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IDENTIFICATION OF THE INTER-BUNCH AND INTRA-BUNCH BEAM DYNAMICS BASED ON DYNAMIC MODAL DECOMPOSITION (DMD)*

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Abstract

The beam dynamics in circular and linear particle accelerators have been studied defining physics-driven / model-driven models that have been used for operation of the machine, diagnostics and feedback system designs. In this paper, a data-driven technique is evaluated to characterize the inter-bunch / intra-bunch beam dynamics in particle accelerators. The dynamic modal decomposition (DMD) is an equation-free, data-driven method capable of providing an accurate decomposition of a complex system into spatiotemporal coherent structures that can be used for short-time future state prediction and control. It does not require knowledge of the underlying governing equations and only uses snapshots in time of observables from historical, experimental, or black-box simulations. The application of the DMD algorithm to particle accelerator cases is illustrated by examples of the collective longitudinal motion of the bunches in a circular storage ring and the transverse motion of a bunch circulating in an accelerator.

INTRODUCTION

The beam dynamics in circular and linear particle accelerators have been studied defining a framework for design and operation of machines as well as the background for future research in the topic [1]. Based on this framework, multiple studies were conducted in order to delineate models of the beam dynamics to create diagnostic tools and design feedback systems to stabilize the beam and improve the machine performance [2]. These physics-driven / model-driven models are commonly used during the operation of the machine and their parameters are obtained via measurements to provide both diagnostic tools to the control room operators and design tools to set the feedback systems.

There is another option to create models for dynamic systems that does not requires the previous knowledge of the physical system. Data-driven modeling and control of complex systems is a field that is having a large impact in engineering and physical sciences. Those complex systems generally evolve on a low-dimensional attractor that can be characterized by spatiotemporal coherent structures. In this paper, we present the dynamic mode decomposition (DMD), and apply it to characterize the intra-bunch and inter-bunch beam dynamics. As example, the analysis of the coupled longitudinal beam dynamics of bunches in a circular accelerator is presented. Results from simulation are compared with results using traditional methods based on model-driven analysis. The analysis of the transverse dynamic of a single bunch is used as an example to show the application of the DMD algorithm to characterize the intra-bunch motion.

DYNAMIC MODAL DECOMPOSITION

Generalities

The DMD method originated in the fluid dynamics community as a method to decompose complex flows into a simple representation based on spatiotemporal coherent structures [3]. The particular characteristic of DMD is that it is an equation-free, data-driven method capable of providing an accurate decomposition of a complex system into spatiotemporal coherent structures that can be used for short-time future state prediction and control. DMD has a number of uses, classified in three primary tasks:

- **Diagnostics.** In particular, the algorithm extracts key low-rank spatiotemporal features of many high-dimensional systems, allowing for physically interpretable results in terms of spatial structures and their associated temporal responses.
- **State estimation, future-state prediction, and system identification.** Another application of the DMD algorithm is associated with using the spatiotemporal structures that are dominant in the data to construct dynamical models of the underlying processes observed.
- **Control.** The ultimate goal of the algorithm is to define viable and robust control strategies directly from the data sampling or the models identified by the algorithm. This is the most challenging task due to the dynamics associated is nonlinear and the DMD creates a linear model based on the data taken.

Background

The main objective is to characterize the intra-bunch / inter-bunch dynamics of the beam based on measurements. The beam dynamics can be represented by a set of ordinary differential equations (ODE),

$$\frac{dx(t)}{dt} = f(x(t)) \qquad \text{with } x \in R^{2n} \text{or } x \in C^n$$

ODEs in general are used to describe the inter-bunch dynamics. Partial differential equation (PDF) are used to represent

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the intra-bunch dynamics of a bunch in the accelerator,

$$\frac{\partial x(z,t)}{\partial t} = g(\frac{\partial x(z,t)}{\partial z}, x(z,t))$$

with $x \in R^{2n}$or $x \in C^n$, and $z \in R^m$. If due to the measurements, the state variables $x(z,t)$ are not evaluated continuously in the space variable z, otherwise at discrete locations in z, then the PDE becomes an ODE. If the data is collected by sampling the state variables at $t = k\Delta T$, with $k = 1, ..., N$ for a total of N measurement times, the ODEs in continuous time domain is transformed in a discrete equation $x_{k+1} = F(x_k)$, with $x_k = x(t)\big|_{t=k\Delta T}$. The beam dynamics can be simplified more if the analysis is conducted around the operation point, then by linearizing $F(x_k)$, the equation can be approximated by $x_{k+1} = Ax_k$. All the information about the beam dynamics is represented by the matrix A. This matrix defines a mapping between the subsequent samples x_k and x_{k+1}.

