IBIC 17
INTERNATIONAL BEAM INSTRUMENTATION CONFERENCE
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Review of Beam-Based Techniques of Impedance Measurement

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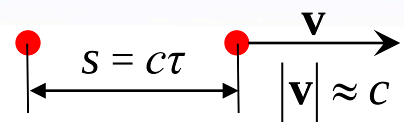
NSLS-II

Wake Fields and Impedances

Wake function: point charge (δ–function) response

$$W_{\parallel}(\tau) = -\frac{1}{q}\int_{-\infty}^{\infty} E_z(t,\tau)\,dt, \qquad \mathbf{W}_{\perp}(\tau) = \frac{1}{qr}\int_{-\infty}^{\infty} \left[\mathbf{E}(t,\tau) + \mathbf{v}\times\mathbf{B}(t,\tau)\right]_{\perp} dt$$

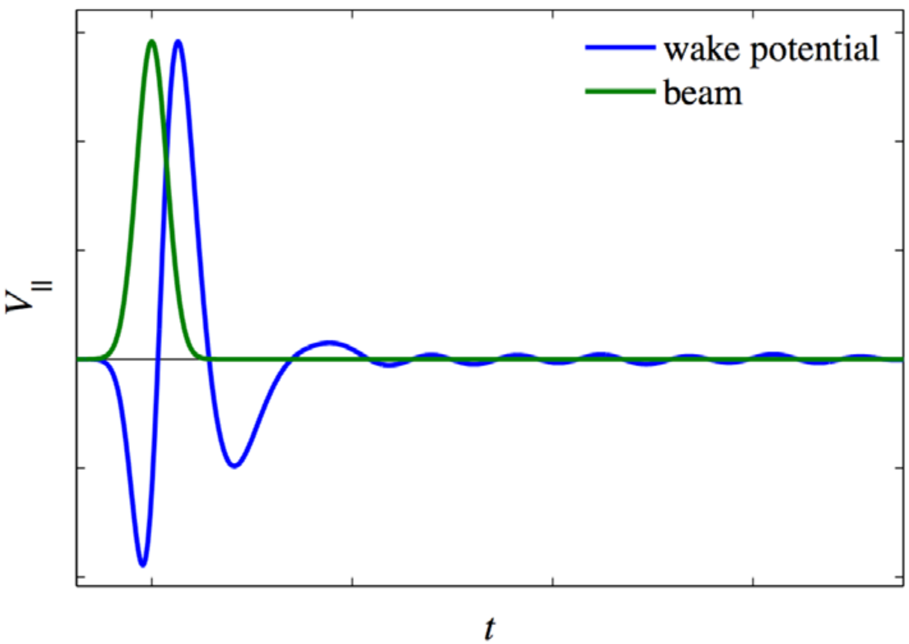


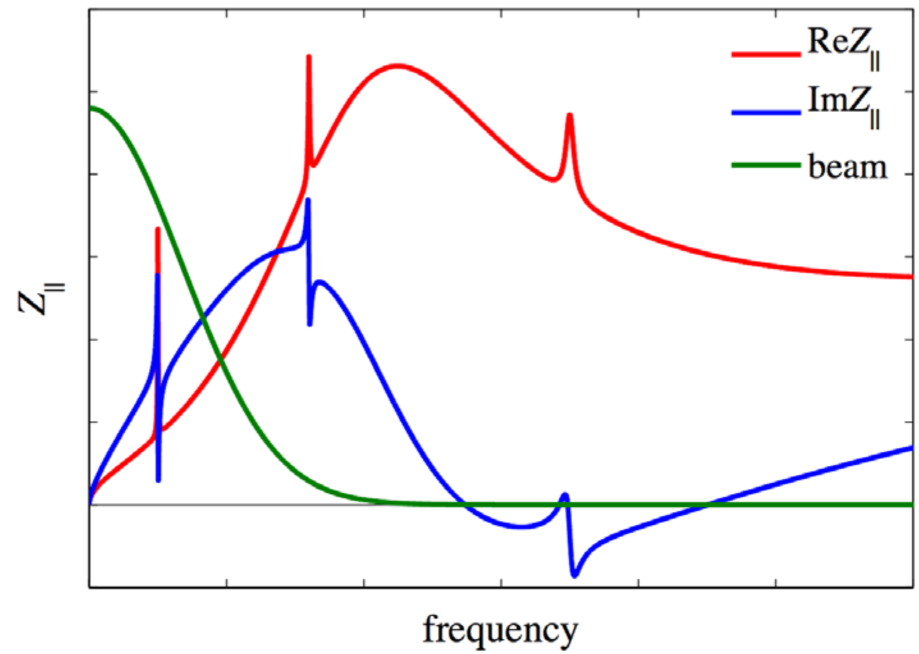
Wake potential: $\lambda(t)$ beam response

