RESONANCE DRIVING TERM EXPERIMENTS: AN OVERVIEW

R. Bartolini,

Diamond Light Source Ltd, Oxfordshire, UK.

Abstract

The frequency analysis of the betatron motion is a valuable tool for the characterization of the linear and non-linear motion of a particle beam in a storage ring. In recent years, several experiments have shown that resonance driving terms can be successfully measured from the spectral decomposition of the turn-by-turn BPM data. The information on the driving terms can be used to correct unwanted resonances, to localize strong non-linear perturbations and provides a valuable tool for the construction of the non-linear model of the real accelerator. In this paper we introduce the theory, the computational tools and we give a review of the resonance driving terms experiments performed on different circular machines.

INTRODUCTION

The characterization of the non-linear single particle dynamics in a circular accelerator by means of the frequency analysis of the betatron motion has been investigated by several authors since the eighties. Early attempts by Ando [1] and Bengtsson [2] already provided the basic idea of the technique, namely, the comparison between the beam orbits obtained by numerical tracking or measured at a BPM, with the analytic expression of the betatron oscillations in presence of non-linear resonance driving term in the Hamiltonian of motion. In particular, Bengtsson [2] provided a classification of the frequency lines present in the spectrum of the betatron oscillations as the result of well defined resonance driving term in the Hamiltonian to the first perturbative order. Bengtsson's work benefited from numerical techniques conceived to improve the precision of the FFT in the determination of the frequencies of the spectral lines devised by Asseo [3]. Bengtsson's approach is based on the perturbative analysis of Hamiltonian flows in the single resonance approximation (Poincare–Von Ziepel procedure): only the largest Fourier component of a single resonance driving term is considered in the Hamiltonian, therefore the information on the s-dependence of the driving term is lost. Furthermore the use of Hamiltonian flows renders very cumbersome the extension of the theory to higher order. The topic was revisited from a Normal Form map perspective by Bartolini and Schmidt [4] which provided explicit analytical expressions of the orbit and devised a procedure to establish a one-to-one correspondence between spectral lines and resonance driving terms based on a order by order procedure to clearing the data from the contribution of lower order driving terms. The work of Bartolini and Schmidt used extensively an improved algorithm for the frequency analysis of the betatron orbits [5, 6] based on the Laskar's NAFF algorithm [7] for the decomposition of a quasi-periodic signal into its discrete frequency components. More recently Tomas [8] has extended the theory by clarifying how the spectral line reflects the variation of the resonance driving term along the ring, thus allows the localization of multipolar errors in the ring.

In parallel with the development of the theory, many experiments have been carried out to test the applicability of this technique to real experimental data. The first extensive campaign of measurements aimed at the detection of resonance driving term from the frequency analysis of excited betatron oscillations was performed at the CERN-SPS in the late nineties [9]. Since then, several experiment undertaken on many different machines have demonstrated that spectral lines excited by resonance driving term can be identified, their amplitude dependence and their s-dependence can be assessed and compared with what is expected from the machine model. This provides a valuable tool to correct unwanted resonance, localize multipolar errors and in general to gain a better insight into beam dynamics of a circular accelerator.

Recently, further efforts have been made towards the use of this technique to calibrate the non-linear model of the machine and possibly correct the non-linear beam dynamics by compensating the non-linear field errors distributed in the ring. These applications have been proved only on tracking data [10] and still lack a full experimental verification. However, if proved successful, the frequency analysis of the betatron motion will provide a valuable tool complementary to LOCO [11] and the FMA [12] to help building a realistic on-line non-linear model of the ring.

THEORETICAL BACKGROUND

The betatron motion of a charged particle in a circular accelerator is determined by the sequence of linear and non-linear magnetic elements that constitute the lattice of the ring. The motion of the charged particles can be equivalently described in terms of Hamiltonian flows [13] or symplectic one turn maps [14]. This is true as far as collective or dissipative terms are neglected, in particular, in electron machines, one neglects dissipative and quantum effects due to the emission of synchrotron radiation.