The DMD algorithm estimates the matrix A and produces a low-rank eigendecomposition of the matrix A that optimally fits the measured trajectory x_k for $k = 1, 2, \ldots, N$ in the least-square sense so that

$$\|x_{k+1} - Ax_k\|_2$$

is minimized across all the points for $k = 1, 2, \ldots, N-1$ [3]. To minimize the approximation error across all the snapshots from $k = 1, 2, \ldots, N$, it is possible to arrange the N snapshots into two large data matrices:

$$X = \begin{bmatrix} | & | & & | \\ x_1 & x_2 & \ldots & x_{N-1} \\ | & | & & | \end{bmatrix}$$

$$X' = \begin{bmatrix} | & | & & | \\ x_2 & x_3 & \ldots & x_N \\ | & | & & | \end{bmatrix}$$

The local linear approximation $x_{k+1} = Ax_k$ may be written in terms of these data matrices as $X' = AX$. The best fit A matrix is given by

$$A = X'X^{\dagger}$$

where $X^{\dagger}$ is the Moore-Penrose pseudoinverse.

To solve the pseudo inverse in case the state dimension n is large, the DMD algorithm circumvents the eigendecomposition of A by considering a rank-reduced representation in terms of a *Proper Order Decomposition*-projected matrix $\tilde{A}$. The algorithm takes the singular value decomposition (SVD) of X, $X = U\Sigma V^*$, where $*$ denotes the conjugate transpose, $U \in C^{n\times n}, \Sigma \in C^{n\times N-1}, V \in C^{N-1\times N-1}$ (In case $x_k \in C^n$ and $X \in C^{n\times N-1}$). The matrices U and V are unitary and the singular values of X are, in descendent order, located in the diagonal of Σ. If the data presents a low order structure, the singular values will decrease sharply to zero defining a limited number of dominant modes. In this case, it is possible to truncate the singular values of X and reduce the system taking into account only those r dominant modes. The decomposition of the matrix X can be approximated by defining $U \in C^{n\times r}, \Sigma \in C^{r\times r}, V \in C^{N-1\times r}$. Thus, the matrix A can be obtained by using the pseudoinverse of X obtained via the SVD:

$$A = X'V\Sigma^{-1}U^*$$

In practice, it is more efficient computationally to estimate $\tilde{A}$, the $r \times r$ projection of the full matrix A onto the *Proper Order Decomposition* modes:

$$\tilde{A} = U^*AU = U^*X'V\Sigma^{-1}.$$

The matrix $\tilde{A}$ defines a low-dimensional linear model of the dynamical system:

$$\tilde{x}_{k+1} = \tilde{A}\tilde{x}_k$$

where the low- and high-dimensional states are related by $x_k = U\tilde{x}_k$.

The matrix $\tilde{A}$ can be eigendecomposed by

$$\tilde{A}W = W\Lambda$$

where the columns of W are the eigenvectors and Λ is a diagonal matrix containing the corresponding eigenvalues λ_i, with $i = 1, \ldots, r$. It is possible to reconstruct the eigendecomposition of A from W and Λ. The eigenvalues of A are given by Λ and the eigenvectors of A are given by the columns of Φ

$$\Phi = X'V\Sigma^{-1}W$$

Based on the low-rank approximation of both the eigenvalues and the eigenvectors, the solution of the states can be estimated for all time in the future. The estimation is given by

$$x_k \approx \sum_{i=1}^{r} \phi_i e^{\lambda_i k\Delta T} b_i = \Phi e^{\Lambda k\Delta T} b \qquad (1)$$

where b_i is the initial condition of each mode, Φ is the matrix whose columns are the eigenvectors ϕ_i and b is a vector of the coefficients b_i.

ANALYSIS OF INTER-BUNCH DYNAMICS

To apply the DMD algorithm to characterize the inter-bunch dynamics, the collective effects of the longitudinal bunch dynamics of a full circular accelerator is analyzed. The example corresponds to the electron ring of the Electron Ion Collider (EIC) under design at Brookhaven National Laboratory. The ring operates at E = 10 GeV and the beam current is $I_{b_{DC}}$ = 2.5 A and there are 17 RF stations operating at 1.27 MV.

The longitudinal inter-bunch dynamic is defined by the coupling between the individual bunches in the ring through the total machine impedance distributed along the ring. In

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particular if the analysis is focused on the low-order longitudinal modes of the beam dynamics, the main source of coupling among bunches is the RF station impedance. Results from simulations consider the RF station configured with the LLRF feedback system optimally setup for impedance minimization but it does not include any one-turn delay (comb) filter. Additionally, there is no longitudinal feedback to stabilize the beam. In this example the beam motion is unstable.

Given the analysis is focused on the low-order longitudinal mode dynamics of the beam, the bunches are represented by a reduced number of macrobunches to reduce the complexity in the simulation. They represent the charge of several bunches, keeping the total current in the ring equal to nominal. In the simulations, the full ring is represented by 20 macrobunches.

