# Virtual Pepper-Pot Technique for 4D Phase Space Measurements.


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## Abstract

A novel method for 4-dimensional transverse beam phase space measurement is proposed at the Photo Injector Test facility at DESY in Zeuthen (PITZ) for ongoing beam coupling studies. This method is called Virtual Pepper-Pot (VPP), because key principles of the pepper-pot mask scheme are applied. The latter approach is of limited use in high-brightness photo injectors, because of technical reasons. At PITZ a slit scan method instead is the standard tool for reconstruction of horizontal and vertical phase spaces. The VPP method extends the slit scan technique with a special post-processing. The 4D transverse phase space is reconstructed from a pepper-pot like pattern that is generated by crossing each measured horizontal slit beamlet with all measured vertical slit beamlets. All elements of the 4D transverse beam matrix are calculated and applied to obtain the 4D transverse emittance, 4D kinematic beam invariant and coupling factors. The proposed technique has been applied to experimental data from the PITZ photo injector optimization for 0.5 nC bunch charge. Details of the VPP technique and results of its application will be discussed.


## Theory

**4D transverse beam matrix**

A main theoretical tool to describe the 4D beam dynamics is the 4D transverse beam matrix

$$\sigma^{4D} = \begin{pmatrix} \langle xx\rangle & \langle x'x\rangle & \langle yx\rangle & \langle y'x\rangle \\ \langle xx'\rangle & \langle x'x'\rangle & \langle yx'\rangle & \langle y'x'\rangle \\ \langle xy\rangle & \langle x'y\rangle & \langle yy\rangle & \langle y'y\rangle \\ \langle xy'\rangle & \langle x'y'\rangle & \langle yy'\rangle & \langle y'y'\rangle \end{pmatrix}$$

with elements $\langle uu\rangle$ and $\langle uv\rangle$ representing a variance of $u$ and a covariance between $u$ and $v$ respectively.

**Emittance and emittance invariant**

The projected horizontal emittance (and analogycally for vertical emittance) is defined as

$$\epsilon_{x,\mathrm{scaled,normalized}} = f_{\mathrm{scaling}}\beta\gamma\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

with relativistic factors product $\beta\gamma$ and scaling factor defined as the ratio of the full beam size to the reconstructed beam size

$$f_{x,\mathrm{scaling}} = \frac{\sigma_{x,\mathrm{fullbeam}}}{\sigma_{x,\mathrm{slitscan}}}$$

The 4D emittance is defined as

$$\epsilon_{\mathrm{4D,scaled,normalized}} = f_{x,\mathrm{scaling}} f_{y,\mathrm{scaling}} (\beta\gamma)^2 \sqrt{\det(\sigma^{4D})}$$

A transverse emittance invariant is defined as

$$I_2^{(2)} = \epsilon_x^2 + \epsilon_y^2 + 2\begin{vmatrix} \langle xy\rangle & \langle x'y\rangle \\ \langle xy'\rangle & \langle x'y'\rangle \end{vmatrix}$$

**X-Y correlation and coupling factors**

A correlation value between horizontal phase space and vertical phase space is introduced as

$$\rho_{4D} = \sqrt{1 - \left(\frac{\epsilon_{4D}}{\epsilon_x\epsilon_y}\right)^2} \quad [2]$$

Its value ranges between 0 (no correlation) and 1 (fully correlated) in analogy to Pearson's coefficient.

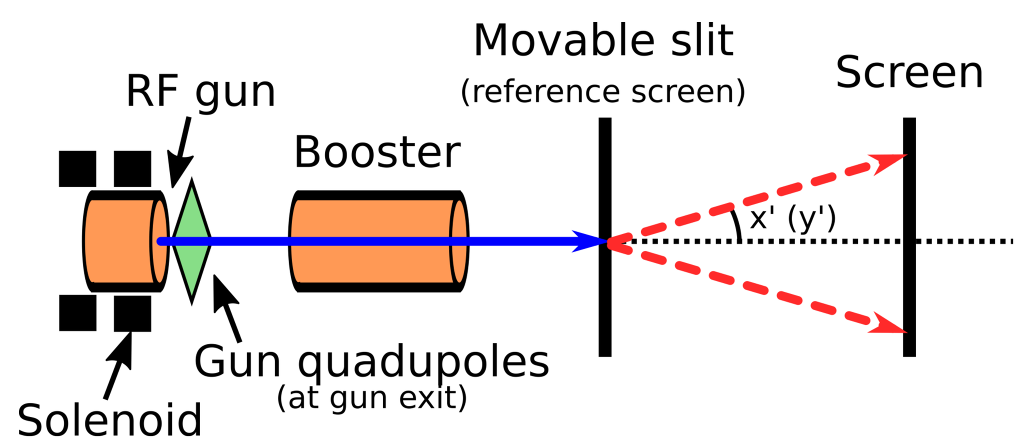
Two coupling factors are shown for comparison.