As for any non-linear Hamiltonian system, the motion of a charged particle can be characterized in terms of regular and chaotic orbits. Regular orbits can be stable or unstable: the motion on regular stable orbits is quasi-periodic and can be decomposed in a series of harmonic spectral lines with discrete frequencies [15]:

$$x(n) = \sum_{k=1}^{\infty} a_k e^{i(2\pi\nu_k n + \varphi_k)}$$

The frequencies ν_k are linear combinations of the betatron tunes ν_x in the horizontal and ν_y in the vertical plane. The amplitude of the spectral lines decreases rapidly (exponentially if the Hamiltonian is analytic) with the order of the linear combination of the betatron tunes, therefore the decomposition will contain effectively only a limited number of spectral lines. For chaotic orbits this decomposition breaks down and a broad continuous set of frequencies will be excited in the spectrum.

While the motion in the linear element of the ring is integrable and can be described in terms of Courant-Snyder variables as rotations on circles in the phase space, the non-linear motion of a charged particle in a ring is generally non-integrable. However, perturbative approaches can be used to analyze the non-linear motion and extract useful semi-analytical dynamical quantities to characterize and control the particle dynamics. The perturbative parameter is typically the gradient of the non-linear element in the Hamiltonian flows or the amplitude from the origin in the map formalism.

In the non-resonant Normal Form approach the one turn map M is conjugated towards a simpler map U that depends only on the action variables but not on the angles. The conjugation is performed with a symplectic change of variables Φ according to the scheme in Fig. 1:

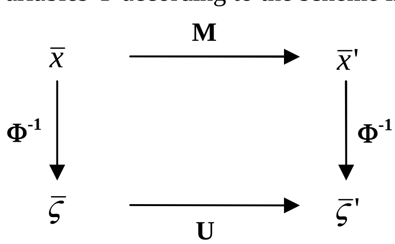

Fig. 1: Schematic of the transformation of the one turn map M into its Normal Form U.

The transformation Φ and the Normal Form map U can be expressed as a Lie series with generating function F and Hamiltonian H respectively according to

$$\Phi = e^{:F:} \qquad U = e^{:H:}$$

The generating function F and the Hamiltonian H can be written as a sum of homogeneous polynomials in the ζ variables, e.g.

$$F = \sum_{jklm} f_{jklm} \zeta_x^+ \zeta_x^- \zeta_z^+ \zeta_z^- = F_3 + F_4 + \ldots$$

where F_k is a polynomial of order k.

The coefficient f_{jklm} of the generating function are related to the coefficient h_{jklm} of the Hamiltonian by:

$$f_{jklm} = \frac{h_{jklm}}{1 - e^{2\pi i[(j-k)\nu_x + (l-m)\nu_y]}}$$

and the h_{jklm} are the resonance driving terms which are determined by the distribution of magnetic elements along the ring, e.g. the coefficients h_{3000}, h_{1020}, h_{1002} are all proportional to the sextupoles strength and are responsible for the excitation of the third order resonances (3,0), (1,2), (1,-2) respectively.

The motion in the normalized coordinate given by the map U is trivial since it is an amplitude dependent rotation; therefore after N turns we have,

$$\varsigma_x(N) = \sqrt{2I_x}\, e^{i(2\pi\nu_x N + \psi_{x0})}$$

where I_x is the horizontal action, ψ_{x0} is the horizontal initial phases. An analogous expression is valid for the vertical plane.

To derive the motion in the original coordinate x we have to transform back the orbit from the ζ variable to the x variables with the transformation Φ. To the first perturbative order in the amplitude, the expression for the beam oscillation in Courant-Snyder variable, e.g. in the horizontal plane, can be cast in the form:

$$\begin{aligned} \hat{x}(n) - i\hat{p}_x(n) &= \sqrt{2I_x}\, e^{i(2\pi\upsilon_x N + \psi_{x0})} \\ &- 2i \sum_{jklm} j f_{jklm} (2I_x)^{\frac{j+k-1}{2}} (2I_y)^{\frac{l+m}{2}} \cdot \\ &\cdot e^{i[(1-j+k)(2\pi\upsilon_x N + \psi_{x0}) + (m-l)(2\pi\upsilon_y N + \psi_{y0})]} \end{aligned} \qquad (1)$$