$$V(\tau) = \int_{0}^{\infty} W(t)\lambda(\tau - t)\,dt$$

Impedance: frequency-domain transfer function

$$Z(\omega) = \int_{-\infty}^{\infty} W(t)e^{-i\omega t}\,dt \qquad Z(\omega) = \frac{\tilde{V}(\omega)}{\tilde{\lambda}(\omega)}$$





Longitudinal Broad-band Impedance

What Can We Measure?

Single-bunch effects are dependent on integral parameters combining the impedance $Z_{\|}$ and the bunch power spectrum h:

Effective impedance

$$\left(\frac{Z_{\|0}}{n}\right)_{\text{eff}} = \frac{\int_{-\infty}^{\infty} Z_{\|}(\omega)\frac{\omega_0}{\omega}h(\omega)d\omega}{\int_{-\infty}^{\infty} h(\omega)d\omega}$$

Loss factor

$$k_{\|} = \int_{-\infty}^{\infty} V_{\|}(t)\lambda(t)dt = $$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} Z_{\|}(\omega)h(\omega)d\omega$$

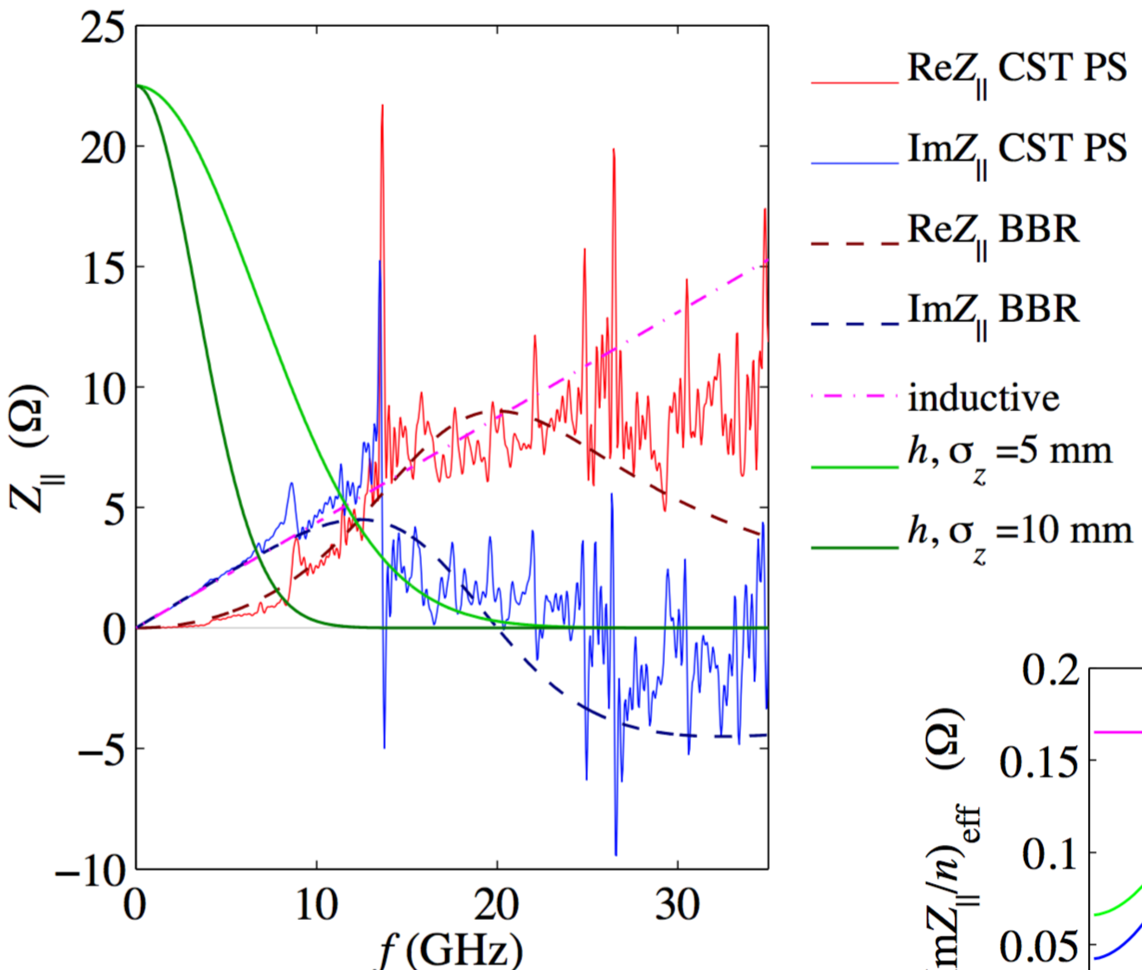
$h(\omega) = \tilde{\lambda}(\omega)\tilde{\lambda}^*(\omega)$ is the bunch power spectrum