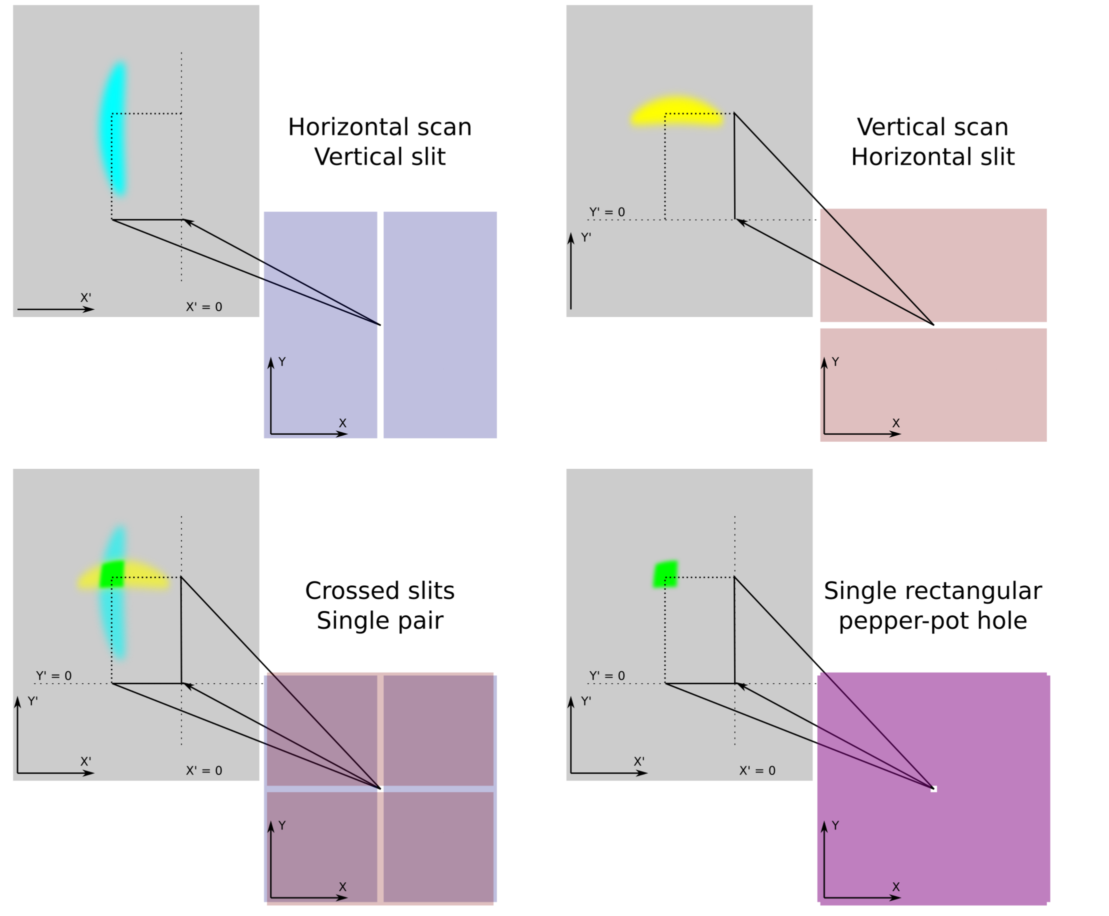
**Coupling 1** $$C = \sqrt{\frac{\epsilon_x\epsilon_y}{\epsilon_{4D}}} - 1 \quad [3]$$

**Coupling 2** $$t = \frac{\epsilon_x\epsilon_y}{\epsilon_{4D}} - 1 \quad [4]$$

## Intoduction

**The Virtual-Pepper Pot technique is analysis technique for measurement of 4D transverse phase space and 4D projected emittance of space-charge dominated electron beams. A step called beamlet crossing is crucial in the analysis. Images of beamlets from single slit scan in both horizontal and vertical direction are combined. The generated image by beamlet crossing resembles an image of a single aperture.**

