrt{(\omega/c)^2 - k_{\perp,n}^2} \qquad (8)$$

where $k_{\perp,n}$ is the eigenvalue obtained from the solution of (3). For decaying modes when $(\omega/c)^2 < k_{\perp,n}^2$, $k_{z,n}$ is imaginary and must be taken as

$$k_{z,n}(\omega) = j\sqrt{k_{\perp,n}^2 - (\omega/c)^2} \qquad (9)$$

Transferring (7) back into time domain and integrating E_z along a path to infinity leads to

$$\begin{aligned}
&\int_0^\infty E_z(x,y,z,t=(z+s)/c)dz \\
&= \int_0^\infty \left[\int_{-\infty}^{\infty}\sum_n C_n(\omega)E_n(x,y)e^{jk_{z,n}(\omega)z}e^{-j\omega\frac{z+s}{c}}d\omega\right]dz \\
&= \sum_n E_n(x,y)\int_{-\infty}^{\infty} C_n(\omega)\left[\int_0^\infty e^{j\left(k_{z,n}(\omega)-\omega/c\right)}dz\right]e^{-j(\omega/c)s}d\omega \\
&= \sum_n E_n(x,y)\int_{-\infty}^{\infty} C_n(\omega)\frac{1}{j\left(\omega/c - k_{z,n}(\omega)\right)}e^{-j(\omega/c)s}d\omega \\
&= \sum_n E_n(x,y)W_n(s)
\end{aligned} \qquad (10)$$

where

$$W_n(s) = \int_{-\infty}^{\infty} C_n(\omega)\frac{1}{j\left(\omega/c - k_{z,n}(\omega)\right)}e^{-j(\omega/c)s}d\omega \qquad (11)$$

is calculated by an inverse Fourier transform (IFFT). It is worth noting that decaying modes contribute to the indirect wake potential. However, as higher order modes decay faster while propagating in the waveguide, their contribution becomes smaller if the full wave solution is extended into the outgoing pipe for a certain distance (one or two mesh window size). In this way, the number of modes to be considered is greatly reduced. Finally, the wake potential is calculated by

$$W_{//}(s) = W_{//,d}(s) - \frac{1}{Q}\sum_n E_n(x,y)W_n(s) \qquad (12)$$

where $W_{//,d}(s)$ is the direct wake potential obtained from the FIT solution.

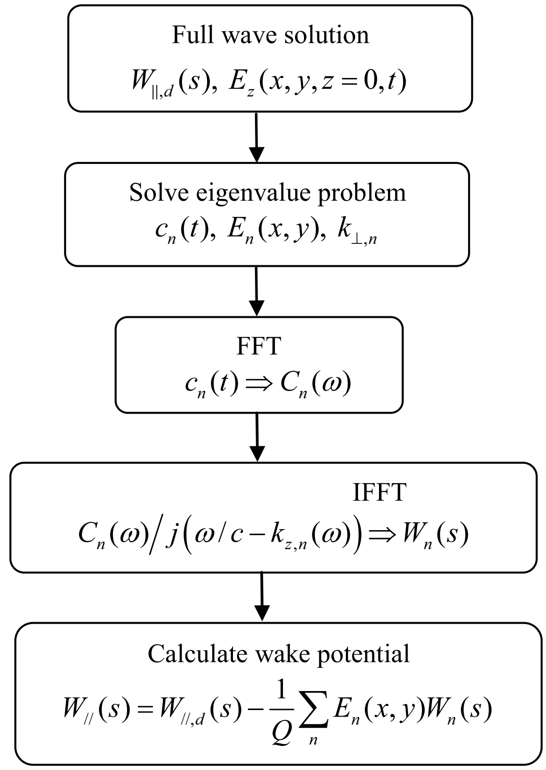

Figure 2: Flow chart of the procedures.

Fig. 2 shows a flow chart of the eigenmode expansion method. In the second step, the eigenvalue problem could be solved either numerically or analytically with known waveguide modes of the beam pipe. The latter one is more computationally efficient but has limited applications. Furthermore, an analytical solution is not consistent with the discrete FIT systems and thus actually less accurate than a numerical solution in some sense. Special attention has to be paid in the third step in calculating the frequency domain eigenmode coefficient $C_n(\omega)$. As we use moving mesh solution for the direct wake fields, $c_n(t)$ is available only within the mesh window. In this case, direct Fourier transform of $c_n(t)$ suffers from serious Gibbs effect. To avoid this problem, appropriate extrapolation of $c_n(t)$ is necessary. We tried two possible extrapolations, the first is a linear extension with Gaussian window, given by

$$c_n(t_w + m\Delta t) = \left[k_{tw}\left(1 - \frac{m}{M_{ext}}\right)\cdot\Delta t + c_{n,m-1}\right]\exp^{-\left(\frac{m\Delta t}{\tau}\right)^2},$$
$$m = 1 \ldots M_{ext} \qquad (13)$$

where k_{tw} is the slope of the coefficient signal at the last point of the time window, M_{ext} is the number of extension points. The second extrapolation is done by using the autoregressive (AR) analysis [13]. Both of the two schemes work well. Examples are shown in the next section to describe these extrapolations.

NUMERICAL RESULTS

We show some numerical examples to test the efficiency of the eigenmode expansion method.

