WEPPO42

Measurement of the Second Moments of Transverse Beam Distribution with Solenoid Scan

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Abstract
Measurement of the dependence of the beam size on profile monitor vs. strength of a focusing element is widely used for measurement of the beam parameters. Such measurements are mostly used for the separate planes and assumption that beam satisfied Gaussian distribution. In many linear accelerators the transverse beam dynamics is coupled between planes and distribution is far from the Gaussian. We developed measurement technique of the second moments of beam distribution which does not rely on any assumptions.

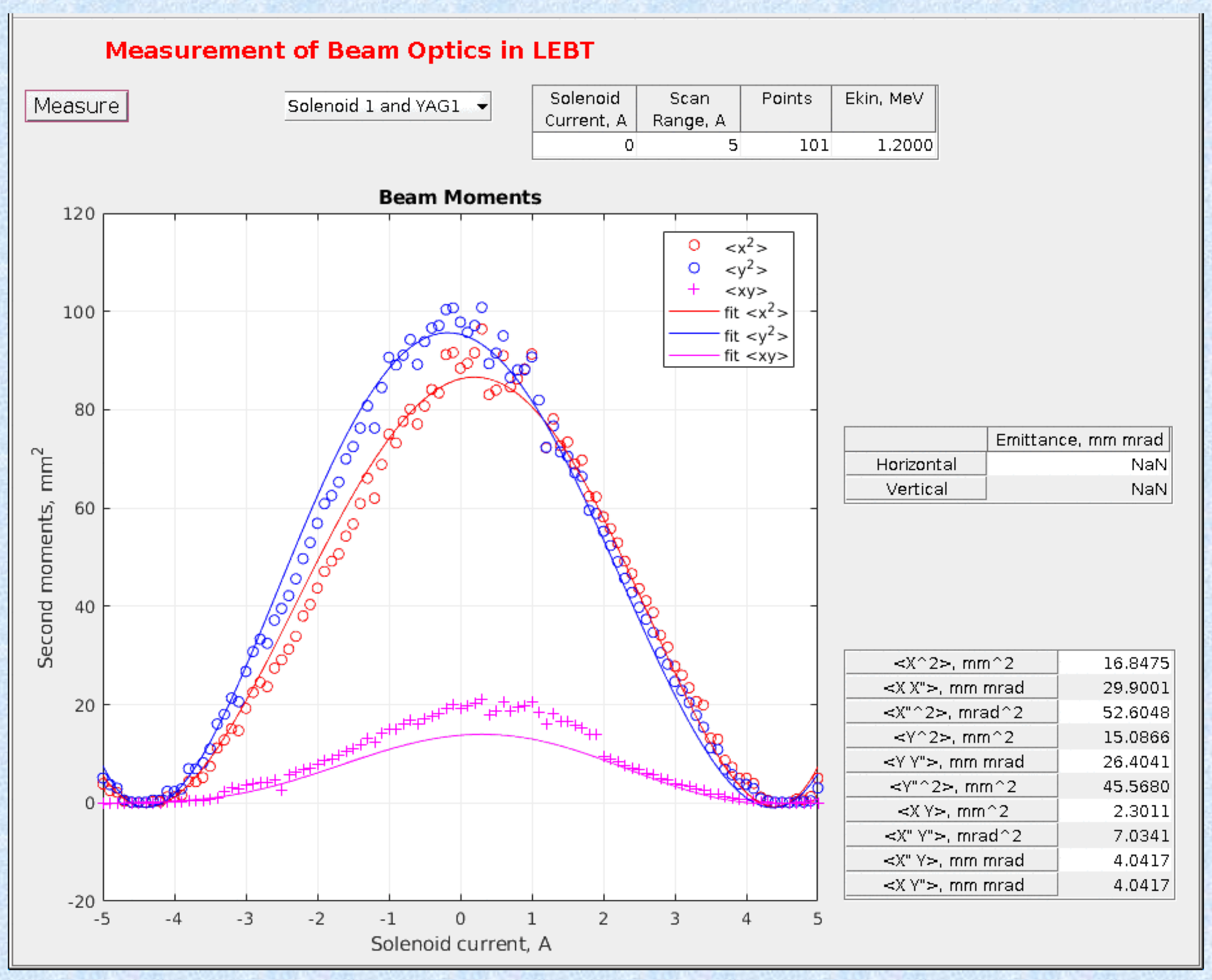
$$\langle x^2\rangle_1 = R_{11}^2\langle x_0^2\rangle + 2R_{11}R_{12}\langle x_0x'_0\rangle + R_{12}^2\langle x_0'^2\rangle + R_{13}^2\langle y_0^2\rangle + 2R_{13}R_{14}\langle y_0y'_0\rangle + R_{14}^2\langle y_0'^2\rangle + 2R_{11}R_{13}\langle x_0y_0\rangle + 2R_{12}R_{14}\langle x'_0y'_0\rangle + 2R_{11}R_{14}\langle x_0y'_0\rangle + 2R_{12}R_{13}\langle x'_0y_0\rangle$$