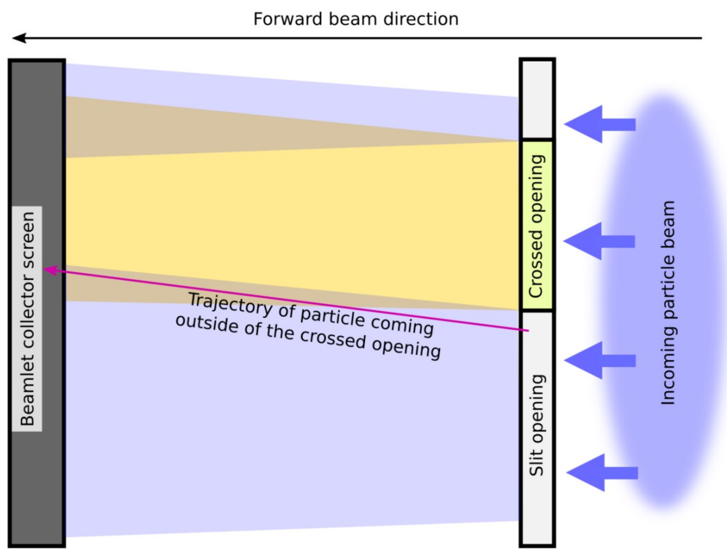



## Challenges

**Foreign charge**

For any virtually crossed slit pair only a beam charge $Q_0$ passes through to the second screen. During measurement there is no slit crossing and the second screen image has $Q_0$ mixed with the rest of the passing charge $Q_f$. The $Q_f$ is called foreign charge, because it is not part of the virtually crossed slit charge. The images of $Q_0$ and $Q_f$ on the second screen can overlap.

