MODELING HIGH-CURRENT INSTABILITIES IN PARTICLE ACCELERATORS *

M. Migliorati, A. Schiavi, Dip. Energetica, Università di Roma "La Sapienza", Italy
G. Dattoli, ENEA, UTS - Tecnologie Fisiche Avanzate, Frascati, Rome, Italy

Abstract

Methods exploiting integration techniques of Lie algebraic nature have been successfully employed in the past to develop charged beam transport codes, for different types of accelerators. These methods have been so far applied to the transverse motion dynamics, while the longitudinal part has been treated using standard tracking codes.

In this contribution we extend the symplectic technique to the analysis of longitudinal and coupled longitudinal and transverse motion in charged beam transport with the inclusion of the non linear dynamics due to the wake field effects. We use this method to model different types of instabilities due to high current.

We consider in particular the case of coherent synchrotron radiation instabilities and their implication in the design and performances of high current accelerators. We discuss either single pass and recirculated devices. As to this last case, we also include the effect due to quantum noise and damping.

INTRODUCTION

Integration techniques based on Lie algebraic methods are commonly adopted to describe charged particle beam transport [1, 2]. These methods are mainly used to treat transverse beam dynamics, without the inclusion of collective effects, responsible of non linearities. Transport non linear problems are usually studied using tracking methods and indeed several tracking codes exist that treat longitudinal and transverse beam motion with the inclusion of wake fields (see, e. g., ref. [3]).

Even though these codes allow a clear understanding of the physics characterizing the instabilities, there are difficulties to distinguish between threshold effects due to non linearities and those induced by the numerical noise caused by the macro particle model.

The extension of the Lie algebraic methods to the non linear case is ideally suited to treat wake fields - beam interaction. The advantage with respect to tracking codes is a very smooth evolution of the beam distribution function that allows to reduce, and in some cases to completely eliminate, the effect of numerical noise. Furthermore it preserves the symplecticity of the problem in a natural way, without particle losses due to numerical artifacts.

The technique, that uses the exponential operators commonly adopted in quantum mechanics, is, to some extent, analytical and flexible enough to allow the introduction of several effects occurring to the beam in transport elements and caused by different kind of wake fields. We have exploited this flexibility to study several problems related to the interaction of the beam with wake fields. In the next section we deal with the study of the longitudinal beam dynamics in recirculated systems, and of the longitudinal motion coupled to the transverse one through the dispersion function in a magnetic compressor. Both cases take into account non linear effects due to the coherent synchrotron radiation (CSR). We then show how the method can be extended to beam dynamics in storage rings by including quantum noise and the damping mechanism due to the synchrotron radiation.

* This work has been partially supported by the EU Commission in the sixth framework program, contract no. 011935 EUROFEL-DS2.

VLASOV EQUATION

The beam transport is modelled by a Vlasov equation

$$\frac{\partial}{\partial s}\rho = H\rho \qquad \text{with} \qquad \rho|_{s=0} = \rho_0 \qquad (1)$$

describing the longitudinal evolution along the coordinate s of a charged beam density distribution ρ.

The operator H contains the physical properties of the problem. It may be specified by simple differential operators, when describing the bunch evolution through magnetic systems [4], or by integral operators, when accounting for non linear problems associated with the effects of wake fields on the beam.

Eq. (1) is an initial value problem whose formal solution, for s-independent H, can be written as

$$\rho = \exp(Hs)\rho_0 \ , \qquad (2)$$

where ρ_0 is the beam distribution at $s = 0$.

In the following of this section we discuss the solution of eq. (1) for a beam passing through a magnetic system and undergoing CSR effects.

