Quadrupole Scan Transverse Emittance Measurements at ELBE

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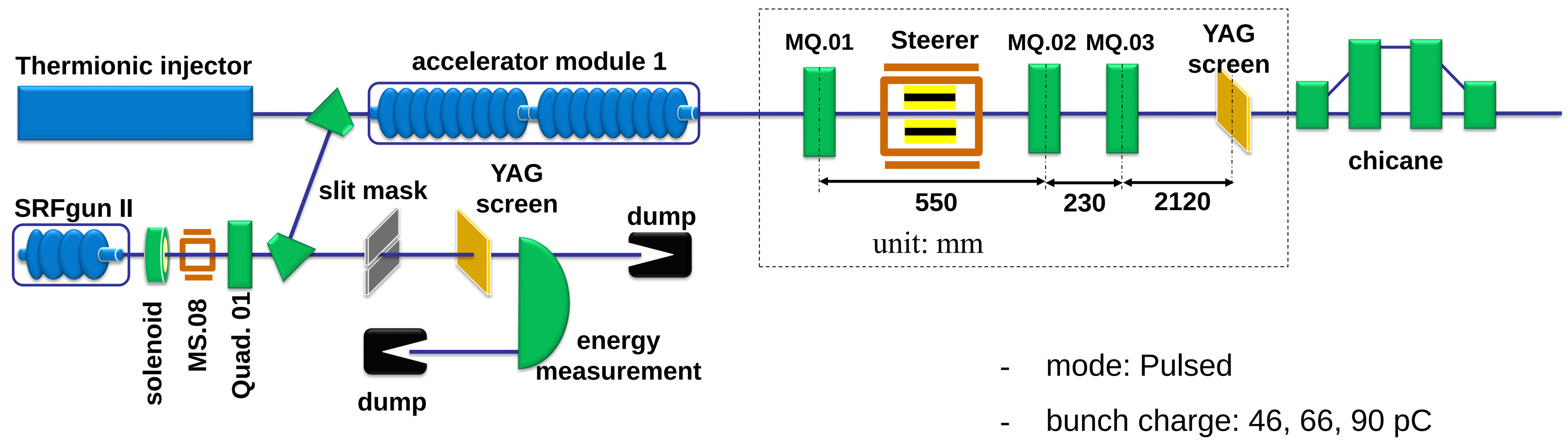
Introduction

Two quadrupoles and one screen are used for beam transverse emittance measurements at HZDR ELBE. The emittance calculated with two different methods, one with thin-lens approximation and the other one without this approximation. The results are compared and analized. To analyze the measurement error, quadrupole calibration is need. Two aspects about quadrupole analysis are made. The first one is thin-lens approximation error and the second one is quadrupole's converged or diverged ability in reality.

