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BPM RESOLUTION STUDIES AT PETRA III

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Abstract

In order to measure the noise level of a BPM system from beam generated orbit data, the correlated beam jitter has to be removed from the position signals. There exist different ways to extract the BPM noise, as the "three-BPM" correlation method or the model-independent principal components analysis (PCA). Both methods will shortly be reviewed. Based on a PCA, the resolution of the PETRA III *Libera Brilliance* based BPM system was measured. The results are presented together with first measurements in view of an updated BPM system for the future PETRA IV project at DESY.

INTRODUCTION

PETRA III is a third–generation synchrotron light source currently operated at 6 GeV by DESY Hamburg, Germany [1, 2]. Since 2016 DESY has been pursuing R&D towards upgrading the machine to a fourth–generation one, PETRA IV, being diffraction limited up to X–rays of about 10 keV [3] and expected to start operation in 2027. For this new machine a good resolution of the button–type BPMs of about 10-20 µm in turn–by–turn and 100 nm in stored beam mode (at 300 Hz bandwidth) will be required [4].

The position resolution $\sigma_{x,y}$ of a button–type BPM is determined by two factors, the monitor constant $K_{x,y}$ and the signal–to–noise ratio SNR [5]:

$$\sigma_{x,y} \propto K_{x,y}/\sqrt{SNR}\,. \tag{1}$$

While $K_{x,y}$ is defined by the pickup geometry (mainly beam pipe diameter, but also button size), SNR depends on geometry (button size defines signal strength), infrastructure (cable length, attenuators...), and quality of the read–out electronics. In the following, the main focus will be on the performance of the read–out electronics. As first step towards a new BPM system for PETRA IV it was decided to measure the achievable resolution of the existing PETRA III *Libera Brilliance* electronics from the commercial supplier Instrumentation Technologies [6].

Usually the design of modern ADCs integrated in electronic devices is driven by the telecommunication market, therefore they are well adapted for cw signals. Beam generated signals from a button–type pickup however are far away from being comparable to a cw signal, therefore it is preferable to perform the resolution study based on orbit data from the electron beam. Beam generated signals however contain two different kinds of jitter. For one thing it is the beam jitter, i.e. a real change of beam angle and position caused by fluctuations in the accelerator (caused by ground motion, energy fluctuation...). This kind of jitter is seen be several or even all BPMs simultaneously because of the correlation established by the particle beam optics. On the other hand it is the noise of the BPM electronics which is the quantity of interest and has to be measured. In case of BPM noise there exist no correlation between adjacent BPM readings. Consequently a correlation analysis is a powerful tool in order to disentangle both jitter sources. In the next section, two common methods which are in use in the accelerator community are briefly described, hereafter a principal component analysis is applied for the determination of the PETRA III BPM resolution.

CORRELATION ANALYSIS

Two schemes are sometimes used for BPM investigations, the "three BPM" correlation method and the Principal Component Analysis (PCA). Examples can be found in Refs. [7] from KEK-B (Tsukuba, Japan) and [8] from SSRF (Shanghai, China). Their underlying ideas will briefly be sketched hereafter. A further method described in Ref. [9] is an extension of the "three BPM" method, but will not be covered.

"Three BPM" Correlation Method

In the "three BPM" method, position readings from three adjacent BPMs are considered, assuming that no non–linear elements are inbetween the monitors. As indicated in Fig. 1, the readings are connected by the transport matrices according to

$$\begin{pmatrix} y_3 \\ y_3' \end{pmatrix} = M_\alpha \begin{pmatrix} y_1 \\ y_1' \end{pmatrix}, \quad \begin{pmatrix} y_2 \\ y_2' \end{pmatrix} = M_\gamma \begin{pmatrix} y_1 \\ y_1' \end{pmatrix}, \quad M_\alpha = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix},$$

and M_γ respectively. While a BPM delivers only position information, position readings from both transport equations can be combined in order to get rid of y_1', resulting in

$$\tilde{y}_2 = \left(\gamma_{11} - \frac{\alpha_{11}\gamma_{12}}{\alpha_{12}}\right) y_1 + \frac{\gamma_{12}}{\alpha_{12}} y_3 \quad = \quad X_{21} y_1 + X_{23} y_3\,.$$

