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MEASUREMENT OF THE SECOND MOMENTS OF TRANSVERSE BEAM DISTRIBUTION WITH SOLENOID SCAN*

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Abstract

Measurement of the dependence of the beam size on profile monitor vs. strength of a focusing element is widely used for measurement of the beam parameters. Such measurements are mostly used for the separate planes and assumption that beam satisfied Gaussian distribution. In many linear accelerators the transverse beam dynamics is coupled between planes and distribution is far from the Gaussian. We developed measurement technique of the second moments of beam distribution which does not rely on any assumptions. The theory and experimental results are presented.

INTRODUCTION

The charged beam in the accelerator is usually is described by Twiss parameters α, β, γ, and emittance ε. Transverse motion in each plane is considered decoupled and parameters at particular point (α_0, β_0, γ_0, ε_0) can be found by solving a set of *n* linear equations:

$$\begin{pmatrix} \sigma^2_{x(1)} \\ \sigma^2_{x(2)} \\ \cdots \\ \sigma^2_{x(n)} \end{pmatrix} = \begin{bmatrix} S^2_{11(1)} & -2S_{11(1)}S_{12(1)} & S^2_{12(1)} \\ S^2_{11(2)} & -2S_{11(2)}S_{12(2)} & S^2_{12(2)} \\ \cdots & \cdots & \cdots \\ S^2_{11(n)} & -2S_{11(n)}S_{12(n)} & S^2_{12(n)} \end{bmatrix} \begin{pmatrix} \beta_0\varepsilon_x \\ \alpha_0 e_x \\ \gamma_0\varepsilon_x \end{pmatrix} \quad (1)$$

where S is a transfer matrix between the point of interest and observation point (profile monitor).

$$\begin{pmatrix} x_1 \\ x'_1 \\ y_1 \\ y'_1 \end{pmatrix} = \mathrm{S} \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \end{pmatrix} \quad (2)$$

The index in the parentheses indicates measurement with varying transfer matrix by the strength of focusing element and/or utilized profile monitor. All four parameters can be resolved using equation connecting Twiss parameters

$$\beta\gamma = 1 + \alpha^2$$

In general case Twiss parametrization is not applicable and one can utilize only the measurements of the second moments of the transverse distribution $<x^2>$, $<x'^2>$, $<xx'>$, $<y^2>$, $<y'^2>$, $<yy'>$, $<xy>$, $<xy'>$, $<x'y>$, and $<x'y'>$ (here we assume that all the first moments are equal zero). Profile monitor provides three observables $<x^2>$, $<y^2>$, and $<xy>$. For the n measurements Eq. 1 transforms into

$$\begin{pmatrix} \sigma^2_{x(1)} \\ \sigma^2_{y(1)} \\ \sigma_{xy(1)} \\ \cdots \\ \sigma^2_{x(n)} \\ \sigma^2_{y(n)} \\ \sigma_{xy(n)} \end{pmatrix} = \mathbf{R} \begin{pmatrix} \langle x^2 \rangle \\ \langle xx' \rangle \\ \langle x'^2 \rangle \\ \langle y^2 \rangle \\ \langle y'^2 \rangle \\ \langle yy' \rangle \\ \langle xy \rangle \\ \langle x'y' \rangle \\ \langle x'y \rangle \\ \langle xy' \rangle \end{pmatrix} \quad (3)$$

where **R** is $3n \times 10$ matrix formed using elements of the transfer matrix S in a manner similar used in [1] (we do not show it explicitly due to its size). On the left size of the Eq. (3) is columnar vector with experimental observables, and on the right side ten seconds moments of the beam at the point of interest. For the illustration we show the dependence of σ_x^2 from transport matrix and ten moments:

$$\sigma_x^2 = S_{11}^2\langle x^2\rangle + 2S_{11}S_{12}\langle xx'\rangle + S_{12}^2\langle x'^2\rangle + S_{13}^2\langle y^2\rangle + 2S_{13}S_{14}\langle yy'\rangle + S_{14}^2\langle y'^2\rangle + 2S_{11}S_{13}\langle xy\rangle + 2S_{12}S_{14}\langle x'y'\rangle + 2S_{11}S_{14}\langle xy'\rangle + 2S_{12}S_{13}\langle x'y\rangle \quad (4)$$

