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INVESTIGATIONS OF SPATIAL PROCESS MODEL FOR THE CLOSED ORBIT FEEDBACK SYSTEM AT THE Sis18 SYNCHROTRON AT GSI *

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Abstract

A closed orbit feedback system is under development at the GSI SIS18 synchrotron for usage during the whole acceleration cycle including the acceleration ramp. Singular value decomposition (SVD) is the most widely used technique in global closed orbit correction for eigenmode decomposition, mode selection and pseudo-inversion of Orbit Response Matrix (ORM) for robust calculation of corrector magnet strengths. A new faster inversion technique based upon Discrete Fourier Transform (DFT) has been proposed for SIS18 ORM exploiting the circulant symmetry, a class of matrices which can be diagonalized by the DFT using only one row or column of the matrix. The existence of a clear relationship between SVD modes and singular values to DFT modes and coefficients for such matrices has been described. The DFT based decomposition of circulant ORM gives hints on physical interpretation of SVD and DFT modes of perturbed closed orbit in a synchrotron. As a first practical application, DFT modes were used to provide robustness against sensor failures such as one or two malfunctioning BPMs.

INTRODUCTION

GSI's SIS18 synchrotron will be the booster ring for the SIS100 synchrotron at the upcoming FAIR facility. SIS18 upgrade is underway to cope with higher beam intensities planned for the FAIR facility [1,2]. The closed orbit feedback system aims to supplement the SIS18 upgrade efforts by stabilizing the beam orbit during the full acceleration cycle. There are many of challenges for the closed orbit feedback system (COFB) for SIS18,

- The change of lattice from triplet to doublet in course of the acceleration ramp leads to orbit and tune changes [3]. Figure (1) shows a plot of typical position movement during the acceleration ramp.

- Influence of power supply ripple on orbit especially visible in horizontal plane. A ≈ 2 kHz bandwidth system would be required to suppress these ripple. Figure 2 shows the phase shifted ripple of two consecutive cycles.

- Several users (up to 16) can be supported in "parallel" operation presently at SIS18 where the users can request dynamic changes in beam energy and intensity. This often leads to non-reproducible orbits due to magnet hysteresis. Strategies to avoid the hysteresis problems are under discussion at FAIR [4].

- BPM failures due to radiation shower inside the accelerator tunnel is a regular occurrence especially during high intensity operation. Feedback systems should be robust to such sensor/actuator failures.

- Intensity-dependent tune movements are already seen at moderate intensity operation of SIS18. Such effects are expected to aggravate during FAIR operation and the COFB should be prepared to deal with them.

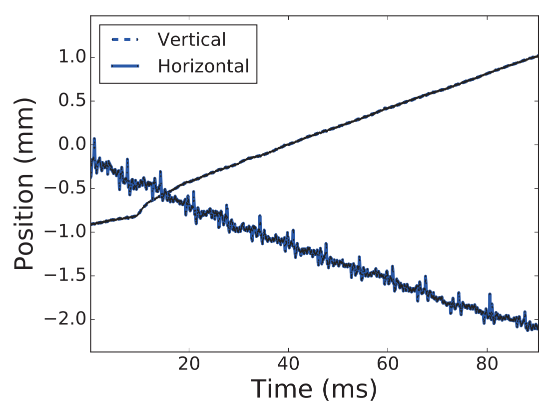

Figure 1: Position in both planes measured at BPM in section 8 during first 90 ms in the acceleration ramp.

A feed-forward orbit correction system can overcome some of these challenges but non-reproducible cycle-to-cycle effects such as hysteresis, intensity-dependent orbits and power supply ripple cannot be easily accounted for. This calls in for a fast feedback system which is robust to uncertainties in machine model and unavailability of sensors.