The tilde indicates that the position at location 2 is an estimated one. It can be calculated from the readings of BPM_1 and BPM_3 with knowledge of the transport matrix elements which are comprised in the coefficients X_{21}, X_{23}. At the other hand, y_2 can directly be measured at BPM_2, the difference

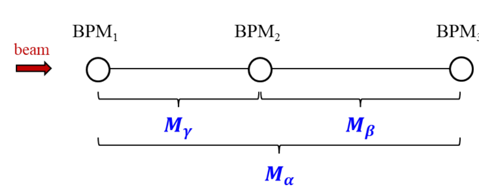

Figure 1: Principle scheme of the "three BPM" correlation method. In order to eliminate the beam correlated jitter in BPM measurements, N position readings of three adjacent BPMs have to be recorded.

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$\Delta = y_2 - (X_{21}y_1 + X_{23}y_3)$ respectively the spread in Δ are measures of the BPM noise contribution to the actual position reading. Using normal error propagation, this spread can simply be expressed as $\sigma_\Delta^2 = \sigma_2^2 + X_{21}^2\sigma_1^2 + X_{23}^2\sigma_3^2$ with σ_i the resolution of each BPM ($i = 1\ldots3$). Under the assumption that all BPMs have the same resolution σ, this relation is rewritten in the form

$$\sigma = \sigma_\Delta/\sqrt{1 + X_{21}^2 + X_{23}^2}\,.$$

The spread σ_Δ is not a parameter which can be deduced from a single measurement. However, it can be determined from a series of consecutive BPM measurements by considering the variance of the difference distribution Δ. This leads to the final equation by which the BPM resolution can be determined from N position measurements of three adjacent BPMs:

$$\sigma = \sqrt{\frac{1}{N-1}\frac{\sum_{i=1}^{N}\left[y_{2,i} - (X_{21}y_{1,i} + X_{23}y_{3,i})\right]^2}{1 + X_{21}^2 + X_{23}^2}}\,. \qquad (2)$$

For the evaluation of the BPM resolution according to Eq. (2), the coefficients X_{21}, X_{23} must be known. Either they are known in advance from the particle beam optics (i.e. the transport matrix elements of M_α, M_γ must be known), or they have to be deduced from the same measurement. In the latter case the BPM readings can be grouped to form a matrix equation in the form

$$\begin{pmatrix} y_{2,1} \\ \vdots \\ y_{2,N} \end{pmatrix} = \begin{pmatrix} 1 & y_{1,1} & y_{3,1} \\ \vdots & \vdots & \vdots \\ 1 & y_{1,N} & y_{3,N} \end{pmatrix} \begin{pmatrix} X_0 \\ X_{21} \\ X_{23} \end{pmatrix}. \qquad (3)$$

The parameter X_0 in the coefficient vector X is introduced in order to take into account a possible offset in Δ. However, it should be close to zero, otherwise the variance of the Δ distribution in Eq. (2) has to be extended, see Ref. [10]. Equation (3) can be solved for X, for example by forming the Moore–Penrose pseudo inverse which results in a least–square estimate for the missing elements of the coefficient vector. Based on Eqs. (2) and (3), the resolution of a single BPM can be determined. By grouping adjacent BPMs together and repeating this method with all groups, the resolution of all BPMs can be measured. This approach was applied for example in Ref. [7].

However, this method has certain restrictions. Firstly it is assumed that the formalism of linear beam optics can be applied which is certainly not the case for all BPMs in a modern diffraction limited light source. On the other hand the derivation of Eq. (2) required that adjacent BPMs have the same resolution. For PETRA III this is not the case, due to the diversity of pickup types and cable lengths the BPMs operate at rather different signal–to–noise levels [11]. Finally it may happen that some of the adjacent BPMs grouped together show a weak correlation. This is the case for a phase advance close or equal to $n\pi/2$ (n an odd integer) between the BPMs. In this situation the uncertainty in the deduced BPM resolutions is large, especially if the coefficients X_{21}, X_{23} must be determined from the same measurement. To overcome these difficulties a PCA is preferable.