This expression can be directly compared with the frequency decomposition of a quasi-periodic signal and the values of the coefficients f_{jklm} at a given BPM can be obtained from the spectral decomposition of the betatron oscillation signal using an enhanced FFT algorithm such as those developed in the SUSSIX [6] or NAFF [7] codes. The rule relating the resonance (m, n) to the spectral line {m’, n’} is quite simple and reads

- horizontal plane { –n + 1, –m }
- vertical plane { –n, –m + 1 }

or in terms of the indices of the resonance driving term h_{jklm} we have:

- horizontal plane { 1 – j + k, m – l }
- vertical plane { k – j, 1 – 1 + m }

therefore, as an example, the resonance driving term of the (3, 0) resonance, h_{3000}, will appear in the amplitude of the {–2, 0} spectral line in the horizontal plane.

The detection of the spectral lines provides therefore information on the resonance driving terms that are relevant in the particle motion and allows the measurement and the correction of their effect.

Bartolini and Schmidt [4] showed that a one to one correspondence between spectral line and resonance driving terms can be built by using an order by order procedure, where the order of the parameter is the power in the amplitude dependence of the spectral lines. In general, a given spectral line is fed by different multipoles at different orders, and therefore the amplitude and phase of the spectral lines is the result of the complex vector sum of all the different contributions. As an example, the (4, 0) resonance, i.e. the {–3 , 0} spectral line, gets contributions from the octupoles in first order, via the F_4 polynomial of order 4 in the generating function and from the sextupoles in second order via F_3. However these contributions have different amplitude dependences and the F_3 contribution to the spectral lines can be completely removed once the F_3 terms have been determined by the analysis of the {–2, 0}, {0, ±2} and {0,0}. In fact, the knowledge of F3 can be used to transform the tracking data or the beam data to remove all F_3 contributions at any order in the amplitude with dedicated software program such as DaLie [16]. As a result the new data will be free of third order resonances and the fourth order resonances will have only the residual contribution of F_4. In this way the determination of the spectral lines excited at different order in the amplitude gives in principle the possibility to recover the terms in the one turn map to any given order.

Another interesting application of the frequency analysis of the betatron motion consists in the localization of non-linear multipolar errors. Tomas [8] has shown that the generating function coefficients f_{jklm} vary along the ring whenever a multipole is met. This behaviour is reflected in corresponding variation of the amplitude and phase of the spectral line with the s-position along the ring and therefore it can be used to localize the s-position of a multipolar kick. The variation of a resonance driving term in presence of a multipolar kick can be easily understood by considering the contribution of each single multipole to the driving term itself. If we consider the resonance driving term h_{jklm} before and after the multipole kick, we realize that the total driving term can be represented as a vector in the complex plane which is made up of two contributions: the contribution of the single multipole kick κ_{jklm} and the contribution of the rest of the machine λ_{jklm} (see Fig. 2). Before the multipole, the complex vector of the total driving term is the vector sum of $\kappa_{jklm} + \lambda_{jklm}$ while after the multipole it is the sum of $\lambda_{jklm} + \kappa_{jklm} \cdot \exp(-2\pi i[(j-k)\nu_x + (l-m)\nu_z]$ where the contribution of the multipole, appearing at the end of the ring, is multiplied by the resonant phase advance.

Applying the decomposition to all BPMs in the ring allows the reconstruction of the whole s-dependence of the driving term. This feature is very important since it allows the localization of an unwanted multipolar kick. It is interesting to note that, if the machine tunes are exactly on resonance, the driving term will not change, therefore the spectral lines will not change and it will not be possible to localize the multipolar kick. Therefore this technique works best when the tunes are not close to the resonance we want to analyze.