The backbone of the closed orbit feedback correction is the orbit response matrix (ORM). It is the effect of corrector magnets on the transverse position of closed orbit measured at the locations of BPMs. Equation (1) describes a generalized orbit response matrix (one for each transverse plane) [5]

$$\mathbf{R}_{mn} = \frac{\sqrt{\beta_m \beta_n}}{2\sin(\pi Q)} \cos\left(Q\pi - |\mu_m - \mu_n|\right) \qquad (1)$$

where β and μ denote the beta function and phase advance while subscripts m and n mark the locations of BPMs and

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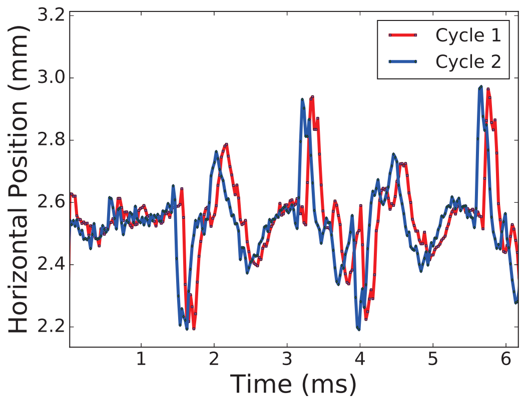

Figure 2: Measured horizontal position of two consecutive cycles at injection plateau.

correctors respectively. Q is the tune which is defined as the number of betatron oscillations in one turn around the ring. SIS18 has 12 BPMs and 12 correctors per plane resulting in a 12×12 matrix. BPMs and correctors are placed in a symmetric manner in y plane such that each BPM is placed at periodic optics location in each cell and the phase difference between two adjacent BPMs is also constant. Similar periodicity also holds for the correctors. The ORM is regarded as the static or spatial process model [6] of the synchrotron in the context of closed orbit feedback system but during acceleration ramp this "static" model varies with time because of the increase of quadrupole magnet strengths with the increase of beam energy. The beta functions and phase advances at the locations of the BPMs and correctors change during ramp. Ideally, the quadrupole strengths at SIS18 (focusing, defocusing and triplet quadrupole families) are varied in a way to keep the tune constant but measurements have shown that the tune also drifts during ramp [3].

Such a tune drift imposes an extra model mismatch on top of systematic change of ORM during ramp. The calculation of corrector magnet strengths requires the calculation of the pseudo-inverse of ORM. Singular value decomposition is the most commonly used technique for the decomposition of ORM particularly in storage rings [7] where the particles are stored at almost constant energy with fixed quadrupole settings. Only model errors and uncertainties are expected, a topic that has been dealt by S. Gayadeen [6], and SVD based decomposition was found to have limitations in terms of quantifying the model uncertainties. Another technique based on Fourier analysis of Eq. (1) [8] was used instead to quantify model uncertainties in [6], but an exact relationship between SVD singular values and Fourier coefficients was not reported.

In this paper, a Discrete Fourier Transform (DFT) based decomposition and inversion of the ORM is suggested exploiting the symmetry of the SIS18 synchrotron and a clear relationship of singular values and modes of SVD with Fourier coefficients and modes of DFT has been established. The use of Fourier modes for the closed orbit correction in case of missing BPMs has been demonstrated using MAD-X [9].

SVD OF SIS18 VERTICAL ORM

SVD is a well known numerical technique for the analysis of multivariate data [10] and is closely related to a classical dimension reduction method called principal component analysis (PCA). The first use of SVD as ORM inversion tool was proposed in 1989 [7]. Lately, it has become a de-facto process for ORM inversion in almost all synchrotron light facilities.

An SVD decomposition of a general $m \times n$ matrix $\mathbf{R}$ is given as

$$\mathbf{R} = \mathbf{U}\mathbf{S}\mathbf{V}^T \tag{2}$$

The diagonal elements of matrix $\mathbf{S}$ are known as singular values while the columns of $\mathbf{U}$ and $\mathbf{V}$ are left and right singular vectors (also called SVD modes) for the corresponding singular values. Matrices $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices such that their inverses are equal to their transpose. The pseudo-inverse of ORM is easily computed by taking inverse of individual singular values.

$$\mathbf{R}^+ = \mathbf{V}\mathbf{S}^+\mathbf{U}^T \tag{3}$$

Small singular values are cut at this point to remove weakly coupled modes which can lead to corrector saturation [7].