$h(\omega) = e^{-\omega^2\sigma_t^2}$ for Gaussian bunch

Measurable effects

- bunch lengthening
- energy spread growth (microwave instability)
- coherent energy loss (heat load)
- synchronous phase shift

Longitudinal Impedance Models

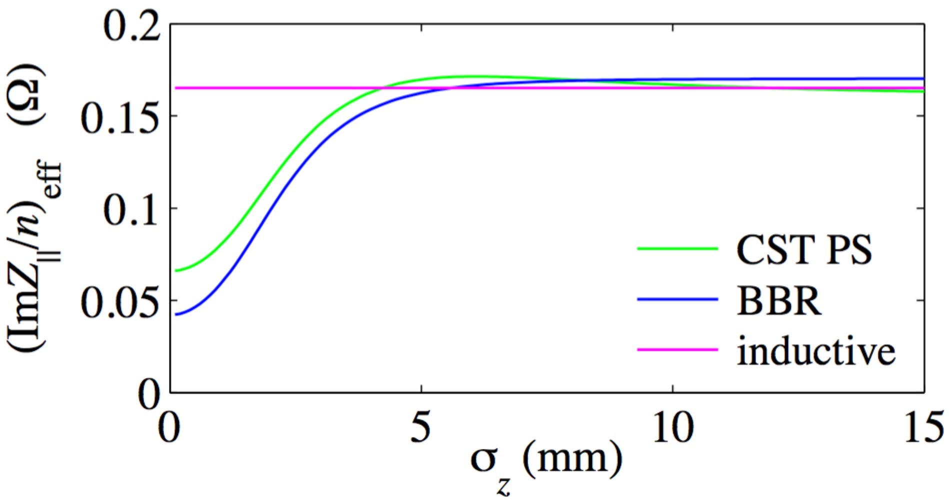


Broad-band resonator

$$Z_{\parallel}^{\text{bbr}} = \frac{R_s}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