Figure 1 shows the unstable longitudinal motion of all the macrobunches. To compare the DMD technique with the traditional model-driven analysis used to study the bean motion and stability, the data is also processed by transforming the longitudinal beam motion into the modal domain using the *n even-filled bunch* base [2]. The motion in the modal domain is

$$\varphi_m(t) = \frac{1}{n}\sum_{\ell=1}^{n} \phi_{B_\ell}(t) e^{-j2\pi \frac{m\ell}{n}}$$

with $\varphi_m(t)$ motion for the m-mode, $\phi_{B_\ell}(t)$ longitudinal motion around the synchronous phase of the ℓ-bunch and n the total number of macrobunches. Figure 2 depicts the beam motion in the modal domain, where a slow growing mode 0 and dominant unstable modes -3, -2, and -4 can be observed.

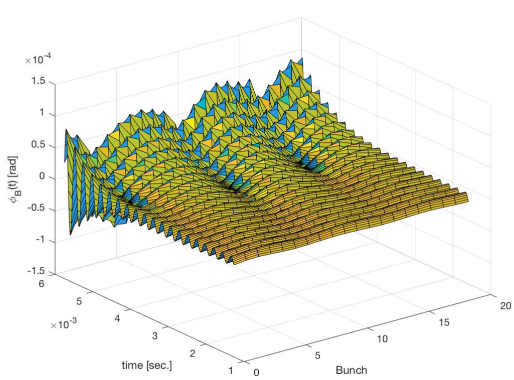

Figure 1: Longitudinal motion of the bunches.

Applying the DMD algorithm described in the previous section to the data, it is possible to extract the dominant modes of the beam dynamics. These modes are characterized by the eigenvalues

$$\lambda_{-3} = 1198 + j2\pi 4.674e3\ sec^{-1}$$
$$\lambda_{-2} = 818 + j2\pi 4.389e3\ sec^{-1}$$
$$\lambda_{-4} = 106 + j2\pi 4.611e3\ sec^{-1}$$

To validate the model obtained by the DMD algorithm, the time evolution of the longitudinal displacement obtained

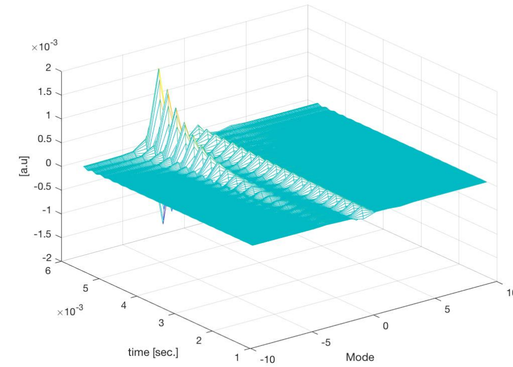

Figure 2: Motion in the modal domain.

by the estimation defined by Eq. (1) is compared with the original data displayed in Fig.1. Figure 3 depicts the estimated phase angle for all the macrobunches, matching the time evolution of the original data shown in Fig.1.

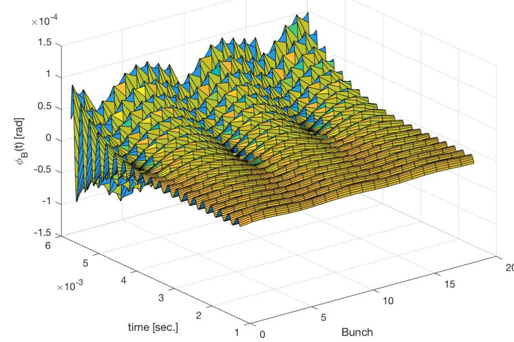

Figure 3: Estimated longitudinal motion of the bunches.

ANALYSIS OF INTRA-BUNCH DYNAMICS

The application of the DMD technique to analyze the intra-bunch dynamics is based on measurements of the transverse bunch motion in the SPS ring at CERN. To measure the vertical motion along the bunch, the acquisition stage of a 3.2 GSamp/sec processing system [4–6] is synchronized with the bunch and able to acquire 16 samples along the 5 nsec bucket. The signal from the exponential pick-up installed in the ring is processed to equalize the cable attenuation and determine the dipole motion of the bunch. This acquired signal corresponds to the product of the vertical motion $y(z)$, at each particular location in the longitudinal dimension z of the bunch, and the bunch charge $Q(z)$ at that particular coordinate.

During these tests, three different lattices have been used the $Q20$, $Q22$, and $Q26$, in particular for this measurement the $Q22$ was set in the machine. Some characteristic parameters of the bunch motion due to this lattice are, vertical fractional tune ≈ 0.185 and longitudinal fractional tune ≈ 0.011.