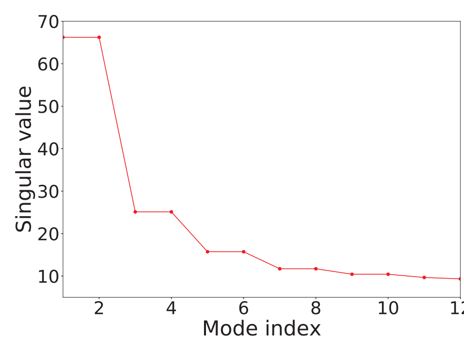

Figure 3: Singular values of simulated SIS18-ORM in y-plane.

Figure 3 shows the plot of singular values of simulated SIS18-ORM in y-plane while first eight columns of $\mathbf{U}$ (black) and $\mathbf{V}$ (red) matrices are plotted in the Fig. 4. Eq. (1) shows that spatial closed orbit perturbations are mainly dominated by the set tune of the machine ($Q_y = 3.27$ for SIS18). One can see that the first two columns of each matrix corresponding to highest equal singular values have sinus and cosinus shapes respectively, with a discrete spatial frequency $k = 3$ while columns corresponding to second highest singular values have spatial frequency $k = 4$. This behavior of columns of $\mathbf{U}$ and $\mathbf{V}$ matrices is an example that SVD decomposes the ORM into two spaces; BPM space and corrector space and each space is further decomposed into different spatial frequency "modes" which are available in the form of columns of $\mathbf{U}$ and $\mathbf{V}$ [7]. The singular values in the diagonal matrix relate the corresponding modes of one space to that

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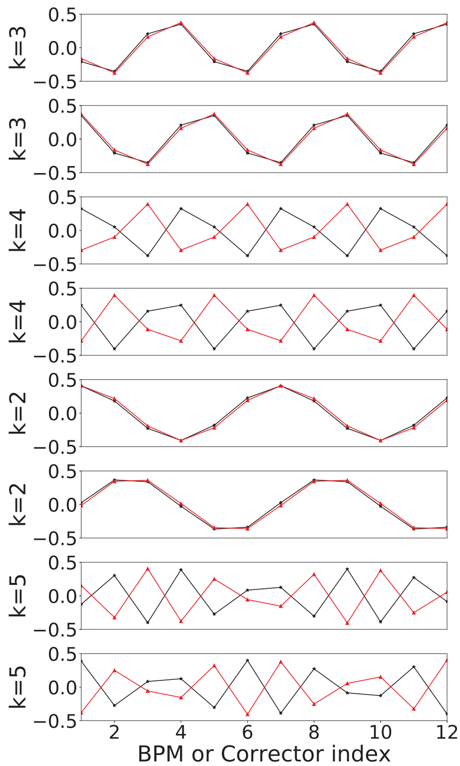

Figure 4: Columns of U (black) and V(red) matrices generated by SVD of simulated SIS18-ORM in y-plane.

of the other. Modes with spatial frequencies far from the tune frequency have less contribution in the closed orbit perturbations as becomes evident from their relatively smaller associated singular values.

There exists a unique phase relationship between the BPM and corrector modes for each frequency as evident from Fig. 4. For example, for $k = 2$ and $k = 3$ there is a small phase difference while for $k = 4$ and $k = 5$ the phase difference between **U** and **V** modes is larger. This frequency-dependent phase relation between two spaces is not physically explained by SVD.