Recirculated beam dynamics without damping

To study the longitudinal beam dynamics in a magnetic system under the effect of short range wake fields, the operator H can be cast in the form

$$H = \eta\varepsilon\frac{\partial}{\partial z} + kW(z;s)\frac{\partial}{\partial \varepsilon} \ , \qquad (3)$$

with η the slip factor, ε the relative energy variation from the nominal value E_0, z the longitudinal coordinate, $k = Ne^2/E_0$, N the number of particles in the bunch, e the electron charge, and $W(z;s)$ the longitudinal wake potential per unit length of the path. The wake potential is a convolution of the wake field times the bunch distribution

$$W(z;s) = \int\int W_{||}(z-z')\rho(z',\varepsilon';s)dz'd\varepsilon' \ . \quad (4)$$

Here we neglect diffusion effects due to quantum fluctuation and radiation damping. This hypothesis is valid whenever we treat phenomena with characteristic times short compared to the synchrotron damping time. Under this assumption, the solution of the Vlasov equation after a longitudinal step δs reads (note that with respect to the ordinary Vlasov equation, the propagation coordinate s plays the role of time)

$$\rho(z,\varepsilon;\delta s) = \exp\left(\eta\delta s\varepsilon\frac{\partial}{\partial z} + k\Omega(z;\delta s)\frac{\partial}{\partial \varepsilon}\right)\rho_0 \ , \quad (5)$$

with

$$\Omega(z;\delta s) = \int_0^{\delta s} W(z;s')ds' \ . \quad (6)$$

In writing the above solution, we are neglecting any contribution due to time ordering corrections, that arises whenever the operator H is explicitly time-dependent and does not commute with itself at different times. With these assumptions we neglect third order terms in δs when evaluating eq. (5) [5].

The exponential operator in eq. (5) consists of two non commuting parts. We decouple it using the operator splitting technique, i. e. we approximate the exponential operator for small step intervals δs as

$$\exp(A+B) = \exp\left(\frac{1}{2}A\right)\exp(B)\exp\left(\frac{1}{2}A\right) \ . \quad (7)$$

It is therefore possible to calculate the action of the previous operator on the initial distribution function by using the relation

$$\exp\left(\alpha\frac{\partial}{\partial z}\right)\exp\left(\beta\frac{\partial}{\partial \varepsilon}\right)f(z,\varepsilon) = f(z+\alpha,\varepsilon+\beta) \ . \quad (8)$$

The distribution function $\rho(z,\varepsilon;\delta s)$ at the position δs can then be expressed in terms of the distribution function $\rho(z,\varepsilon;0)$ at $s=0$ thus giving

$$\rho(z,\varepsilon;\delta s) = \rho\left(z+\eta\varepsilon\delta s + \frac{1}{2}\eta\delta s k\Omega(z+\frac{1}{2}\eta\varepsilon\delta s,\delta s),\right.$$
$$\left.\varepsilon + k\Omega(z+\frac{1}{2}\eta\varepsilon\delta s,\delta s);0\right) \ . \quad (9)$$

As an example, we have numerically solved eq. (9) with the simulation code TEO (Transport by Exponential Operators), by using the CSR wake fields of an ultra relativistic particle in a long magnet and in the steady state regime, that is

$$W_{||}(z) = \frac{1}{4\pi\varepsilon_0}\frac{2}{R^{2/3}}\frac{1}{(3z)^{4/3}} \ , \quad (10)$$

where ε_0 is the dielectric constant of vacuum, and R the curvature radius of the bending magnet.

The initial distribution is sampled on a uniform Cartesian grid, and the simulation domain in phase space is monitored at runtime and dynamically expanded in order to follow the distortions induced by the beam dynamics. Throughout the simulation, the resolution of the mesh is kept constant in order to avoid spurious numerical noise due to remapping. The simulations are carried out with an adaptive algorithm for the step size which controls the accuracy of the computed solution.

A comparison with a tracking code [6] is shown in Fig. 1, where we present the longitudinal distribution function $f(z) = \int\rho(z,\varepsilon)d\varepsilon$ obtained in the stable regime with the tracking code and with TEO. In this last case, the smoothness of the distribution function is evident.

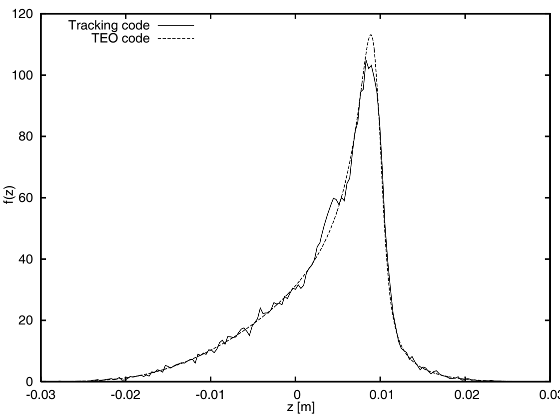

Figure 1: Normalized longitudinal distribution function in stable regime.

