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ROBUST EMITTANCE MEASUREMENTS*

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Abstract

The quadrupole scan is commonly used for measurement of beam emittance. The found dependence of the beam size vs. quadrupole strength is fitted with parabola, which coefficients are used for emittance calculations. The measurement errors can cause substantial variations in the emittance value. Sometimes the fitted parabola has negative minimum value, making impossible emittance calculation. We propose more robust data processing modifying the quadrupole scan procedure using or weighted fit for parabola. The experimental results are presented.

INTRODUCTION

Emittance serves as measure of the transverse phase space occupied by the beam as well as a measure of beam quality in many applications. By definition [1] emittance is

$$\varepsilon_x^2 = \sigma_x^2\sigma_{x'}^2 - \sigma_{xx'}^2 \quad (1)$$

where σ_x^2 and $\sigma_{x'}^2$ are variances, and $\sigma_{xx'}$ is covariance of beam distribution.

Commonly used technique is scanning of a focusing element such as a quadrupole or a solenoid and measuring dependence of the transverse beam size on a profile monitor [2, 3]. The obtained data are fit with a parabola which coefficients are used for calculating emittance. In some cases, even slight changes in the data can cause substantial variation of the emittance value. This is due to that we are subtracting two large numbers. Example of such measurement is shown in Fig. 1.

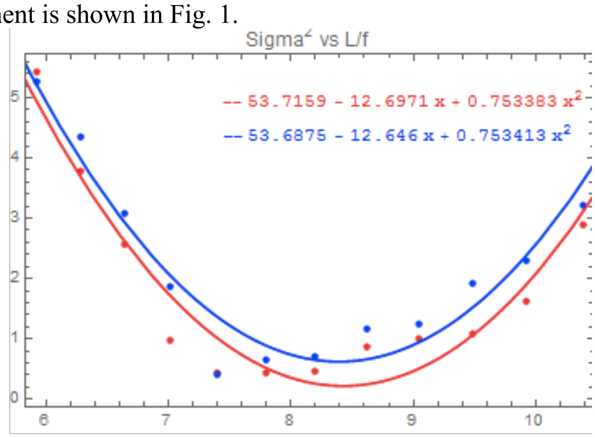

Figure 1: The horizontal axis shows ratio of distance from the solenoid center to the profile monitor and solenoid's focal length. Vertical axis shows the square of the r.m.s. beam size in mm^2.

The maximal difference between the parabolic fit coefficients is 0.2%. Distance from the solenoid to the profile monitor is 3.63 m. Emittance for the blue curve is 0.19 mm mrad, and for the red curve it is 0.11 mm mrad, difference is factor of 2.

Moreover, due to the statistical variations the value of the fitted parabola minimum can be less than zero making the data set unusable for obtaining value of the emittance.

MODIFICATION OF SCAN

We are going to modify the procedure to avoid such subtracting. We can present beam angular distribution as sum of correlated and uncorrelated parts, so that for each particle $x' = \alpha x + \tilde{x}'$, where α is correlation factor and $\tilde{x}'$ is uncorrelated angular spread. Correlation factor can be found from the covariance:

$$\alpha = \sigma_{xx'}/\sigma_x^2 \quad (2)$$

Beam emittance can be found from Eq. (1)

$$\varepsilon_x^2 = \sigma_{x0}^2\sigma_{\tilde{x}'0}^2 \quad (3)$$

where σ_{x0}^2 is r.m.s beam size at focusing element, and $\sigma_{\tilde{x}'0}^2$ is uncorrelated angular spread.

After passing through a focusing element with focal length F and drift with length L the beam size at the observation point can be found

$$\sigma_x^2 = \left(1-\frac{L}{F}\right)^2\sigma_{x0}^2 + 2\alpha L\left(1-\frac{L}{F}\right)\sigma_{x0}^2 + L^2(\alpha^2\sigma_{x0}^2 + \sigma_{\tilde{x}'0}^2) \quad (4)$$

The minimum size is observed when $1 - L/F = -\alpha L$ and minimal size is:

$$\sigma_{xmin}^2 = L^2\sigma_{\tilde{x}'0}^2 \quad (5)$$

So, from $1 - L/F_{min}$ corresponding to the minimal beam size one can find the first order correlation factor α between transverse positions and angles of the particles in the bunch, and the uncorrelated angular spread can be calculated using the minimal beam size.