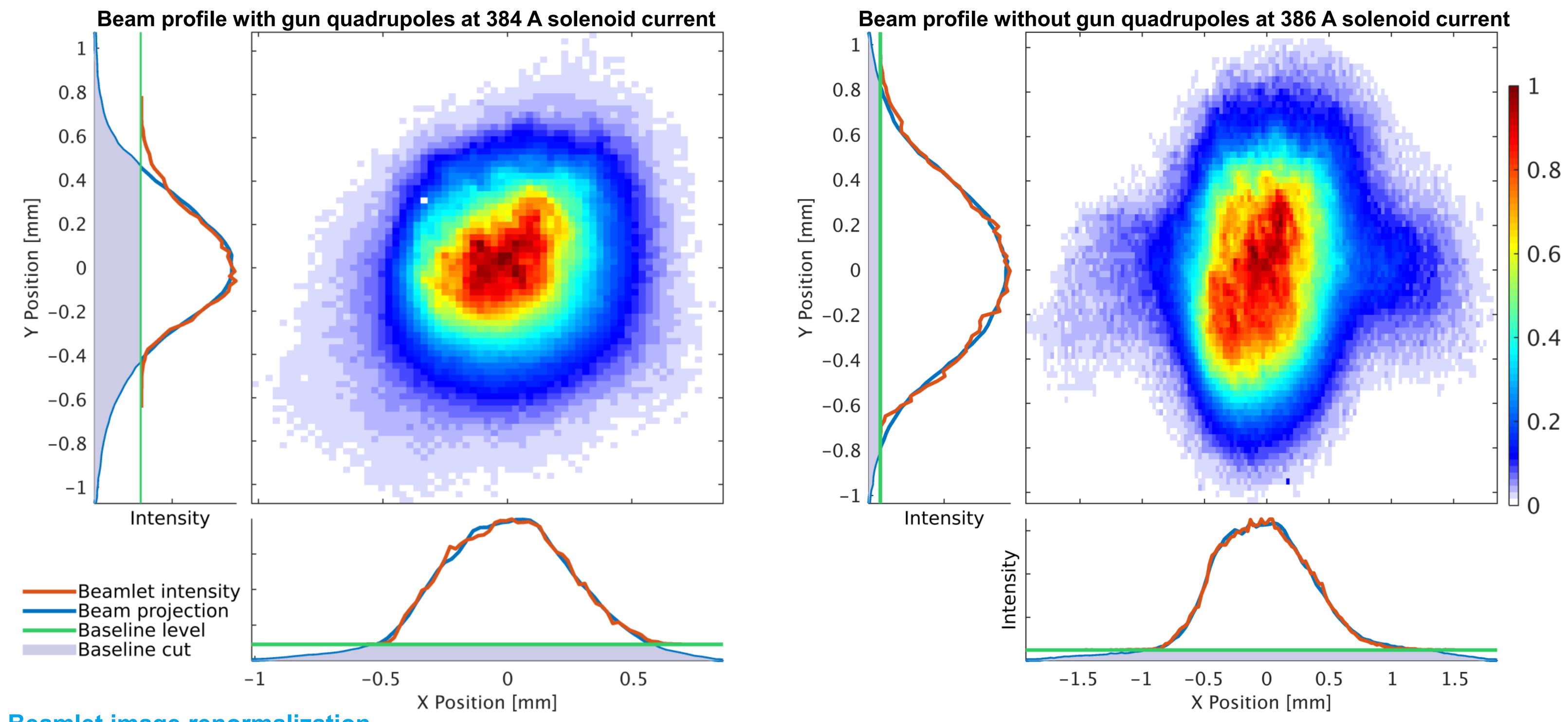



The beamlet crossing step has to isolate $Q_0$ from $Q_f$ as much as possible. The measured beamlet images of any vertical slit and any horizontal slit have separate foreign charges $Q_{fx}$ and $Q_{fy}$. The pixel-wise minimum operation is used for beamlet crossing.

$$Q_{\mathrm{cross}} = \min(Q_x, Q_y) = Q_0 + min(Q_{fx}, Q_{fy})$$

**Charge cut**

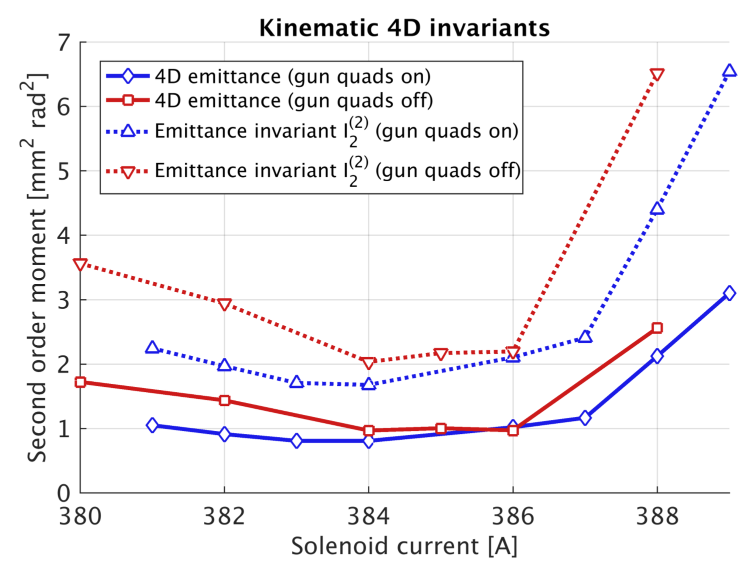
At PITZ, the charge cut value estimates what fraction of the beam is not included in the analysis after a single slit scan measurement. A fit to a reference beam image is used to get the charge cut.