In order to investigate the effect of the CSR on the beam dynamics for a Gaussian bunch under different conditions, we introduce the new phase space variables z' and ε' corresponding to z and ε normalized to the initial rms bunch length σ_{z0} and rms energy spread $\sigma_{\varepsilon 0}$ respectively. By inserting the above quantities in the Vlasov equation, we obtain

$$\frac{\partial\rho'}{\partial s'} - \frac{N'}{N}\varepsilon'\frac{\partial\rho'}{\partial z'} - F(z';s')\frac{\partial\rho'}{\partial\varepsilon'} = 0 \ , \quad (11)$$

with

$$s' = \frac{2Ne^2}{3^{1/3}4\pi\varepsilon_0 E_0 R^{2/3}\sigma_{\varepsilon 0}\sigma_{z0}^{4/3}}s \ ,$$
$$N' = \frac{3^{1/3}4\pi\varepsilon_0\eta E_0 R^{2/3}\sigma_{\varepsilon 0}^2\sigma_{z0}^{1/3}}{2e^2} \ , \quad (12)$$
$$F(z';s') = \int_{-\infty}^{z'}\frac{1}{(z'-x)^{1/3}}\frac{\partial f'(x;s')}{\partial x}dx \ .$$

An important feature of the above equation is that $F(z';\varepsilon')$ is a dimensionless form factor independent on the machine parameters. For an initial Gaussian bunch distribution it is also independent on bunch dimensions and, as a consequence, the evolution of the distribution function depends only on the two parameters s' and N'/N. For two different Gaussian initial distribution functions, the beam dynamics of the normalized ρ' is exactly the same for the same values of s' and N'/N.

The above observations simplify the parametric study of the longitudinal evolution of a Gaussian bunch influenced by the CSR wake field. The variable s' is related to the distance the bunch has to travel in order to allow the microbunching effect to grow, and the variable N'/N is related to the intensity of this effect.

Under these conditions, we performed simulations with an initial Gaussian distribution function, with different values of N'/N, and for a distance s' long enough to develop the microbunching effect when above threshold. The initial distribution function was perturbed with white noise such that the obtained threshold corresponded to the strongest unstable modes that could be excited. The simulations show that above threshold there is not a single unstable mode, but a range of wavelengths of the order of a fraction of σ_{z0} that are unstable and change as the distribution function propagates in the machine.

Beam dynamics in a magnetic bunch compressor

By using the same algorithm with small modification in the Vlasov equation, we have simulated the evolution of the distribution function in a magnetic compressor composed by dipoles and drifts elements as shown in Fig. 2.

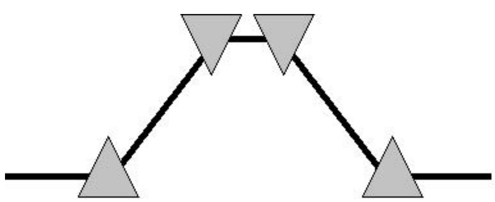

Figure 2: Layout of magnetic compressor.

The Vlasov equation in a drift does not change the longitudinal distribution function but only the energy when the wake field is active. As a consequence, the evolution of the distribution function can be simply obtained from eq. (9) by imposing $\eta = 0$.

Concerning the behavior of the beam in a bending magnet, the Vlasov equation remains that of eq. (5) with the exception that the slip factor depends on the s coordinate $\eta(s) = D(s)/R(s)$, where D is the dispersion function and R the curvature radius.

The modular nature of the exponential operator method allows to generate a beam line of dipoles and drifts along which the distribution function evolves. It is therefore possible to simulate the longitudinal dynamics inside a magnetic bunch compressor. Also in this case the symplecticity of the solution is preserved and the advantages with respect to tracking codes are even more evident.