Principal Component Analysis

A PCA is a method of multivariate statistics with the aim to convert a set of correlated variables into a set of linearly uncorrelated ones, the so called Principal Components (PCs). PCAs are intended for cleansing of correlations in data sets (e.g. in order to structure large data sets or for data compression), and therefore they are well suited for disentangling beam correlated jitter from noise.

The determination of PCs (more specifically the orientation of the principle axes) is explained in an illustrative way based on the first PC: in order to find the axis orientation of the PC showing the greatest correlation, the axis of the new coordinate system is rotated such that the overall data variance with respect to it is maximized. In order to increase the sensitivity to axis rotation, as first step the data have to be centered. If the correct orientation is found, the first PC contribution is removed from the data and the process is repeated in order to find the subsequent PCs with the condition that the axes orientations are perpendicular to each other (i.e. the PCs are uncorrelated).

From mathematical point of view the task is to form the covariance matrix $C \propto MM^\top$ from the $(m \times n)$ data matrix M. C is always real, symmetric and square. Any matrix of this type has a spectral decomposition of the form $C = W\Lambda W^\top$ with W an orthonormal matrix (formed by the orthogonal eigenvectors) and Λ a diagonal matrix (main diagonal elements are the eigenvalues). The eigenvectors represent the PCs, the corresponding eigenvalues the amount of variance contained. Finally, the eigenvectors have to be arranged starting from the largest eigenvalue and going to the smallest one, in order to sort the PCs in descending order.

There exist an alternative numerical method for a PCA which is applied in most cases, the Singular Value Decomposition (SVD). Instead of diagonalizing the covariance matrix C, an SVD can directly be applied to the data matrix in the form $M = U\Sigma V^\top$. Σ is a diagnonal matrix containing the singular values in descending order. There exits a direct relation between the singular values of Σ and the eigenvalues of Λ: $\Lambda_i = (n-1)^{-1}\Sigma_i^2$, i.e. they are fully equivalent. The advantage of using SVD is that the algorithm is numerically more stable, the formation of $MM^\top$ as required for the covariance matrix can cause a loss of precision. A well known example that indicates the dangers of forming $MM^\top$ is the Läuchli matrix [12]. Moreover, benefit of using SVD is that it contains additional useful information in the matrices U, V, see below.

Applying the SVD formalism to the case of BPM resolution determination, the BPM data matrix M has to be constructed. As first step the data have to be centered, i.e. the mean value for each BPM has to be subtracted. Afterwards

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the turn–by–turn readings are arranged in the form

$$M = \frac{1}{\sqrt{nm}} \begin{pmatrix} \mathrm{BPM}_1(\mathrm{turn}_1) & \dots & \mathrm{BPM}_n(\mathrm{turn}_1) \\ \vdots & & \vdots \\ \mathrm{BPM}_1(\mathrm{turn}_m) & \dots & \mathrm{BPM}_n(\mathrm{turn}_m) \end{pmatrix} \quad (4)$$

where the matrix normalization is chosen according to Ref. [13]. While inspecting Eq. (4) it can be seen that each row of M contains the orbit for the respective turn (i.e. information about the space coordinate), while each column contains the turn–by–turn readings for the respective BPM (i.e. information about the time coordinate). This issue is also reflected in the matrices U, V after SVD application: the column vectors of U contain information about the temporal pattern (tune...), the ones of V about the spatial pattern (orbit resp. β-function...). However, the interpretation of U/V as temporal/spatial ones depends on the orientation of the matrix M. Sometimes in literature they are therefore named conversely.