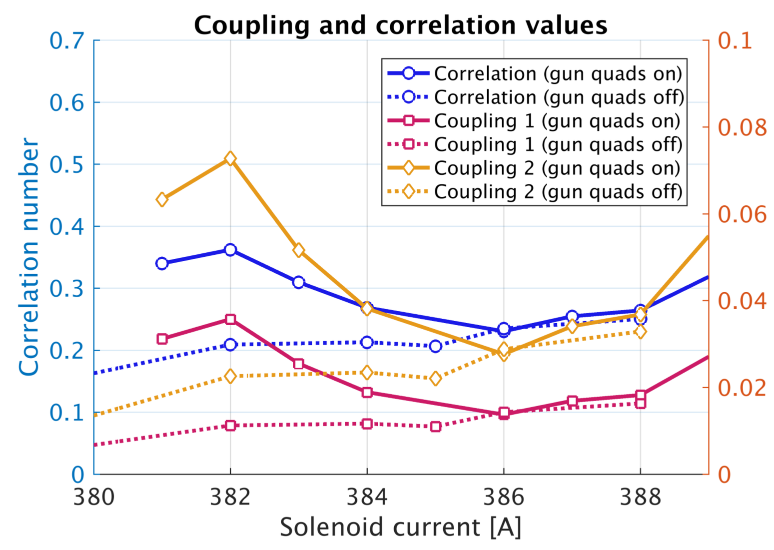


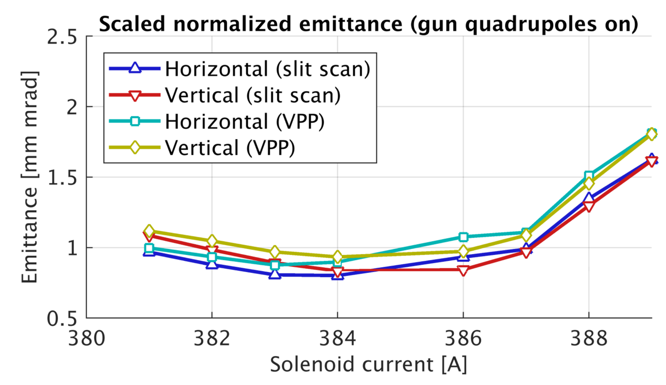

**Beamlet image renormalization**

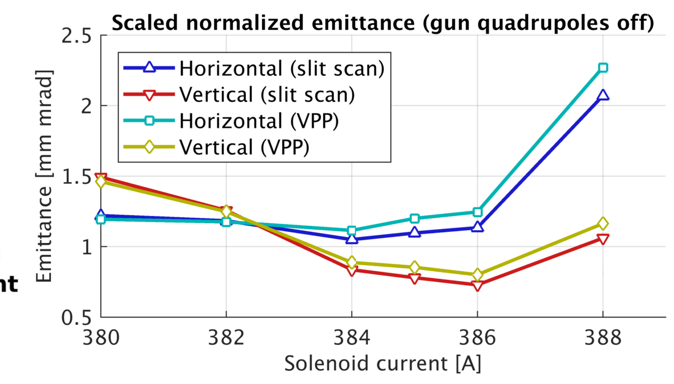
The pixel-wise minimum operation requires equivalently normalized beamlet images. The ratio of the horizontal emittance and vertical emittance from the slit scans is used as a reference point for renormalization.

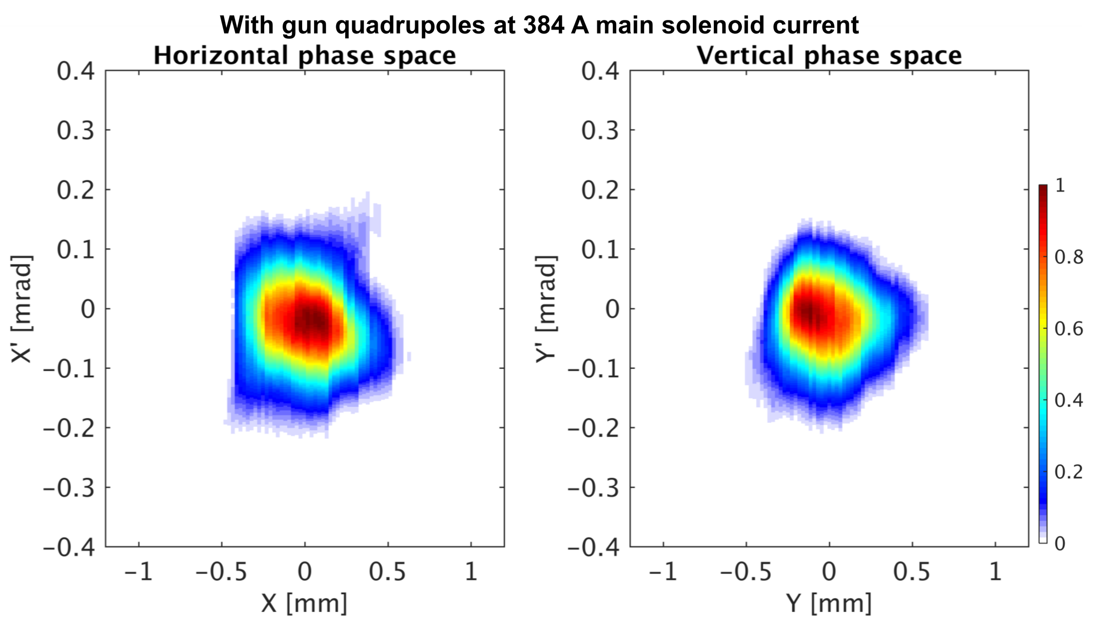
## Results [1]