First, we consider a circular collimator since its wake potential can be found by a 2D simulation with Napoly's integration. The structure and parameters are given in Fig. 3. The length of bunch is 0.2 mm. In Fig. 4, 3D results from the eigenmode expansion method are compared to both 2D result from Napoly's integration and 3D results from Henke's method. As can be seen in Fig. 4, the results agree very well with each other. The size of mesh window is 40 grid points and the number of FFT points used is 128. Fig. 5 shows the coefficients $c_n(t)$ for the two extrapolations of the first three modes. In some cases, the extrapolation by autoregressive analysis requires more extension points since the signal might attenuate slowly.

As a second example, we consider a 3D rectangular collimator as shown in Fig. 6. The structure is taken from the ILC-ESA beam test program [14] (see collimator #4). Fig. 7 shows the wake potential of the collimator, where the bunch length is set to be 1 mm. The result agrees well with that from Henke's method while only the first three modes *i.e.* TM_{11}, TM_{13} and TM_{15} are taken into account.

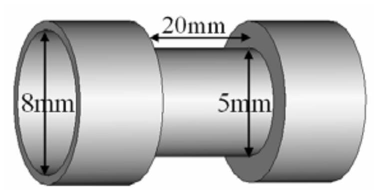

Figure 3: Structure of the circular collimator.

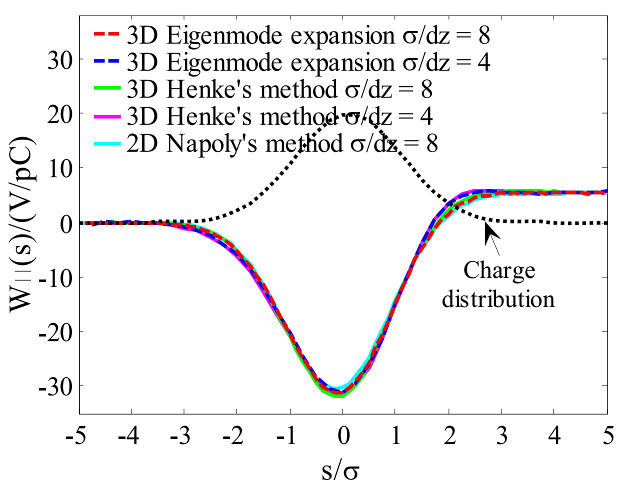

Figure 4: Wake potential of the circular collimator as shown in Fig. 3.

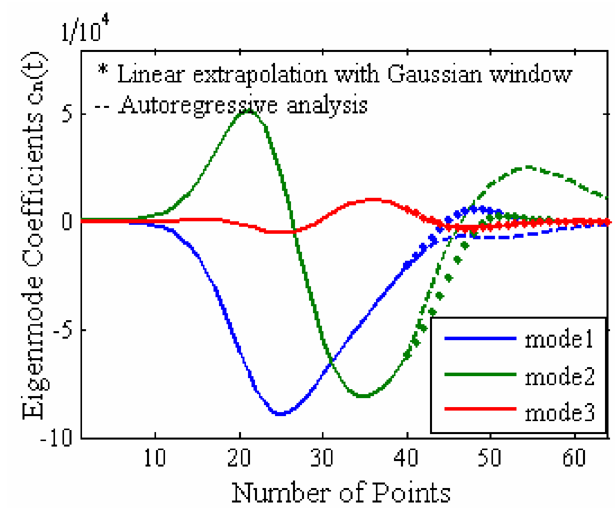

Figure 5: Eigenmode coefficients and extrapolations.

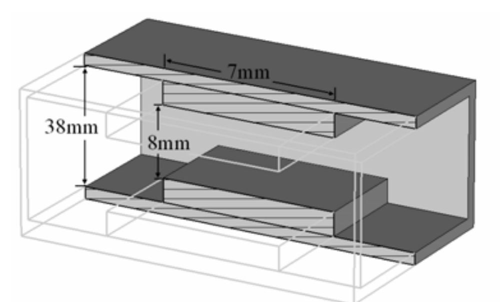

Figure 6: ILC-ESA beam test collimator #4.

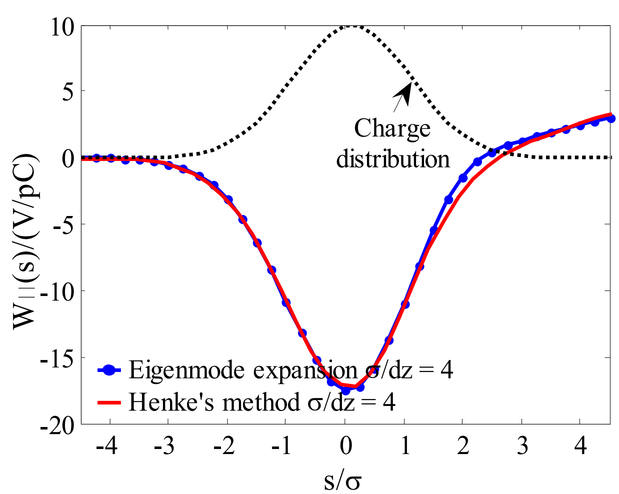

Figure 7: Wake potential of the rectangular collimator as shown in Fig. 6.

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