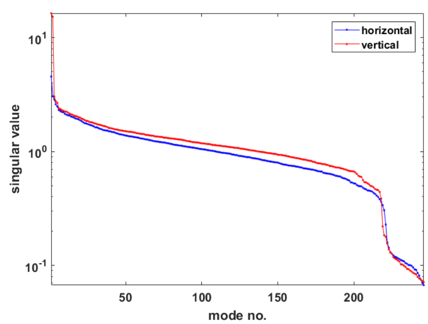

Figure 2: Singular value plot after SVD of the BPM data matrix applying vertical orbit excitation. The modes (or PCs) are ranked according to their information contribution. The first two vertical modes are by far dominant, i.e. they contain most information about the orbit kick.

For illustration purpose the results of a test measurement performed at PETRA III are shown. The machine was filled with 5.6 mA in 960 bunches and the beam was vertically excited by a kicker magnet. For each of the 246 orbit BPMs, 2048 consecutive turns were recorded and analyzed by applying an SVD on the data matrix Eq. (4). Figure 2 shows the plot of the singular values of the Σ matrix. As can be seen, the first two vertical modes are by far dominant due to the excitation in this plane. As indicated before, after SVD the column vectors of U contain information about the temporal pattern. In Fig. 3 the time dependency of the first two vertical modes is plotted together with their corresponding Fourier transforms, the vertical tune being clearly visible. In comparison, spatial information is encoded in the column vectors of V. The β–function is proportional to the first two spatial modes according to $\beta \propto (\Sigma_1 v_1^2 + \Sigma_2 v_2^2)$, see also Ref. [14]. The comparison of extracted (top) and design (bottom) vertical β–function of PETRA III shown Fig. 4 demonstrates that it is well reproduced apart from a scaling factor.

As can be concluded, an SVD based PCA helps to extract useful information about beam and BPM system. In the case described above the BPM electronics noise was removed from the beam generated jitter signals by considering only the modes (PCs) which contribute to the beam signal. In order to determine the BPM resolution, in the following the method is applied just the other way around, i.e. the singular values of modes containing beam information are set equal to zero. The decision whether a mode contributes to the beam signal or not was taken according to the Fourier transform of temporal modes: if a tune signal could be identified, the mode was removed from the analysis. However, it turned out that it is sufficient to simply remove the contribution of the first two PCs without affecting the result.

BPM RESOLUTION MEASUREMENTS

Similar to the measurement described before, 2048 consecutive turns were recorded and analyzed after a vertical orbit excitation, however this time for single bunch filling patterns with various bunch charges. Based on a PCA the correlated beam jitter was removed from the data and the BPM resolution was extracted. Figure 5 shows an example from such a PCA analysis. As can be seen from this figure, different BPMs in the accelerator have different resolutions. However, according to Eq. (1) the resolution depends both on the monitor constant and on the signal–to–noise ratio. In order to account for the geometrical imbalance, in a first step all data are normalized to the common monitor constant of $K_{x,y}$ = 10 mm. In order to take into account the signal level influence, in a second step they are plotted as function of the input signal levels rather than of the location in the accelerator. In Fig. 6 the re–scaled data set of Fig. 5 is plotted as function of the signal level.

Treating all data sets in a similar way which were measured at single bunch currents in the range from 70 µA up to 2.4 mA, the BPM resolution can be represented over a wide range of input levels. In Fig. 7 the result is summarized in a semi–logarithmic representation, all data points measured at the same signal level are averaged and the error is determined by the sample variance. As can be seen the resolution improves with increasing signal level as expected,

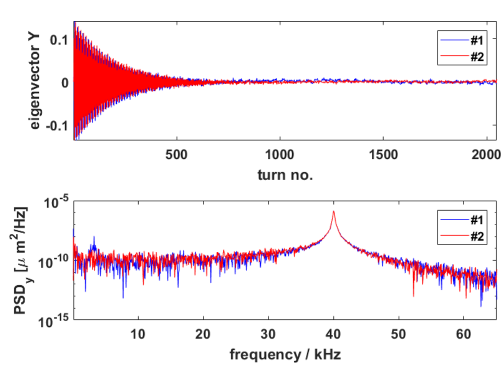

Figure 3: Top: time behavior of the first two modes $u_{1,y} u_{2,y}$ as extracted from the matrix U after SVD. Bottom: the Fourier transform of the modes clearly indicates the PETRA III vertical tune.