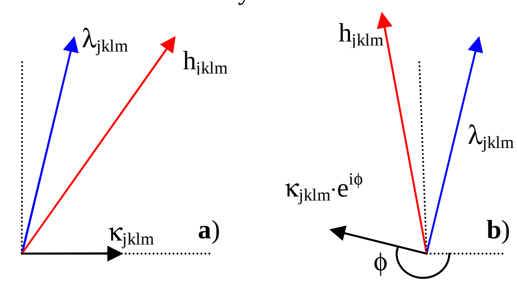

Fig. 2: Addition of complex contribution to the resonance driving term before a) and after b) a single multipole: the contribution of the single multipole (black) appears rotated by the resonant phase advance $\phi = 2\pi[(j-k)\nu_x + (l-m)\nu_z]$. The contribution of the rest of the machine (blue) is unchanged and the total driving term (red) changes according to the location of the multipolar kick.

LIMITS OF THE TECHNIQUES

The technique outlined has its limits. The precision of the frequency decomposition of the signal will be reduced in presence of noise in the BPMs [17] or any other noise source. Nowadays, however, the BPM precision can reach 10 μm rms with few tens of mA bunch trains. BPMs gain, and non-linearities will also affect the precision of the method. The effect of the BPMs gain can be avoided by normalizing the amplitude of the spectral lines to the tune line. The non-linearities in the BPMs response induced by the geometry of the BPM block in the vacuum chamber will modify the measured oscillation especially when considering large oscillations. Bi-dimensional maps that correct the BPM reading (x_{bpm}, y_{bpm}) to the real (x, y) position in the aperture have to be computed to overcome this effect [18].

Another serious limitation is given by the natural decoherence of the betatron oscillations due to the finite size of the bunch and to the amplitude dependent tuneshift. The betatron oscillations excited after the kick will damp after a number of turns that depends on the amplitude dependent tuneshift and the bunch density distribution (typically much shorter than the damping time for electron machines). In some cases only a few hundred turns might be available. Tomas [8] has demonstrated that the amplitude of the spectral lines excited by a resonance of order m will decohere m – 1 times faster than the main tune line. A decoherence factor of (m-1) should be applied to the amplitude spectral line. An alternative solution is provided by the use of a non-destructive continuous excitation of betatron oscillation with an AC dipole: in this case the decoherence issues are virtually eliminated [19].

Another possible source of errors comes from the reconstruction of the particle momentum with two BPMs when the two BPMs are distant and many non-linear elements are present between them. Obviously the distribution of the BPM in the ring is fixed and in general the contribution to a resonance driving term given by two multipole of the same order, with no BPM in between, cannot be distinguished. Only the sum of the complex vector sum of the two multipolar kicks can be detected.

EXPERIMENTS

After the first pioneering experiment at CERN-LEAR [20] the first campaign of measurement specifically dedicate to the detection and characterization of the resonance driving terms was performed at the CERN-SPS between 1999 and 2002 [9, 21-23]. In the first experiment the spectral line {-2, 0} excited by the (3, 0) resonance driving term h_{3000}, was detected and its amplitude dependence at one BPM was found in a reasonably good agreement with the model as shown in Fig. 3.

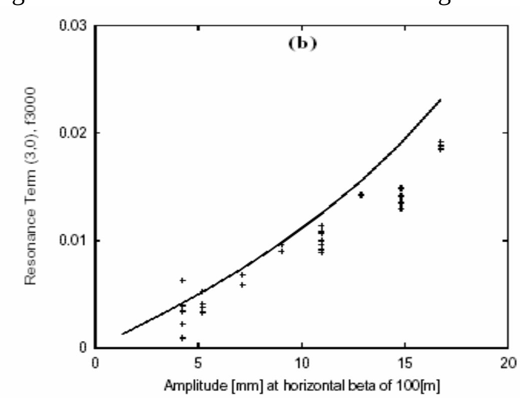

Fig. 3: Amplitude of the {–2, 0} spectral line excited by the (3, 0) resonance as a function of the amplitude of the induced betatron oscillation and comparison with the model [9].