Spatial Model Variation and SVD Based Decomposition

The power of SVD to decompose any matrix into orthonormal basis of left and right orthogonal matrices is in fact a weakness when dealing with uncertainty modeling because SVD is a numerical decomposition technique [10] and there is no direct way of associating uncertainties in lattice parameters to the singular values. One cannot model the singular values as a function of lattice parameters and this problem becomes prominent when lattice parameters change during acceleration ramp as a function of quadrupole magnet strengths. Secondly, singular values do not change independently with lattice settings because every new **R** has a new set of **U**, **S** and **V** matrices. In this situation, either one has to pre-calculate and store the inverses of ORMs for each energy step during the ramp or one has to update and decompose ORM online at each new energy which increases the computational effort. In addition to systematic on-ramp variations, there are also sources of model uncertainties including BPM and corrector scaling errors from imperfect calibrations or drifts in the tune [6, 11].

Harmonic Analysis

Harmonic analysis has been proposed as an alternative to SVD for the uncertainty modeling. Fourier series expansion of Eq. (2) is performed to calculate the Fourier coefficients which are expressed as a function of tune [8, 11] and are given as

$$\sigma_k = \frac{Q}{\pi(Q^2 - k^2)} \tag{4}$$

where Q is the tune and k is the discrete Fourier frequency. There is a qualitative relationship between these Fourier coefficients and SVD singular values such that highest singular value and highest Fourier coefficients both correspond to the tune mode but there is no physical relationship reported between Fourier basis and SVD modes as well as singular values and Fourier coefficients.

EXPLOITING CIRCULANT SYMMETRY OF SIS18 ORM

As mentioned before, BPMs and correctors are placed symmetrically in each cell of SIS18 and as a result the ORM of SIS18 has a special shape in which each row/column is a cyclic rotation of the previous row/column. Such a square matrix is called Circulant matrix [12, 13] which is a special member of the Toeplitz matrices family [14] in which the diagonal elements are identical and each row (or column) is a cyclic rotation of the previous row (or column). The shape of a typical Circulant matrix is the following.

$$\mathbf{R} = \begin{bmatrix} r_0 & r_1 & r_2 & r_3 & . & . & . & . & r_{n-1} \\ r_{n-1} & r_0 & r_1 & r_2 & . & . & . & . & r_{n-2} \\ r_{n-2} & r_{n-1} & r_0 & r_1 & . & . & . & . & r_{n-3} \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ r_1 & r_2 & r_3 & r_4 & . & . & . & . & r_0 \end{bmatrix} \tag{5}$$

One of the special properties of Circulant matrices is that they can be diagonalized with the help of Discrete Fourier Transform (DFT) of any single row / column of the matrix. Let **R** be a square Circulant matrix of dimension N, then it can be decomposed as [12]

$$\mathbf{R} = (1/N)\mathbf{F}\hat{\mathbf{H}}\mathbf{F}^* \tag{6}$$

where $\hat{\mathbf{H}}$ is a diagonal matrix containing the discrete Fourier coefficients on its diagonal positions, **F** is a standard Fourier matrix whose columns are given by

$$F_{k,n} = e^{-j2\pi(k-1)(n-1)/N} \tag{7}$$

In Eq. (7), $n \in [1, .., N]$ and $k \in [1, .., N/2]$ where n is the sampling point, k is the discrete frequency of each Fourier

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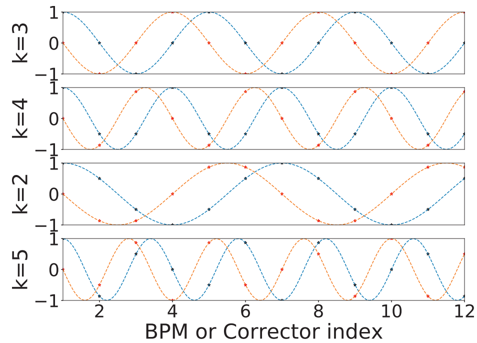

Figure 5: Fourier modes (Real (black dots) and imaginary (red dots) parts of columns of Fourier matrix **F**).

mode and N is the size of the square circulant matrix. In case of ORM, N is the total number of BPMs or corrector magnets. Real and imaginary parts of the first four dominant Fourier modes have been plotted in Fig. 5 (using cosine and sine interpolations between data points). The inverse of **R** can be written as

$$\mathbf{R}^{-1} = (1/N)\mathbf{F}^{*}\hat{\mathbf{H}}^{-1}\mathbf{F} \tag{8}$$

where $\hat{\mathbf{H}}^{-1}$ is the diagonal matrix having inverses of Fourier coefficients at its diagonal positions. Mode truncation is also possible by removing the Fourier coefficients smaller than a threshold along with corresponding columns of **F**. In this case the inverse will be approximated as Pseudo-inverse.