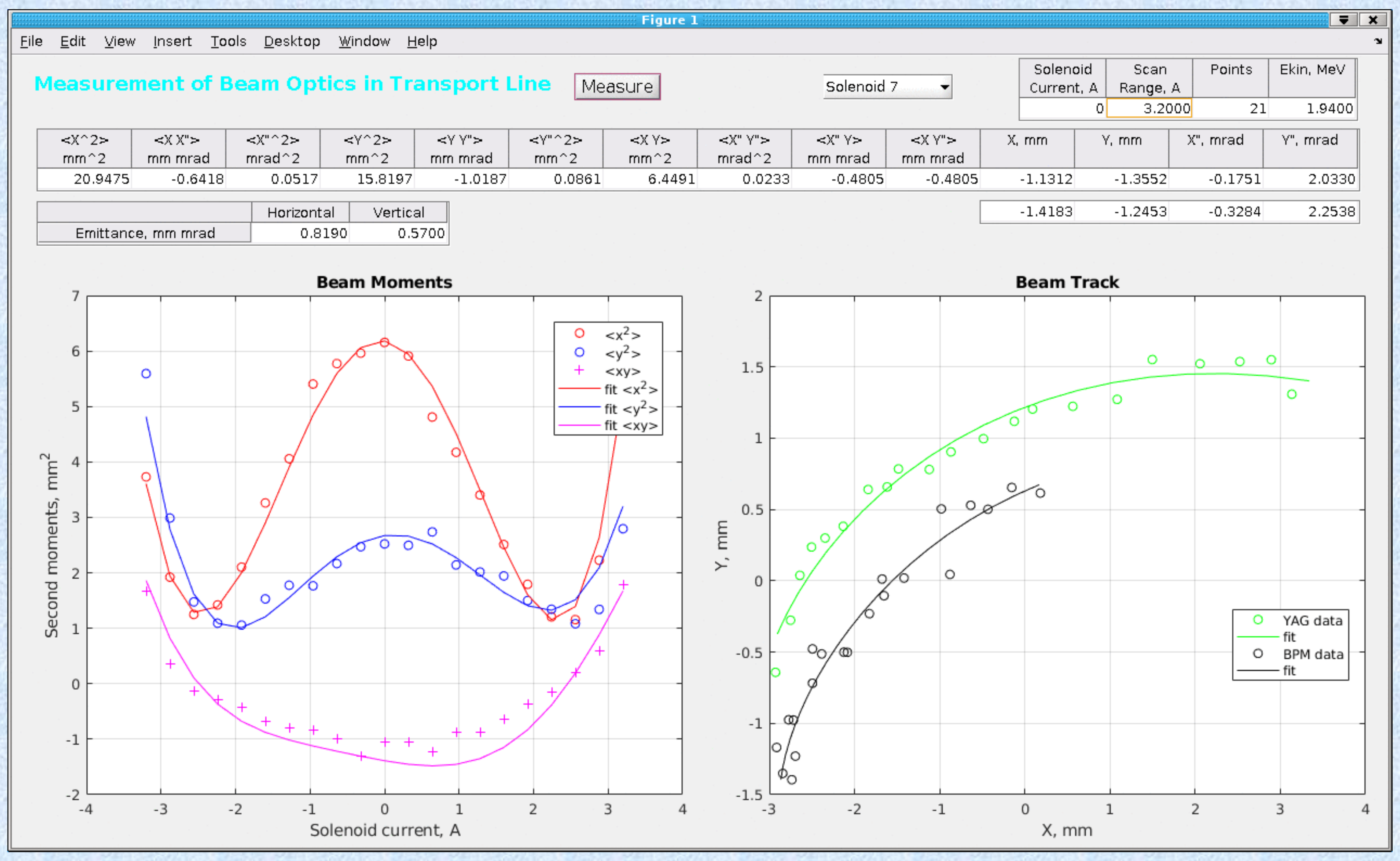
$$\langle y^2\rangle_1 = R_{31}^2\langle x_0^2\rangle + 2R_{31}R_{32}\langle x_0x'_0\rangle + R_{32}^2\langle x_0'^2\rangle + R_{33}^2\langle y_0^2\rangle + 2R_{33}R_{34}\langle y_0y'_0\rangle + R_{34}^2\langle y_0'^2\rangle + 2R_{31}R_{33}\langle x_0y_0\rangle + 2R_{32}R_{34}\langle x'_0y'_0\rangle + 2R_{31}R_{34}\langle x_0y'_0\rangle + 2R_{32}R_{33}\langle x'_0y_0\rangle$$