In subsequent experiments the work has concentrated on the analysis of the s-dependence of the driving term using the CERN-SPS in especially dedicated configurations where the polarity of the eight extraction sextupoles was varied to enhance or reduce the sextupoles induced driving terms. Fig. 4 shows the s-dependence of the {– 2, 0} spectral line along the ring and a comparison between tracking data and measured data. One sextupole was forcefully disconnected to investigate the difference in the spectrum lines pattern. The location of the eight extraction sextupole is clearly put in evidence by the abrupt steps in amplitude of the spectral line. The agreement between the amplitude of the measured spectral line and the tracking data is significantly improved in the bottom graph where the model took into account the disconnected sextupole.

In 2003-2004 [24, 25] similar experiments were performed at RHIC. The first measurements based on the beam excitation with an AC dipole were compared with the results obtained with kicked beam and with the model, showing the potentiality of the AC dipoles [19]. Furthermore, a combination of the signals taken at three consecutive BPMs was devised to single out the contribution to the resonance driving terms generated only by the multipoles present between the selected BPMs. An example of the "three BPMs method" [25] providing the measurement of the local contribution from sextupoles to the (3,0) resonance driving terms with an AC dipole is reported in Fig. 5, with the comparison with RHIC tacking data.

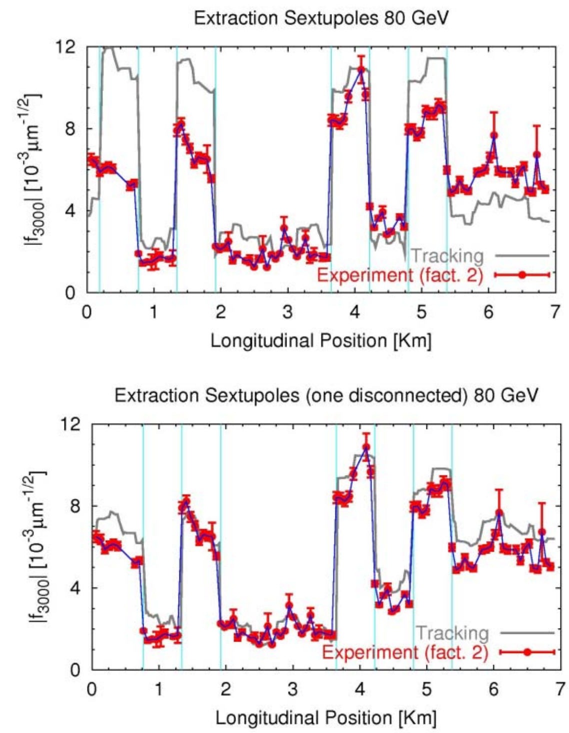

Fig. 4: Amplitude of the {– 2, 0} spectral line as a function of the position along the ring. Comparison with the tracking data for the case where all the eight sextupoles are powered in the model (top) or the first sextupole is disconnected in the model (bottom) [22] (courtesy F. Schmidt).

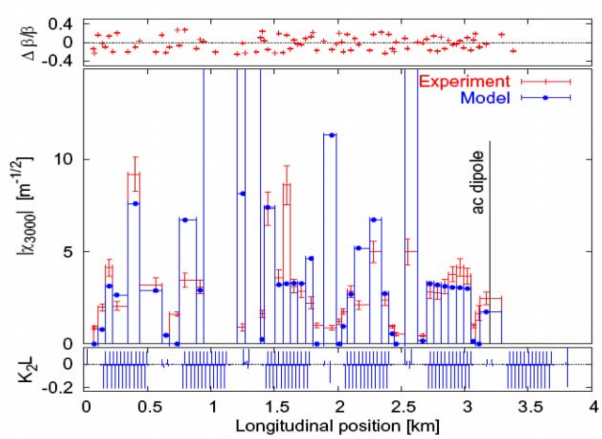

Fig. 5: Local contribution to the resonance driving terms measured with an AC dipole. The top plot shows the β-beating and the bottom plot shows the sextupolar component of the ring [25] (courtesy R. Tomas).