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The motion of the bunch measured is unstable and the transverse displacement in time is depicted in Fig. 4. The data shows the motion for the 16 samples acquired, where sample 0 corresponds to the head of the bunch and sample 16 to the tail. The bunch exhibits an unstable 'head-tail' motion.

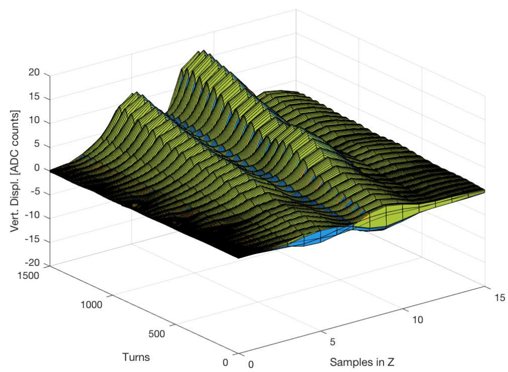

Figure 4: Transverse motion along the bunch.

Applying DMD to characterize and reduce the bunch dynamics, the estimated vertical motion using Eq. (1) for the same initial condition defined in Fig. 4, is depicted in Fig. 5. Comparing both figures, the measurements and the estimated motions match showing that the model extracted by the DMD algorithm defines the characteristics of the bunch for that motion.

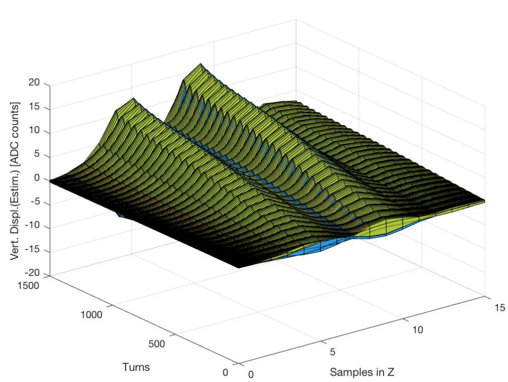

Figure 5: Estimated transverse motion along the bunch.

Further details of this analysis can be presented. Figure 6 depicts the n singular values of the matrix X (or matrix Σ) showing only $r = 2$ dominant modes to characterize this motion. The data displayed in Fig. 6 includes only the 2 dominant modes to approximate the representation. These modes have the following eigenvalues:

$$\lambda_a = 0.0013 + j2\pi 0.196\, 1/rev$$
$$\lambda_b = 0.0001 + j2\pi 0.1965\, 1/rev$$

The associated eigenvectors (or discrete eigenfunctions) for each mode are depicted in Fig. 7. They all have similar shapes. Observing the real part of the eigenvalues, it is possible to conclude that the growth rate of λ_a is dominant

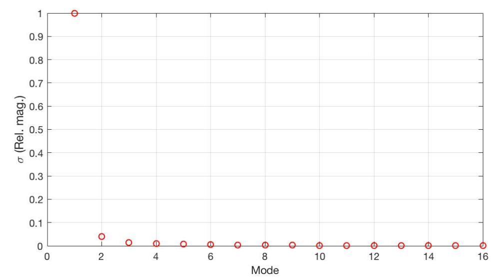

Figure 6: Singular Values of matrix X.

with respect to the one of λ_b. If the bunch motion is free of noise, the analysis will define only one dominant mode characterized by the eigenvalue λ_a. Due to noise and perturbations in the components settling the transverse motion of the bunch, this motion has a modulation that is captured and approximated by the mode defined by the eigenvalue λ_b.

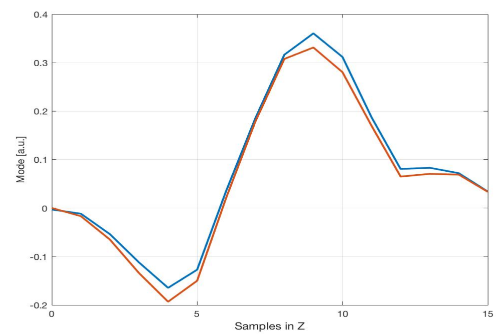

Figure 7: Eigenvectors (discrete eigenfunctions) for the dominant modes.

CONCLUSIONS

The dynamic modal decomposition is applied in this work to identify the inter-bunch / intra-bunch dynamics of the beam in particle accelerators. The method can be applied to create analysis and diagnostic tools and could be extended to identify the beam model to design feedback systems to stabilize the beam motion.

The advantage of this technique is that it is model-free, the characterization of the dynamics does not require the knowledge of the eigenstructure of the system dynamics. Based on measurements or simulation results the eigenvalues - eigenvectors (or discrete eigenfunctions) can be estimated for the dominant modes of the motion.

This technique has been applied successfully in other fields dealing with ordinary or partial differential equations to describe the system dynamics.

Further research is necessary to improve the method and create diagnostic tools and beam model identification.

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