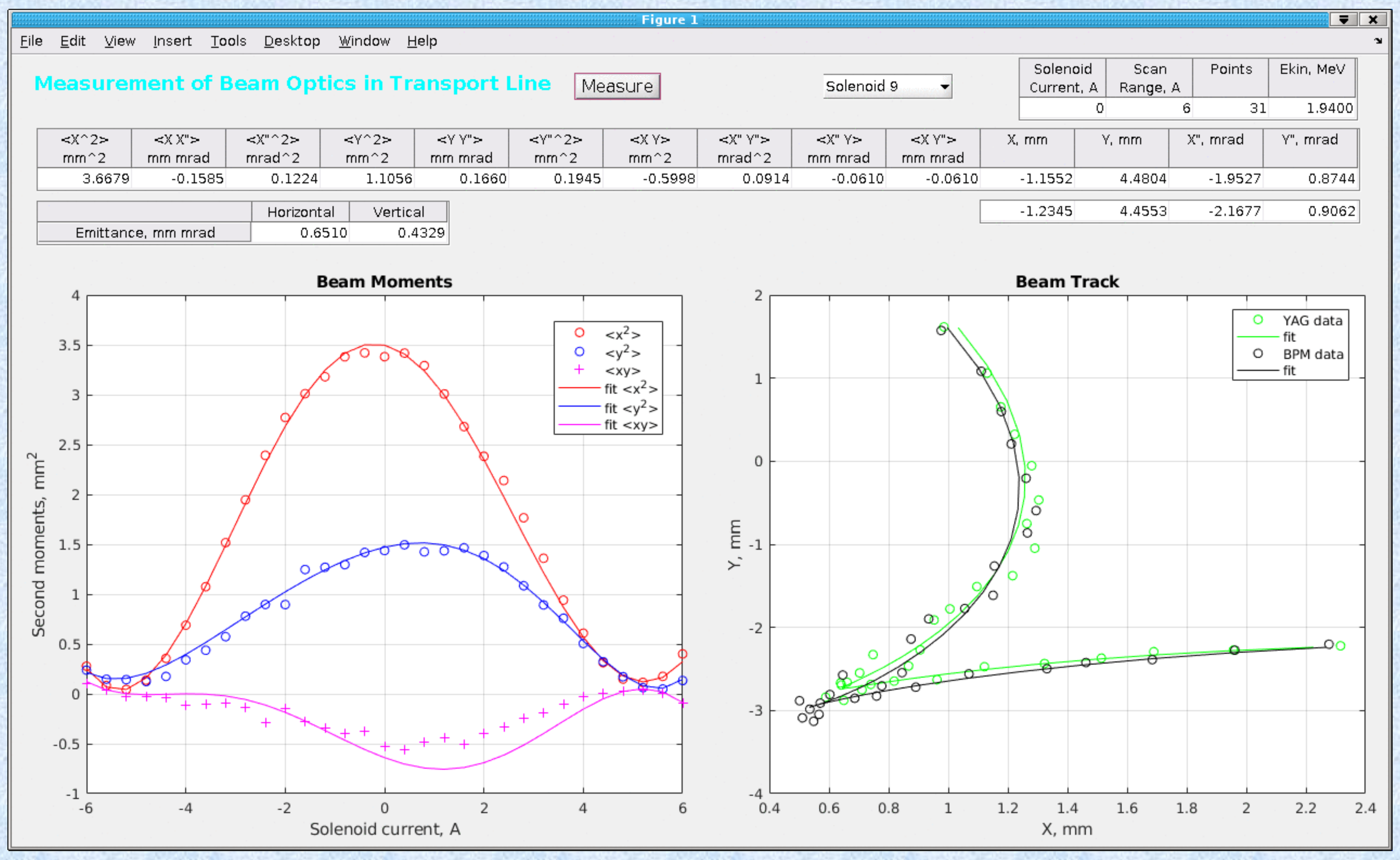
$$\langle xy\rangle_1 = R_{11}R_{31}\langle x_0^2\rangle + (R_{11}R_{32}+R_{12}R_{31})\langle x_0x'_0\rangle + R_{12}R_{32}\langle x_0'^2\rangle + R_{13}R_{33}\langle y_0^2\rangle + (R_{13}R_{34}+R_{14}R_{33})\langle y_0y'_0\rangle + R_{14}R_{34}\langle y_0'^2\rangle + (R_{11}R_{33}+R_{13}R_{31})\langle x_0y_0\rangle + (R_{12}R_{34}+R_{14}R_{32})\langle x'_0y'_0\rangle + (R_{11}R_{34}+R_{14}R_{31})\langle x_0y'_0\rangle + (R_{12}R_{33}+R_{13}R_{32})\langle x'_0y_0\rangle$$