Now one can find the second moments by solving the system of linear equations when at least four measurements were performed. However, matrix R is rank deficient. No matter how many experimental points we have its rank is 9. Analysis showed that this is a fundamental feature of the system containing only solenoids and drifts. Neither transfer matrix of solenoid [2] nor transfer matrix of drift does not change $x'y - xy'$. To resolve these moments, we need to add a quadrupole into the transport line. We do not have any quadrupole in the transport, therefore in the analysis of the experimental results we assumed that $\langle x'y\rangle = \langle xy'\rangle$.

It should be noted that the approach we used is suitable for emittance dominated beams. The substantial space charge forces will introduce systematic errors which are behind the scope of this paper.

EXPERIMENTAL RESULTS

The proposed approach was tested on the two accelerators in the RHIC complex: the first one used for coherent electron cooling (CeC) experiment [3, 4], and the second one for the low energy RHIC electron cooler (LEReC) [5, 6].

The measurements performed at CeC accelerator utilized electron beam generated by 1.25 MeV superconducting RF gun. Bunch length was 375 ps and bunch charge 0.6 nC. The measurements results are shown in Fig. 1. This beam is close to round.

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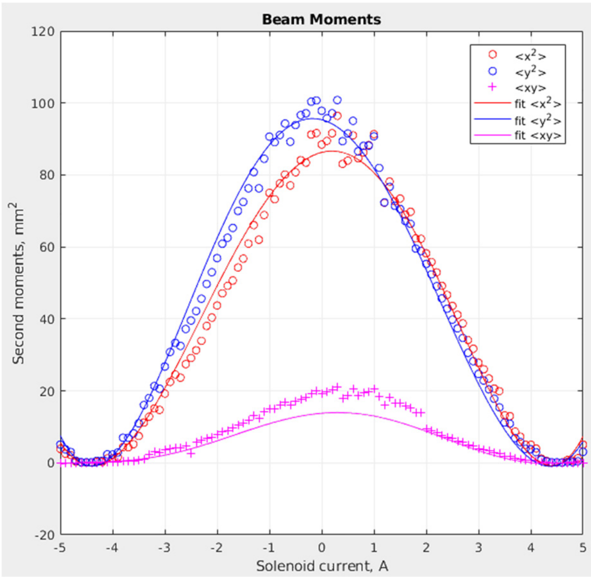


Figure 1: Measurement of the second moments of the 1.2 MeV electron beam distribution in the CeC accelerator with solenoid and screen placed 0.61 m downstream. The found second moments are: $<x^2>=16.8$ mm^2, $<xx'>=29.9$ mm mrad, $<x'^2>=52.6$ mrad2, $<y^2>=15.1$ mm^2, $<yy'>=26.4$ mm mrad, $<y'^2>=45.6$ mrad^2, $<xy>=2.3$ mm^2, $<x'y'>=7.9$ mrad^2, $<x'y>=<xy'>=4.0$ mm mrad.