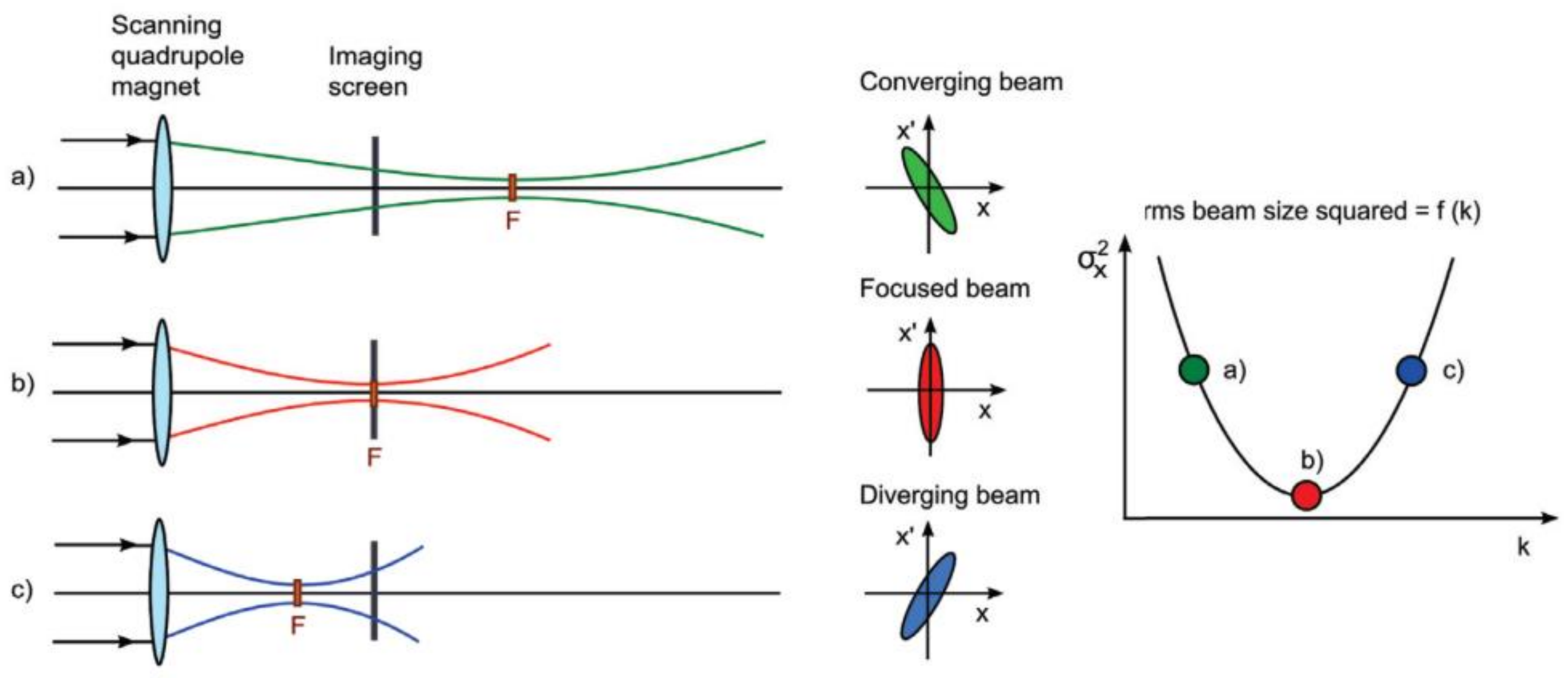
Experimental layout



- mode: Pulsed
- bunch charge: 46, 66, 90 pC
- Energy = 15.885MeV
- Bunch length ~ 2 ps

Transverse Emittance Measurement

Quad-scan method



- change the quadrupole current
- photograph the images on screen
- integrate the distribution and calculate beam rms sizes
- fit beam rms sizes as quadrupole strength

$$\Sigma = \begin{pmatrix}\sigma_{11} & \sigma_{12}\\ \sigma_{21} & \sigma_{22}\end{pmatrix} \quad \Sigma^s = M\Sigma^q M^T$$

$$\sigma_{11} = \langle x_i^2\rangle = \epsilon\beta,\ \sigma_{22} = \langle x'^2_i\rangle = \epsilon\gamma,$$

$$\sigma_{12} = \sigma_{21} = \langle x_i x'_i\rangle = -\epsilon\alpha.\ \langle x_i^2\rangle$$

$$\epsilon = \det(\Sigma) = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2}$$

$$\epsilon_n = \beta\gamma\epsilon$$

Transport matrix

- Converge

$$M = \begin{bmatrix}1 & d\\ 0 & 1\end{bmatrix}\begin{bmatrix}\cos(\sqrt{k}l) & \frac{1}{\sqrt{k}}\sin(\sqrt{k}l)\\ -\sqrt{k}\sin(\sqrt{k}l) & \cos(\sqrt{k}l)\end{bmatrix}$$

$$\sigma_{11}^s = (\sigma_{11}^q + d^2\sigma_{22}^q + 2d\sigma_{12}^q)cos^2(\sqrt{k}l) + \frac{-2d\sigma_{11}^q - 2d^2\sigma_{12}^q}{l}(\sqrt{k}l)\sin(\sqrt{k}l)\cos(\sqrt{k}l) + \frac{d^2\sigma_{11}^q}{l^2}(\sqrt{k}l)^2 sin^2(\sqrt{k}l) + \frac{l^2\sigma_{22}^q}{(\sqrt{k}l)^2}sin^2(\sqrt{k}l) + (2d\sigma_{22}^q + 2\sigma_{12}^q)l\cdot\frac{1}{\sqrt{k}l}\sin(\sqrt{k}l)\cos(\sqrt{k}l) + (-2d\sigma_{12}^q)\ sin^2(\sqrt{k}l)$$