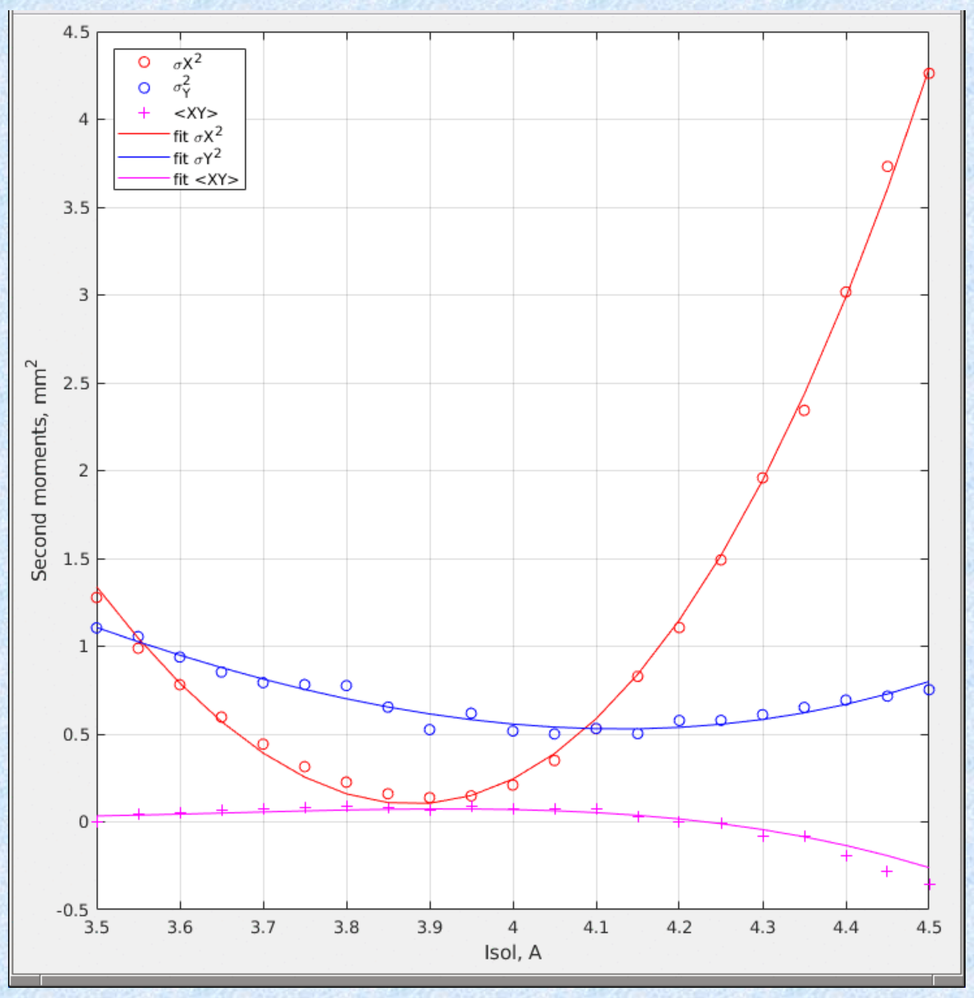
The second moments can be by solving the system of linear equations when at least four measurements were performed. However, matrix U is rank deficient. No matter how many experimental points we have its rank is 9. Analysis showed that this is a fundamental feature of the system containing only solenoids and drifts. Neither transfer matrix of solenoid nor transfer matrix of drift does not change $x'y - xy'$. To resolve these moments, we need to add a quadrupole into the transport line. We do not have any quadrupole in the transport, therefore in the analysis of the experimental results we assumed that $\langle x'y\rangle = \langle xy'\rangle$.
It should be noted that the approach we used is suitable for emittance dominated beams. The substantial space charge forces will introduce systematic errors which are behind the scope of this paper.