Now we need to find beam size at focusing element to calculate beam emittance. For this purpose, we can measure the beam size when focusing element is off ($1/F = 0$):

$$\sigma_x^2 = \sigma_{x0}^2 + 2\alpha L\sigma_{x0}^2 + \alpha^2L^2\sigma_{x0}^2 + \sigma_{xmin}^2 \quad (6)$$

and

$$\sigma_{x0}^2 = \frac{\sigma_x^2-\sigma_{xmin}^2}{(1+\alpha L)^2} = \frac{\sigma_x^2-\sigma_{xmin}^2}{(L/F_{min})^2} \quad (7)$$

Emittance can be found using formula below:

$$\varepsilon^2 = \frac{\sigma_x^2-\sigma_{xmin}^2}{(L/F_{min})^2}\frac{\sigma_{xmin}^2}{L^2} \quad (8)$$

If the beam is converging to the minimal size with focusing element off or close to such condition, then there can be significant error in determination of the beam size at the

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focusing element. In this case we can use the setting of the focusing element providing for substantially different beam size and use the formula below for calculation of the beam size

$$\sigma_{xo}^2 = \frac{\sigma_x^2 - \sigma_{xmin}^2}{(1-L/F)^2 + 2\alpha L(1-L/F) + L^2\alpha^2} \quad (9)$$

If we set $1/F = \alpha$ (passing through the focusing element removes linear correlation) and measured beam size will be

$$\sigma_x^2 = \sigma_{x0}^2 + L^2\sigma_{\tilde{x}'0}^2 \quad (10)$$

Because the beam is converging ($\alpha = -1/L$), this will require defocusing element which cannot be realized with solenoids. But we can set focusing so that waist will be achieved in the middle between focusing element and the profile monitor ($1/F = \alpha + 2/L$). In this case beam size at profile monitor will be close to beam size in the focusing element and Eq. (10) can be used as well.

WEIGHTED FIT

Another approach for solving the problem is to utilize the weighted fit. In most cases well-known formulas or standard functions for the polynomial fit are used. Such approach implies that errors in the measurement of beam size are independent of beam size and are constant. The following error function is used for fit evaluation:

$$\chi^2 = \sum\left(\sigma_{meas}^2 - \sigma_{fit}^2\right)^2 \quad (11)$$

It is possible to measure few data points at the fixed focusing element setting to find the variation of the measured beam size.

$$\chi^2 = \sum\frac{\left(\sigma_{meas}^2 - \sigma_{fit}^2\right)^2}{\sigma_{error}^2} \quad (12)$$

Implementing multiple measurements at each setting of focusing element slows down the process. If we assume that relative error in the beam size measurement is relative constant, then we can modify the error function in the following way:

$$\chi^2 = \sum\left(1 - \frac{\sigma_{fit}^2}{\sigma_{meas}^2}\right)^2 \quad (13)$$

Figure 2 shows the fitting curves for the same set of experimental data points with regur and weighted fits.

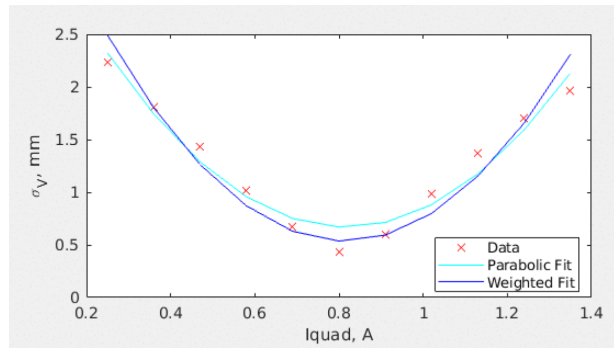

Figure 2: Experimental data of quadrupole scan with regular fit (light blue) and weighted fit (dark blue).

CONCLUSIONS

Two methods are proposed and experimentally tested for increasing robustness of the emittance measurements.

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