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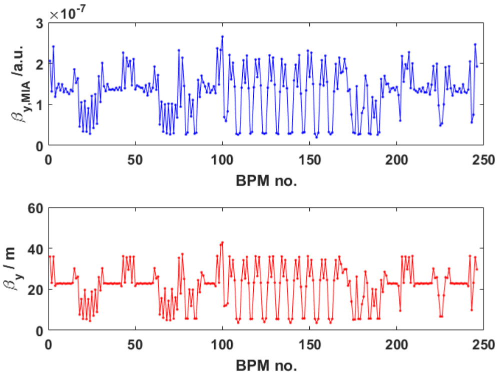

Figure 4: Top: vertical β–function reconstructed from the first two modes $v_{1,y}, v_{2,y}$ as extracted from the matrix V after SVD. Bottom: vertical β–function according to design optics.

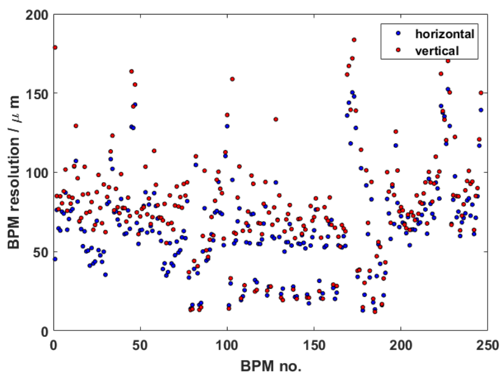

Figure 5: BPM resolution of all PETRA III BPMs for a single bunch filling of 2.01 mA.

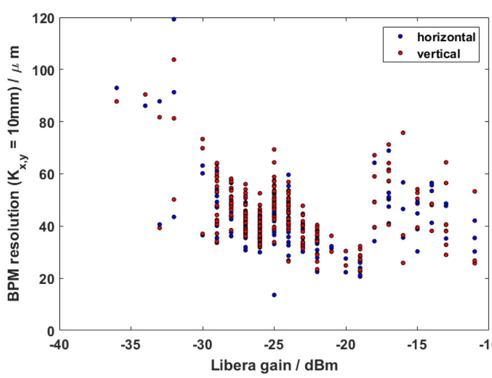

Figure 6: Same data set as in Fig. 5, but this time plotted as function of the input signal level and normalized to the common monitor constant of $K_{x,y}$ = 10 mm.

However, at distinct levels which are indicated by arrows in Fig. 7 the resolution suddenly gets worse. These signal levels correspond to the ones at which internal attenuators are switched in the *Libera Brilliance* according to their default gain scheme described in the manual. Furthermore the best turn–by–turn resolution for single bunch operation which is measured amounts to about 30 µm. As consequence it can be concluded that the *Libera Brilliance* BPM electronics which is used at PETRA III will not be sufficient for the operation of PETRA IV where a resolution about 10-20 µm is required.

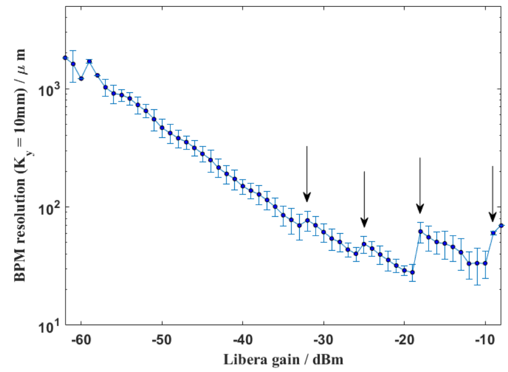

Figure 7: Single bunch turn–by–turn resolution of the *Libera Brilliance*. The arrows indicate signal levels at which internal attenuators are switched.

SUMMARY AND OUTLOOK

Two methods for noise level determination of a BPM system from beam generated orbit data are reviewed, the "three-BPM" correlation method and the principal components analysis (PCA). It is shown that the SVD–based PCA is advantageous because it allows to circumvent specific disadvantages connected with the "three-BPM" method, and it allows to extract additional information (β–function, phase advance, tune. . .) which might be of interest for beam physics.