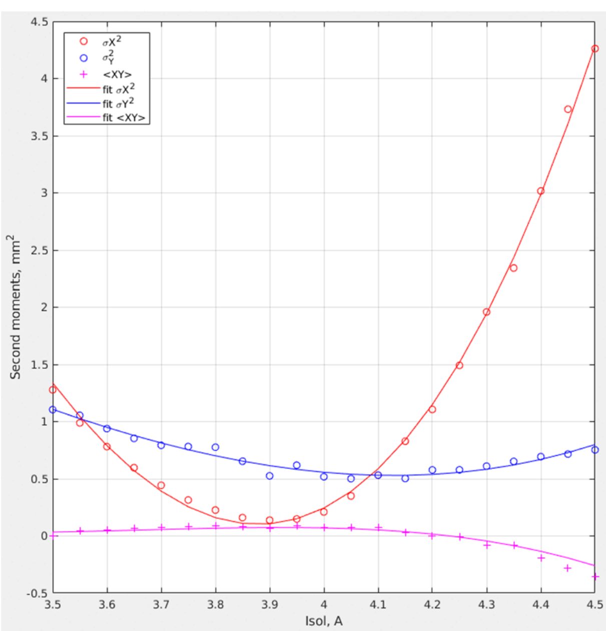
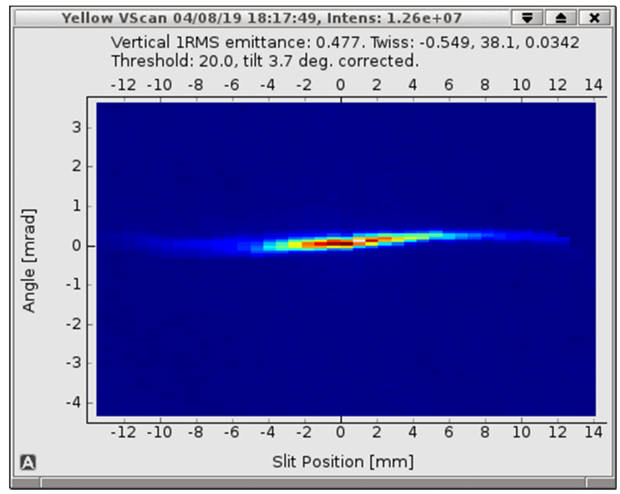
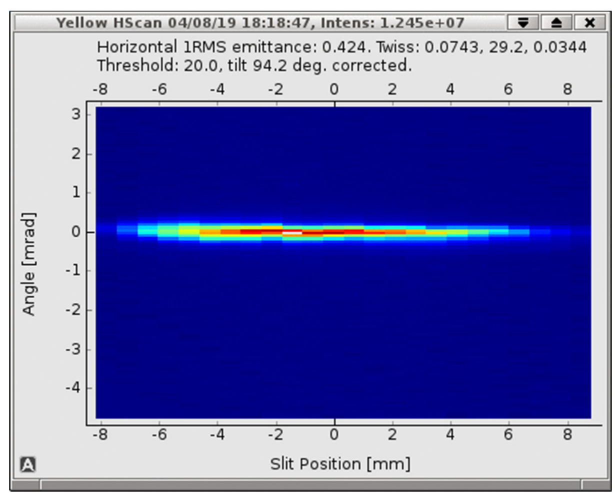


Figure 2: Measurement of the second moments of the 1.6 MeV electron beam distribution in the LeREC accelerator with solenoid and screen placed 0.76 m downstream. The found second moments are: $<x^2>=27.2$ mm^2, $<xx'>=0.81$ mm mrad, $<x'^2>=0.03$ mrad2, $<y^2>=6.1$ mm^2, $<yy'>=0.32$ mm mrad, $<y'^2>=0.05$ mrad^2, $<xy>=6.0$ mm^2, $<x'y'>=0.0$ mrad^2, $<x'y>=<xy'>=0.13$ mm mrad.

For the LEReC the measurements were performed in the transport line equipped with nine solenoids with 1.6 MeV electron beam with 0.2 A peak current. The measurement results are shown in Fig.2. This beam has substantially different parameters in X and Y planes and is not even close to round. The energy of LeREC beam is slightly higher and peak current ten times less, therefore it is much closer to the emittance dominated beam. If the values for the second moments are used for emittance calculations (neglecting cross-plane elements) the value of horizontal r.m.s. emittance is 0.4 mm mrad and the vertical r.m.s. emittance is 0.43 mm mrad. These values are close to the ones obtained with slit scan shown in Fig. 3.



a) vertical plane scan



b) horizontal plane scan

Figure 3: Emittance measurement of the LeREC beam with slit scan (courtesy A. Fedotov).

Both CeC and LeREC systems allow measurement of the beam function at any location of a solenoid, but due to the limited time this possibility was not finalized. One of the problems is that beam size varies considerably and the outer parts of the bunch can be clipped.

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CONCLUSION

We have developed and experimentally confirmed procedure for the measurements of the seconds moments of the transverse beam distribution by changing beam transport optics with solenoid and observing beam profile on the monitor.

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