Equivalence of SVD and DFT for Circulant Symmetry

There exists a mathematical equivalence between SVD and DFT diagonalization in case of circulant matrices [15]. Both techniques decompose ORM into sinus and cosinus modes but with a different distribution of information between component matrices. The diagonal elements of $\hat{\mathbf{H}}$ are complex numbers

$$\sigma_k = \sigma_{rk} + j\sigma_{ik} = \sum_{0}^{N-1} R_n e^{-j2\pi kn/N} \tag{9}$$

while diagonal elements of **S** are scalars equal to the magnitude of the complex Fourier coefficients

$$s_k = |\sigma_k| = \sqrt{(\sigma_{rk})^2 + (\sigma_{ik})^2} \tag{10}$$

Columns of **F** and **F*** contain cosine and sine functions as real and imaginary parts

$$F_k = \cos\left(\frac{2\pi kn}{N}\right) + j\sin\left(\frac{2\pi kn}{N}\right) \tag{11}$$

The columns of **U** and **V** are real valued cosine and sine functions

$$\begin{aligned} V_k &= \sqrt{\frac{2}{N}}\cos\left(\frac{2\pi kn}{N} + \phi_k\right) \\ U_k &= \sqrt{\frac{2}{N}}\cos\left(\frac{2\pi kn}{N} + \phi_k + \phi_{dk}\right) \end{aligned} \tag{12}$$

where ϕ_k is some arbitrary starting phase while ϕ_{dk} is the frequency-dependent phase difference and is given as

$$\phi_{dk} = arg(\sigma_k) \tag{13}$$

The distribution of phase of Fourier coefficients into left and right unitary matrices in case of SVD makes it difficult to quantify the lattice changes into singular values only.

Benefits of DFT over SVD against Model Uncertainties

Some benefits of the DFT approach can be expected over the limitations of SVD in case of SIS18-ORM.

- All the lattice information which is distributed in all three matrices in case of SVD (in the form of phase relationship between **U** and **V** modes and their amplitudes as well as singular values) is contained only in the diagonal matrix in case of DFT (in the form of phase and magnitude of complex Fourier coefficients) and **F** and $\mathbf{F}^{*}$ are standard matrices which are same for any circulant matrix of the same dimension. This means that the lattice changes during acceleration ramp can be easily taken into account by only updating the Fourier coefficients of the response of correctors measured at one BPM only (first row of ORM), with an assumption that all the errors are uniformly distributed (which is true for SIS18).

- Decomposition of ORM into standard DFT modes gives a physical interpretation to the modal structure of perturbed orbit in a synchrotron easier than SVD. It also explains the structure of SVD modes and their mutual relationship. Decomposition of perturbed orbit into standard sine/cosine modes can be utilized to reconstruct a perturbed orbit if one or two data points (BPMs) are missing; an operational scenario in SIS18. This will be demonstrated in the next section.

- For a circulant ORM the computation time in case of DFT is significantly shorter than that of SVD which gives the possibility of online inversion of orbit response matrix. Only diagonal matrix $\hat{\mathbf{H}}^{-1}$ needs to be constructed using inverses of DFT coefficients.