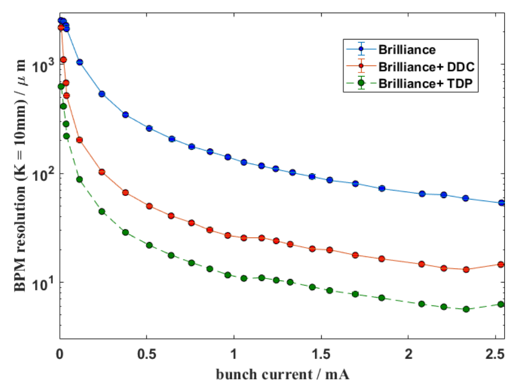

Figure 8: Single bunch turn–by–turn resolution of *Libera Brilliance* and *Brilliance+* in TDP and DDC mode.

Based on a PCA, the resolution of the *Libera Brilliance* based BPM system of PETRA III was measured. It is shown that this read–out electronics will not fulfill the resolution specifications which are required for PETRA IV. First tests in view of alternative BPM systems are underway. As an example Fig. 8 shows the measured resolution of the *Libera Brilliance* compared to the one of the successor model *Libera Brilliance+* which offers two processing modes, Time Domain Processing (TDP) and Digital Down Conversion (DDC). The data were taken with a single BPM, beam generated jitter was removed by summing up and splitting the four pickup signals. As can be seen, the *Libera Brilliance+* operated in TDP mode offers already a much better resolution. Further resolution studies for PETRA IV are an ongoing task and will continue with different types of BPM read–out electronics.

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REFERENCES

[1] K. Balewski *et al.*, "PETRA III: a low emittance synchrotron radiation source", Tech. note DESY 2004-035, 2004.

[2] http://petraiii.desy.de/

[3] C.G. Schroer *et al.*, *J. Synchrotron Rad.*, vol. 25, pp. 1277–1290, 2018.

[4] I. V. Agapov *et al.*, "Status of the PETRA IV project", in *Proc. IPAC'19*, Melbourne, Australia, May 2019, pp. 1404–1407. doi:10.18429/JACoW-IPAC2019-TUPGW011

[5] S.R. Smith, "Beam position monitor engineering", in *Proc. BIW'96*, Argonne, IL, USA, May 1996, *AIP Conf. Proc.*, vol. 390, pp. 50–65, 1997. doi:10.1063/1.52306

[6] Instrumentation Technologies. https://www.i-tech.si/.

[7] M. Arinaga *et al.*, *Nucl. Instr. and Meth.*, vol. A 499, pp. 100-–137, 2003.

[8] Z. Chen *et al.*, *Chinese Physics C*, vol. 38, pp. 077004 1–5, 2014.

[9] N. Baboi *et al.*, "Resolution Studies at Beam Position Monitors at the FLASH Facility at DESY", in *Proc. BIW'06*, Batavia, IL, USA, May 2006, *AIP Conf. Proc.*, vol. 868, pp. 227–237, 2007. doi:10.1063/1.2401409

[10] S. Hiramatsu, "Beam Position Monitors for Circular Accelerators", in *Proc. IBIC'12*, Tsukuba, Japan, Oct. 2012, paper THTA01, pp. 590–601.

[11] I. Krouptchenkov, A. Brenger, G. Kube, F. Schmidt-Föhre, and K. Wittenburg, "Experience with the Commissioning of the Libera Brilliance BPM Electronics at PETRA III", in *Proc. DIPAC'09*, Basel, Switzerland, May 2009, paper TUPD03, pp. 291–293.

[12] P. Läuchli, *Numer. Math.*, vol. 3, pp. 226-240, 1961.

[13] Ch.–X. Wang, "Model independent analysis of beam centroid dynamics in accelerators", SLAC-R-547, 2003.

[14] Ch.–X. Wang, V. Sajaev, and Ch.–Y. Yao, *Phys. Rev. ST Accel. Beams*, vol. 6, pp. 104001, 2003.