The possibility of correcting unwanted resonances with the direct measurement of resonance driving terms was demonstrated in a series of experiments made at CERN-

PS in 2004 [26]. Targeting specifically the systematic skew resonance $3Q_y = 16$ appearing as the {0, –2} spectral line in the vertical plane, by means of a two independent skew sextupoles, it was possible to reduce the resonance driving terms directly from the observation of the spectral line. The vertical phase space plot, before and after correction, is reported in Fig. 6. It is evident that, when the resonance is compensated, the phase space is free from the deformation due to the (0, 3) resonance. The analysis of the resonance content of the CERN-PS, based on the spectral lines, showed that an alternative working point with a vertical integer tune lower by one unit is preferable to the one used in standard operation.

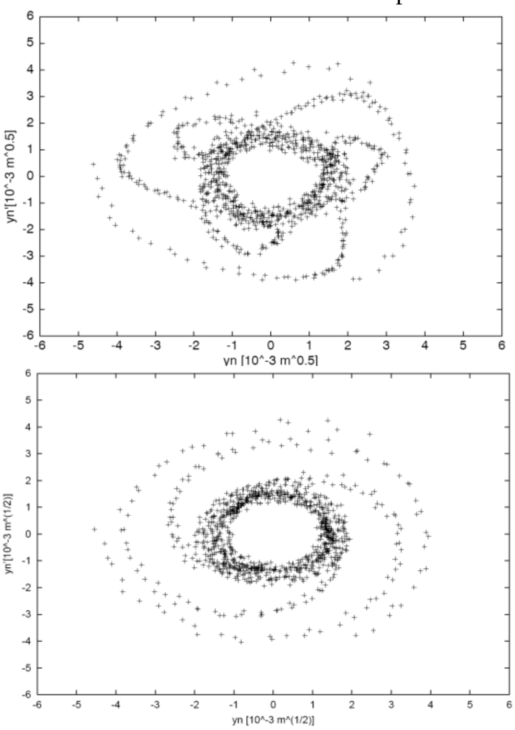

Fig. 6: Normalized vertical phase space of the uncompensated (top) and compensated (bottom) lattice for the CERN-PS [26] (courtesy P. Urschutz).

In the last years, several other machines have attempted an experimental analysis of the resonance driving terms excited in their ring: in particular Tevatron [27], ALS [28], ESRF [29], BESSY-II [30].

At the Tevatron and ALS the experiments were again devoted to the identification of the spectral lines of the main resonance along the ring. At the Tevatron, a strong contribution to the (3, 0) resonance driving term from the sextupole family S6 was put into evidence. In Fig. 7 we report a comparison of the {–2, 0} line measured at the Tevatron and a comparison with tracking data. The spectral line analysis allowed the partial compensation of this effect with an independent sextupole (S6A0). The experiments performed at the ESRF and BESSY-II put in evidence the possibility of measuring also higher order resonance driving terms. At BESSY-II it was possible to detect the spectral line {–3, 0} in the vertical plane excited by the 4th order skew resonance (3, 1) and to verify the correct dependence of the spectral line amplitude with the amplitude of the induced betatron oscillations. According to formula (1), the amplitude of the {–3, 0} vertical spectral line should grow with the 3rd power of the horizontal oscillation amplitude. Indeed, Fig. 8 shows a linear relation between the amplitude of the {–3, 0} line and the cube of the kicker voltage.