$$\begin{pmatrix}\langle x^2\rangle_1\\ \langle y^2\rangle_1\\ \langle xy\rangle_1\\ \cdots\\ \langle x^2\rangle_N\\ \langle y^2\rangle_N\\ \langle xy\rangle_N\end{pmatrix} = \mathrm{U}\begin{pmatrix}\langle x_0^2\rangle\\ \langle x_0x'_0\rangle\\ \langle x_0'^2\rangle\\ \langle y_0^2\rangle\\ \langle y_0y'_0\rangle\\ \langle y_0'^2\rangle\\ \langle x_0y_0\rangle\\ \langle x'_0y'_0\rangle\\ \langle x_0y'_0\rangle\\ \langle x'_0y_0\rangle\end{pmatrix}$$









Measurement of the second moments of the 1.6 MeV electron beam distribution in the LeREC accelerator (0.2 A peak current) with solenoid and screen placed 0.76 m downstream. The found second moments are: $\langle x^2\rangle$=27.2 mm², $\langle xx'\rangle$=0.81 mm mrad, $\langle x'^2\rangle$=0.03 mrad2, $\langle y^2\rangle$=6.1 mm², $\langle yy'\rangle$=0.32 mm mrad, $\langle y'^2\rangle$=0.05 mrad², $\langle xy\rangle$=6.0 mm², $\langle x'y'\rangle$=0.0 mrad², $\langle x'y\rangle$=$\langle xy'\rangle$=0.13 mm mrad.

If the values for the second moments are used for emittance calculations (neglecting cross-plane elements) the value of horizontal r.m.s. emittance is 0.4 mm mrad and the vertical r.m.s. emittance is 0.43 mm mrad.