As an example, we have simulated the beam evolution through the SPARXINO magnetic compressor [7] by using an initial distribution function obtained with the multi particle tracking code PARMELA [8]. In Fig. 3 we show the longitudinal distribution function at the exit of a magnetic compressor obtained with Elegant [3]. Even in absence of

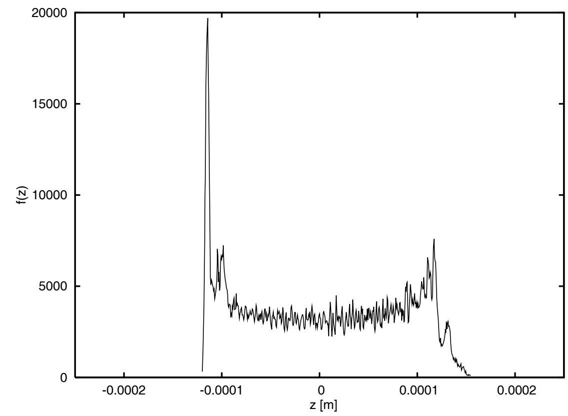

Figure 3: Distribution function at the exit of the magnetic compressor obtained with ELEGANT.

CSR effects, the distribution has a high level of noise due to the macro particle model.

A very smooth distribution function, that well agrees with the previous result, is shown in Fig. 4 and has been obtained with the TEO code by filtering the initial macro particle distribution function with a Savitzky-Golay smoothing filter [9]. Studies of the microbunching effects in the

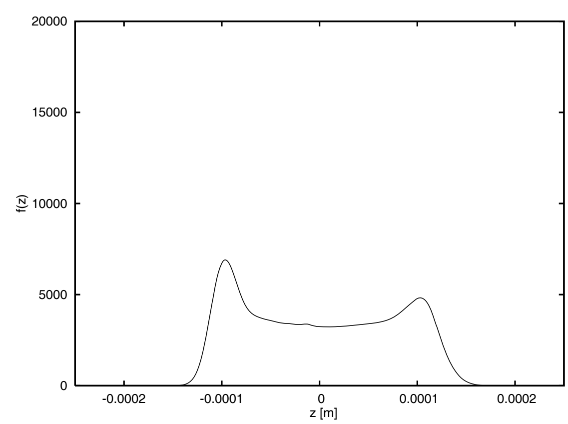

Figure 4: Distribution function at the exit of the magnetic compressor obtained with TEO.

magnetic compressor induced by CSR are in progress.

FOKKER PLANCK EQUATION

In order to study more deeply the longitudinal beam dynamics in a storage ring in presence of short range wake field, we have to include in the equations the effect of quantum fluctuation, radiation damping and RF cavity [10]. The differential equation governing the distribution function is known as Fokker - Planck equation, and, by applying the same operator splitting technique given in eq. (7), we obtain

$$\rho(z',\varepsilon';\delta s') = \exp(\delta s')\exp\left(\frac{\delta s}{2}H\right)$$
$$\exp\left[\delta s'\left(\varepsilon'\frac{\partial}{\partial\varepsilon'}+\frac{\partial^2}{\partial\varepsilon'^2}\right)\right]\exp\left(\frac{\delta s}{2}H\right)\rho_0\ , \quad (13)$$

with

$$\delta s' = \frac{2\delta s}{c\tau_d}\ , \qquad H = f_1(z')\frac{\partial}{\partial\varepsilon'}+\frac{\eta\sigma_{\varepsilon 0}}{\sigma_{z0}}\varepsilon'\frac{\partial}{\partial z'}\ , \quad (14)$$

where τ_d is the radiation damping time, and $f_1(z')$ a function containing the wake potential defined in eq. (4) and the RF voltage.