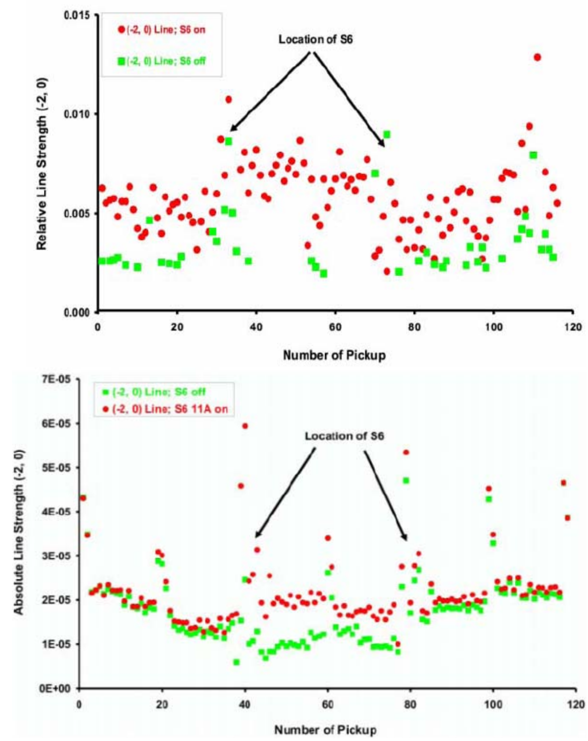

Fig. 7: Tevatron data: sextupole spectral line {–2, 0} as a function of the pick up number with S6A sextupole on (red) and off (green). Experimental data (top) tracking data (bottom). The azimuthal position of the BPMs is shifted in the two graphs, however the location of the S6 sextupoles is explicitly marked [27] (courtesy Y. Alexhain).

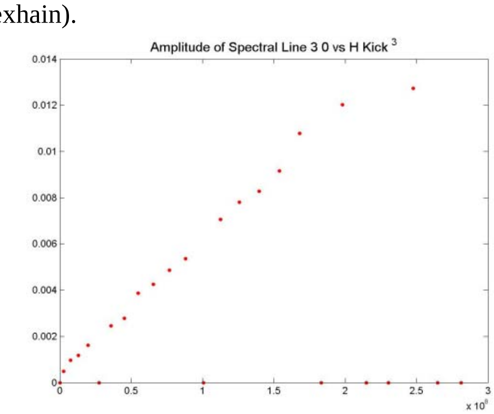

Fig. 8: Amplitude of the {–3, 0} line as a function of the cube of the kicker voltage measured at BESSY-II (courtesy P. Kuske).

NON-LINEAR MODEL CALIBRATION

The experimental results presented show that the spectral analysis of turn-by-turn data provides a wealth of information on the non-linear beam dynamics. It is therefore natural to envisage the possibility of using this technique to obtain a realistic modelization of the non-linear part of the ring directly from the turn-by-turn data acquired at all BPMs in the ring.

A possible schemes was devised in Ref. [10] and it is based on the fitting the non-linear multipole strength along the ring to reproduce the s-dependence of selected spectral lines. Given a particular resonance whose s-dependence pattern differs significantly from what expected from the model, one can build a target vector A_{meas}, whose components are the amplitude and phase of the spectral lines computed at each BPM, and compare it with the same target vector computed from the ideal model A_{model}. The components of the target vector will depend on the value of the particular multipolar elements that excite the corresponding resonances. If the resonance (3,0) is targeted, then the target vector to use is given by the amplitude and phase of the spectral line {–2, 0} which is excited by the sextupoles in the ring:

$$\overline{A}_{meas}^{(-2,0)} = (A_1, ..., A_{Nbpm}; \phi_1, ..., \phi_{Nbpm})$$

where Nbpm is the total number of BPMs in the ring. The distance between the two vectors A_{meas} and A_{model}, e.g.

$$\chi^2 = \sum_{j=1}^{2Nbpm} \left[A_{model}^{(-2,0)}(j) - A_{meas}^{(-2,0)}(j) \right]^2$$

gives a measure of the discrepancy between the real accelerator and the model. This quantity can be minimized by a least square minimization procedure that involves fitting the strength of the magnetic elements directly responsible for the excitation of that particular spectral line. Fig. 9 shows an example of the application of this technique to Diamond tracking data.