- Diverge

$$M = \begin{bmatrix}1 & d\\ 0 & 1\end{bmatrix}\begin{bmatrix}\cos h(\sqrt{k}l) & \frac{1}{\sqrt{k}}\sin h(\sqrt{k}l)\\ \sqrt{k}\sinh(\sqrt{k}l) & \cos h(\sqrt{k}l)\end{bmatrix}$$

$$\sigma_{11}^s = (\sigma_{11}^q + d^2\sigma_{22}^q + 2d\sigma_{12}^q)cosh^2(\sqrt{k}l) + \frac{2d\sigma_{11}^q + 2d^2\sigma_{12}^q}{l}(\sqrt{k}l)\sin h(\sqrt{k}l)\cosh(\sqrt{k}l) + \frac{d^2\sigma_{11}^q}{l^2}(\sqrt{k}l)^2 sinh^2(\sqrt{k}l) + \frac{l^2\sigma_{22}^q}{(\sqrt{k}l)^2}sinh^2(\sqrt{k}l) + (2d\sigma_{22}^q + 2\sigma_{12}^q)\frac{l}{\sqrt{k}l}\sin h(\sqrt{k}l)\cos h(\sqrt{k}l) + 2d\sigma_{12}^q sinh^2(\sqrt{k}l)$$

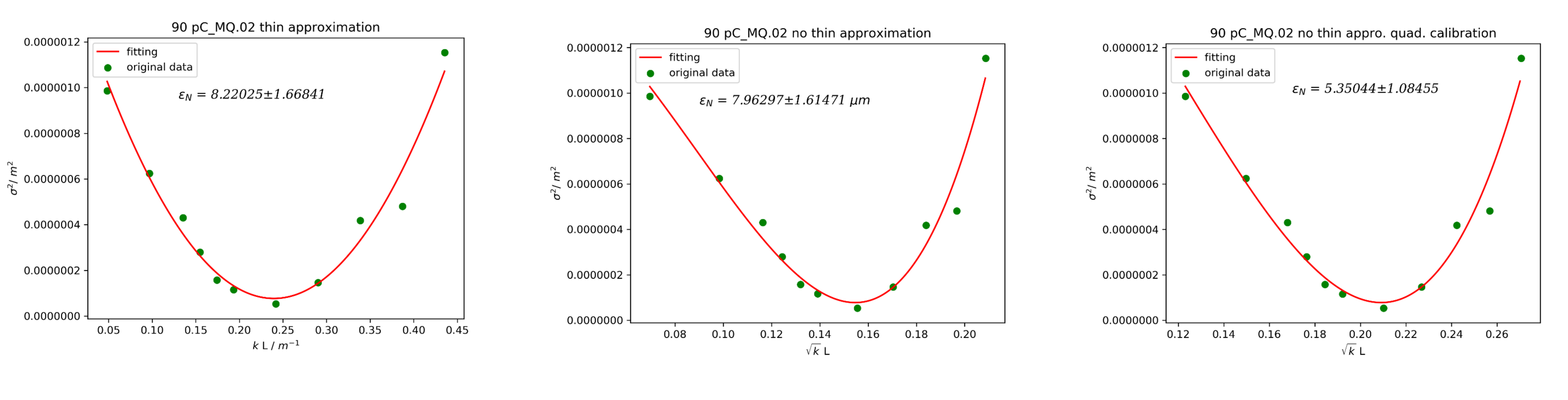
- Thin-lens approximation

$$M = \begin{pmatrix}1 & d\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \mp kl & 1\end{pmatrix}$$

$$\sigma_{11}^s = (\sigma_{11}^q d^2 l^2)k^2 + (2dl\sigma_{11}^q \mp 2d^2 l\sigma_{12}^q)k + \sigma_{11}^q + 2d\sigma_{12}^q + d^2\sigma_{22}^q$$

Emittance results and Conclusions

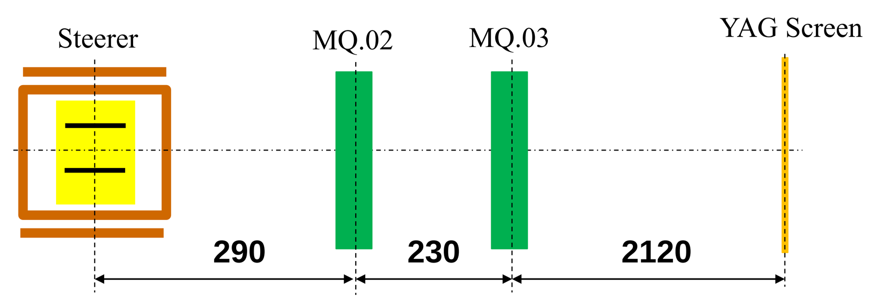
Emittance / um	Thin-lens approximation		No thin-lens approximation		Quad. calibration	
	MQ.02	MQ.03	MQ.02	MQ.03	MQ.02	MQ.03
46 pC	5.30 ± 0.94	10.81 ± 4.55	5.12 ± 0.92	10.39 ± 4.39	3.43 ± 0.62	6.59 ± 2.79
66 pC	4.99 ± 1.16	5.58 ± 2.04	4.81 ± 1.11	5.37 ± 1.97	3.22 ± 0.74	3.40 ± 1.25
90 pC	8.22 ± 1.67	8.18 ± 0.74	7.96 ± 1.61	7.89 ± 0.74	5.35 ± 1.08	5.01 ± 0.48