The structure of eq. (13) is similar to eq. (5). The exponential operators act sequentially on the initial distribution function ρ_0 and the only difference with respect to the analysis of the previous section is the presence of a new exponential operator containing second order derivatives. By applying the rules of ref. [11], we have

$$\exp\left[\delta s'\left(\varepsilon'\frac{\partial}{\partial\varepsilon'}+\frac{\partial^2}{\partial\varepsilon'^2}\right)\right]\rho(z',\varepsilon';0) =$$
$$\frac{\exp(-\delta s')}{2\sqrt{\pi}\left[1-\exp(-\delta s')\right]}\int_{-\infty}^{\infty}\exp\left[\frac{-(\varepsilon'-x)^2}{4\left[1-\exp(-\delta s')\right]}\right]$$
$$\rho(z',\exp(-\delta s')\,x;0)dx\ . \quad (15)$$

The numerical implementation of the solution of the Fokker Planck equation in presence of wake fields is then very similar to what we have already done with TEO. We have to recursively apply the operators at every longitudinal step δs to study the evolution of the longitudinal distribution function. We expect also in this case a very reduced level of noise with respect to tracking codes, and the preservation of the symplecticity.

This characteristics make the application of this technique interesting for the study of the threshold between the potential well distortion and the microwave instability regimes. This aspect of the problem is currently under study.

CONCLUSIONS

We have developed a technique that uses the exponential operators, commonly adopted in quantum mechanics, to simulate the beam dynamics in presence of the non linear contribution due to the wake fields.

The numerical application of the technique has first regarded the solution of the Vlasov equation in a storage ring ignoring quantum fluctuation and radiation damping. The effect of the CSR has been included and the results are in total agreement with a multi particle tracking code. The comparison shows that the method is well suited for the study of the instability due to the very low numerical noise levels it introduces with respect to the tracking code. This feature has allowed to study the parametric dependence of the microbunching threshold produced by CSR for an initial Gaussian bunch perturbed with white noise during the propagation in a storage ring.

By slightly modifying the equations but substantially with the same structure of the algorithm, we have simulated the beam dynamics in a bunch compressor, and the first results, compared with multi particle tracking codes, showed very good agreement and, again, a very low level of numerical noise even in the case of an initial distribution derived from a macro particle distribution.

Finally we have shown how the method can be extended to the case of Fokker - Planck equation, which implies second order derivatives of the distribution function, by changing the evolution operator. This last application allows to study the longitudinal beam dynamics in a storage ring under the effect of the potential well distortion (low current regime) or microwave instability (high current regime) produced by the short range wake fields. The threshold between these two regimes can be studied as well.

ACKNOWLEDGMENTS

The Authors express their sincere appreciation to Prof. L. Palumbo for his help and encouragement.

REFERENCES

[1] D. R. Douglas, A. J. Dragt, "Charged Particle Beam Transport Using Lie Algebraic Methods", 9th Particle Accelerator Conference, Washington, DC, USA , Mar 1981, p. 2522.

[2] H. Grote, F. C. Iselin, "The MAD program (methodical accelerator design) : version 8.10 ; user's reference manual", CERN/SL/90-13 (AP), CERN, 1993.

[3] M. Borland, "ELEGANT" LS-287, ANL, Argonne, IL 60439, USA.

[4] A. J. Dragt, *AIP Conf. Proc.*, **87**, pp. 147-313 (1982).

[5] G. Dattoli, et al., "Study of coherent synchrotron radiation effects by means of a new simulation code base on the non-linear extension of the operator splitting method", submitted to publication.

[6] G. Dattoli, et al., "Electron beam properties and impedance characterization for storage rings used for free electron lasers", NIM A 471, (2001), pp. 403411.

[7] M. Boscolo, et al., "First simulation results on laser pulse jitter and microbunching instability at SPARXINO", Workshop on The Physics and Applications of High Brightness Electron Beams, Erice (Trapani), Italy 9-14 October, 2005.

[8] J. Billen, "PARMELA", LA-UR-96-1835, 1996.

[9] W. H. Press, et al., "Numerical Recipes in Fortran", Cambridge University Press.

[10] M. Migliorati, "Single bunch behavior and microwave instability" Il Nuovo Cimento A 112, N. 5 (1999), pp. 485-490.

[11] G. Dattoli, L. Mezi, "The storage-ring FOKKER-PLANCK equation: inclusion of the free electron laser amplifier contributions", NIM A 436, (1999), pp. 415-429