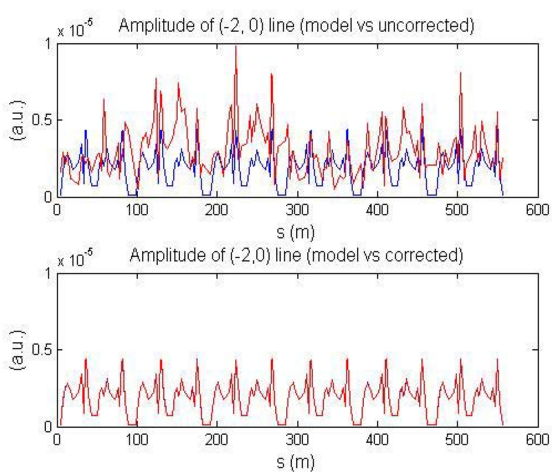

Fig. 9: Comparison of spectral lines {-2,0} from tracking data for the Diamond lattice with sextupole strength errors: top) ideal lattice (blue) and uncorrected lattice (red); bottom) ideal lattice (blue) and compensated lattice (red): the two amplitudes are practically indistinguishable.

A MATLAB numerical code was generated to produce the fit and reconstruct the errors [31]. It is worth mentioning that this method works equally well in the modelisation of the linear part of the ring: the linear optic is inferred from the amplitude of the tune lines, fitting the quadrupoles strengths, and the linear coupling is inferred by targeting the {0, 1} spectral line in the horizontal plane and the {1, 0} line in the vertical plane, fitting the skew quadrupoles in the ring. This method has been successfully applied to tracking data where it reproduced the resonance driving term accurately. However, its effectiveness in dealing with real machine data has still to be proven.

CONCLUSIONS

The identification of the resonance driving terms with the frequency analysis of the betatron motion is a well understood topic. The pioneering experiments at CERN have demonstrated that the resonance driving terms can be measured experimentally and can be used to improve the routine operation of the machine. Furthermore, they can be used to compare the real machine with the model. However, a full reconstruction of the non-linear model of the ring has still to be shown in an experiment. This is the next challenge in the development of this technique.

REFERENCES

[1] A. Ando, Part. Acc., **15**, 177, (1984).
[2] J. Bengtsson, CERN 88-05, (1988).
[3] E. Asseo, CERN PS/LEA 85-3, (1985).
[4] R. Bartolini, F. Schmidt, Part. Acc., **59**, 93, (1998).
[5] R. Bartolini et al., Part. Acc., **52**, 147, (1996).
[6] R. Bartolini, F. Schmidt, CERN SL **98-017**, (1999).
[7] J. Laskar et al., Physica D, **56**, 253, (1992).
[8] R. Tomas, PhD Thesis, Univ. Valencia, (2003).
[9] R. Bartolini et al., PAC99, 1557, (1999).
[10] R. Bartolini, F. Schmidt, PAC05, 1452, (2005).
[11] J. Safranek, NIMA **388**, 27, (1997).
[12] J. Laskar, PAC03, 378, (2003).
[13] A. Schoch, CERN 57-21, (1958).
[14] A. Dragt et al., Ann. Rev. Nucl. Part. Sci., **38**, 455, (1998).
[15] H. Goldstein, Classical Mechanics, II edition (1980).
[16] E. Forest, "The DaLie Code", 1986, unpublished.
[17] R.Bartolini et al., Part. Acc., **55**, 247, (1996).
[18] G. Rehm, private communications.
[19] R. Tomas, PRSTAB, **5**, 054001, (2002).
[20] E. Asseo et al., EPAC88, 541, (1988).
[21] F. Schmidt et al., PAC01, pg. 437, (2001).
[22] M. Hayes et al., EPAC02, pg. 1290, (2002).
[23] F. Schmidt et al., PAC03, pg. 2231, (2003).
[24] R. Tomas et al., PAC03, pg. 2228, (2003).
[25] R. Tomas et al., PRSTAB, **8**, 024001, (2005).
[26] P. Urschutz, EPAC04, 1918, (2004).
[27] Y. Alexahin et al., 2140, EPAC06, (2006).
[28] C. Steier, USPAS, ASU, January 2006.
[29] Y. Papaphilippou et al., ESLS XIII, (2003).
[30] P. Kuske et al., unpublished, (2004).
[31] R. Bartolini, "No-Lobo", unpublished, (2004).