ROBUSTNESS AGAINST MISSING BPMs

If one or two BPMs stop working or measure false beam positions, the calculated corrector strengths can further degrade the closed orbit instead of correction. This is a common operational scenario at SIS18 due to radiation shower inside vacuum pipe and COFB will be either stopped or ORM will be altered to exclude the effect of faulty BPM. We have proposed a method to estimate the orbit position at the location of missing BPM using the beam position at all other BPMs and modal structure of the circulant ORM. The correction is demonstrated for SIS18 in vertical plane using CERN's tool MAD-X [9].

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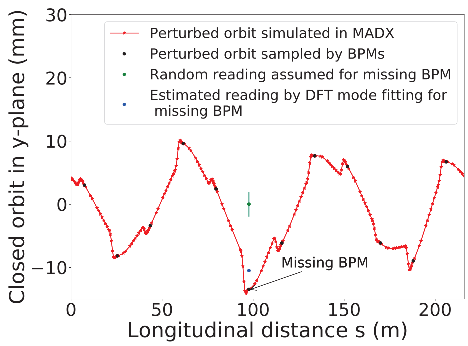

Figure 6: Closed orbit fitting in case of missing BPM.

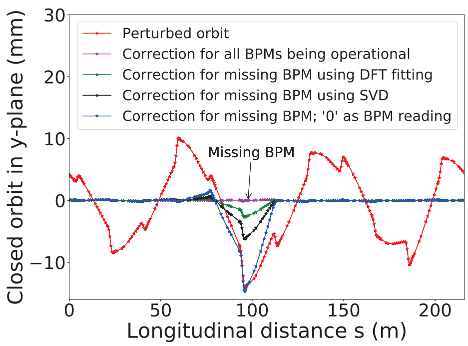

Figure 7: Comparison of SVD and DFT based orbit correction in case of missing BPM.

Figure 6 shows a perturbed closed orbit (red curve) simulated in MAD-X as a result of random misalignments (in a range of -0.85 to 0.85 mm) in all 24 quadrupoles in SIS18. A misaligned quadrupole has an effect of magnetic dipole on the beam and as a result closed orbit is perturbed from its "ideal path".

The black dots in Fig.6 represents the sampling of perturbed orbit at the locations of BPMs while the green dot shows a random orbit position (in a range of -1 to 1 mm) assumed at the location of a missing or faulty BPM. The DFT based modal analysis of ORM predicts that the dominant modes in perturbed orbit consists of cosine/sine functions with discrete frequencies $k = 3, 4, 2, 5$ (in descending order of Fourier coefficients). A combination of cosine functions with above mentioned discrete frequencies was used to estimate the orbit position at the missing BPM location using the Python SCIPY curve fitting module keeping their relative amplitudes and phases as free parameters to be optimized. The fitting algorithm was constrained to keep the fitted curve closest to the orbit positions within an accuracy of 0.01 mm at the working BPM locations while free to chose any value at the location of missing BPM. As a result the optimized orbit position was found to be closer to the actual orbit position within a maximum error of 3 ± 0.048 mm.

Figure 7 shows the orbit correction using the estimated orbit position (green curve). Red curve shows the perturbed orbit while magenta curve shows the corrected orbit when all BPMs are working. Orbit correction using SVD of non-circulant matrix (excluding the row corresponding to the faulty BPM from ORM) is plotted in black color. Orbit correction taking the orbit position "zero" at the missing BPM location and using a circulant matrix has also been plotted in blue curve for comparison. The robustness against missing BPM is shown by the overall improved correction obtained using an estimated beam position instead of using non-circulant matrix. Besides the better global correction one can also get the benefits of circulant symmetry and DFT decomposition (e.g. online decomposition during ramp) even when the symmetry has been broken due to a missing BPM. This approach of estimating the orbit position was also tested for two consecutive missing BPMs scenario and was found to work.

CONCLUSION

A new approach for the decomposition and inversion of orbit response matrix based on Discrete Fourier Transform has been introduced for the closed orbit correction. An equivalence between SVD and DFT coefficients and modes has also been established which gives more physical insight into the concept of BPM and corrector spaces. A practical usage of modes to achieve robustness against one or two missing BPMs has been demonstrated with MADX simulations.

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