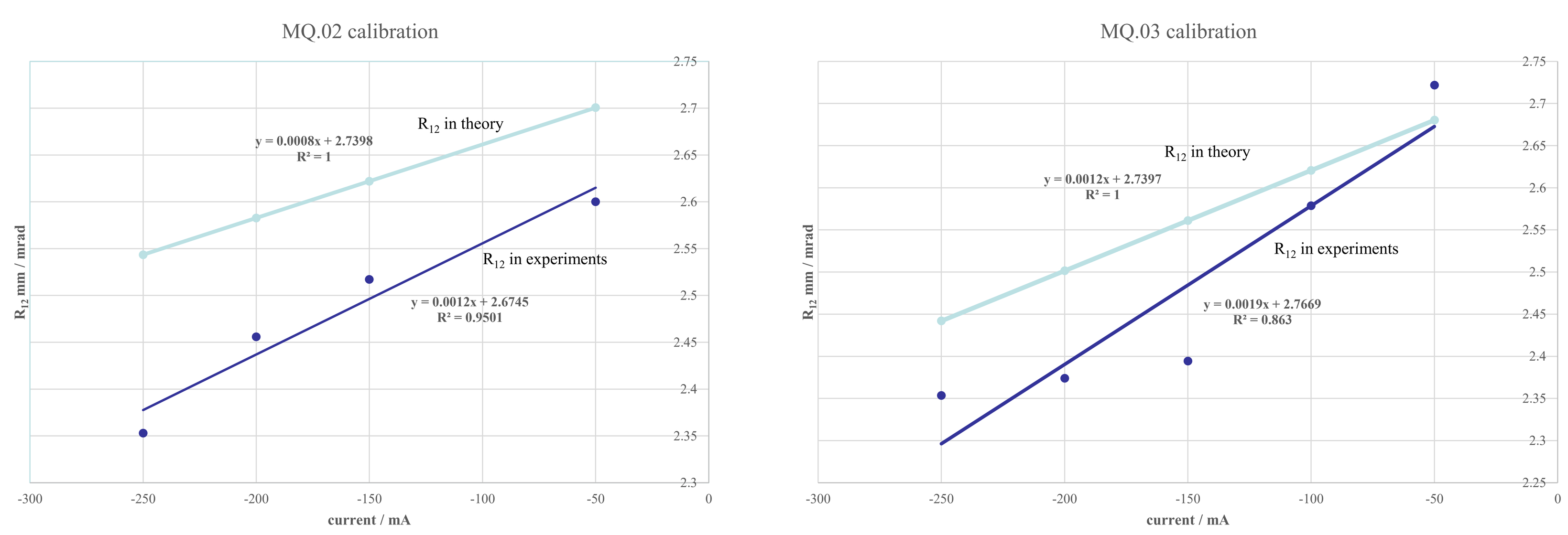
Conclusions:

In quadrupole scans, the traditional fitting way using thin lens approximation to calculate beam emittance will introduce 3% to 5% error compared with thick lens with the distance between quadrupole and screen is about 2 meters. But before using thick lens matrix, one should make sure the beam's status, convergence or divergence, and select the right matrix. Sometimes it is not obvious to distinguish between covergence and divergence by one screen because of overfocus. A more important aspect is about quadrupole calibration. In our measurements, the errors from quadrupole are really big. So it is necessary to calibrate quadrupole before or after quadrupole scan.

Quadrupole calibration



$$\begin{pmatrix}x_s\\ x'_s\end{pmatrix} = \begin{pmatrix}R_{11} & R_{12}\\ R_{21} & R_{22}\end{pmatrix}\begin{pmatrix}x_0\\ x'_0\end{pmatrix} \quad R_{12} = \frac{dx_s}{dx'_0}$$



Acknowledgement

We would like to thank the whole ELBE team for their help with this project. Thank Dr. Houjun Qian from PITZ DESY for his helpful discussion. The work was partly supported by China Scholarship Council, and Fluid Institute of physics, China Academy of Engineering Physics.

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