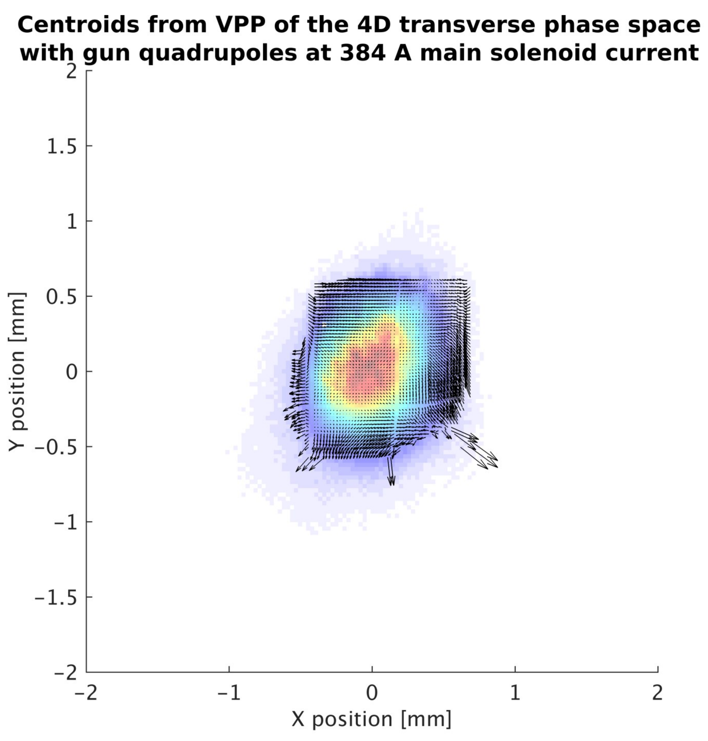


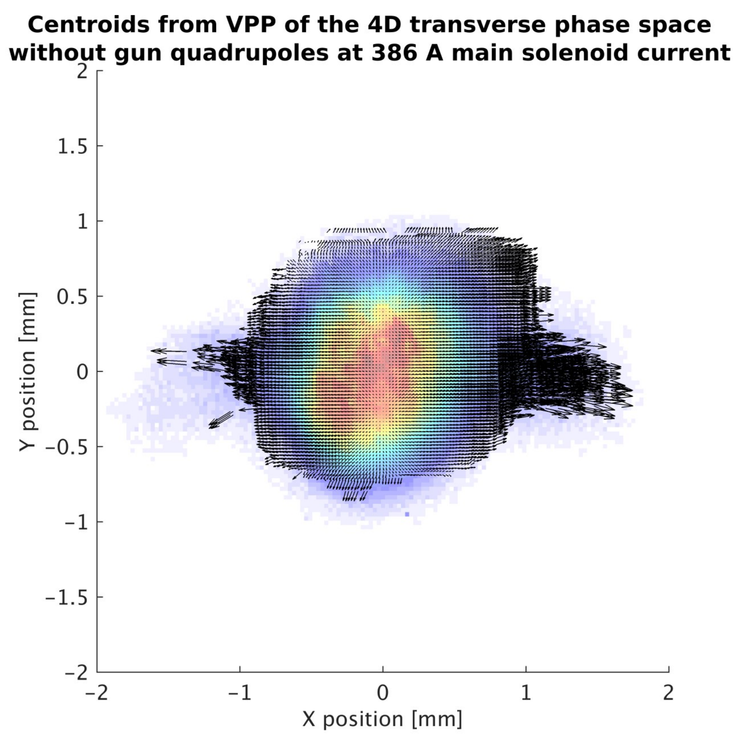



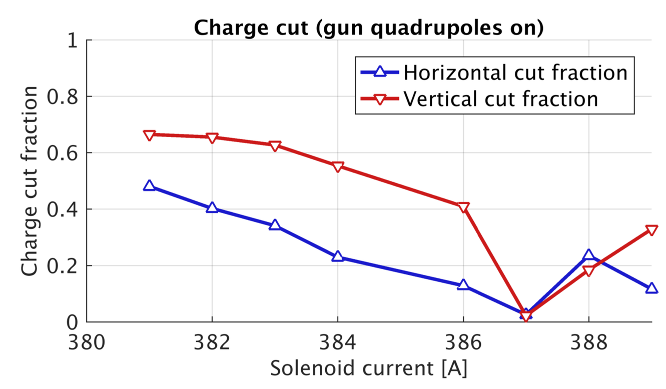



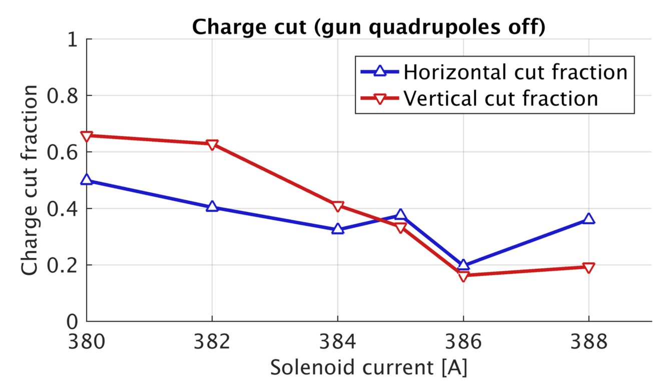



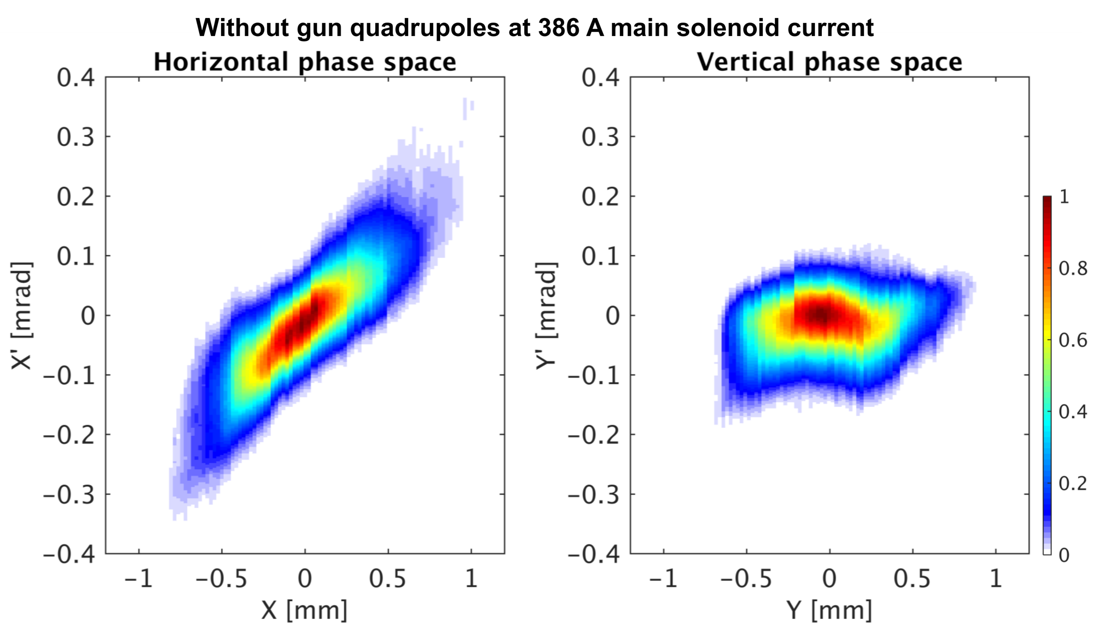




## Conclusion

> 4D phase space information extracted

> Projected emittance from VPP overestimates single slit results (systematic error)

> Analysis of gun quadrupoles in this setup did not reveal coupling compensation, but 4D emittance reduction is obtained

> Further studies on charge cut effects on 4D phase space, 4D emittance, coupling terms are planned


## References

[1] M. Krasilnikov *et al.*, "Electron beam asymmetry compensation with gun quadrupoles at PITZ".

[2] G. Rangarajan, F. Neri and A. Dragt, "Generalized emittance invariants".

[3] E. Prat and M. Aiba, "Four-dimensional transverse beam matrix measurement using the multiple-quadrupole scan technique".

[4] Q. Zhao *et al.*, "Beam Transverse Coupling and 4D Emittance Measurement Simulation Studies for PITZ".