Inductive model

$$\frac{Z_{\parallel}}{n} = i\frac{R_s}{Q}\frac{\omega_0}{\omega_r}$$



Bunch Lengthening

Haissinski equation

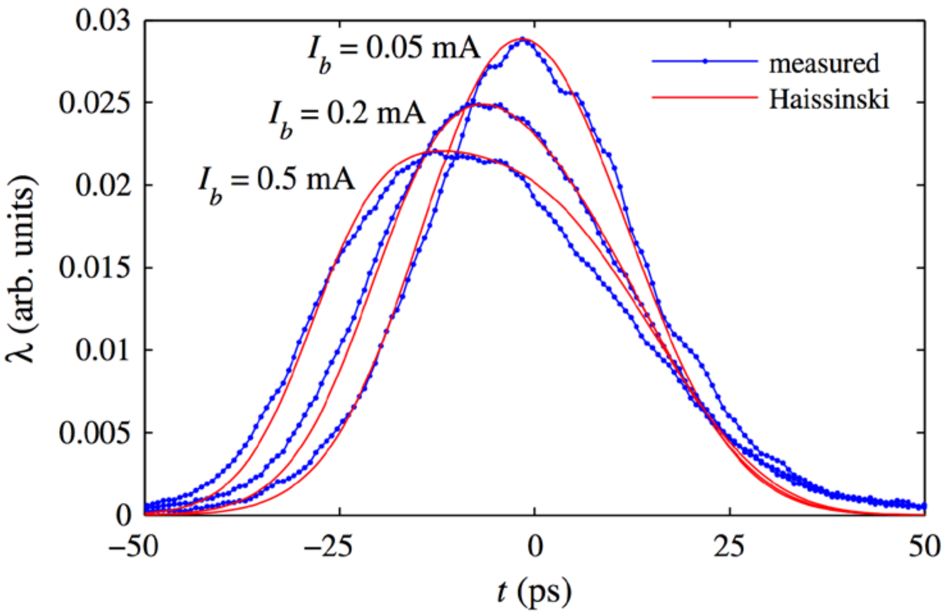
J.Haissinski, Nuovo Cimento 18, 1 (1973)

$$\lambda(t) = K\lambda_0(t)\exp\left[-\frac{\alpha I_b}{\omega_s^2\sigma_0^2 E/e}\int_{-\infty}^{t} S(t+\tau)\lambda(\tau)\mathrm{d}\tau\right]$$

$$\lambda_0(t) = \frac{1}{\sqrt{2\pi}\sigma_0}\exp\left(-\frac{t^2}{2\sigma_0^2}\right)$$

$$S(t) = \int_0^t W_\parallel(\tau)\mathrm{d}\tau$$

PRSTAB 18 064401 (2015)



3-rd order equation for r.m.s. bunch length

B.Zotter, CERN SPS/81-14 (DI), 1981

$$\left(\frac{\sigma_z}{\sigma_{z0}}\right)^3 - \frac{\sigma_z}{\sigma_{z0}} = \frac{\alpha I_b \,\mathrm{Im}(Z/n)_{eff}}{\pi \nu_s^2 E/e}\left(\frac{R}{\sigma_{z0}}\right)^3$$

Instrumentation:

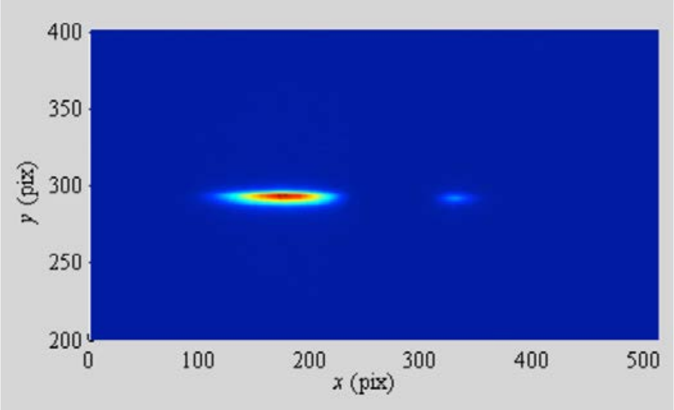
- streak camera
- dissector tube
- button-type BPM (indirect, via bunch spectrum width)

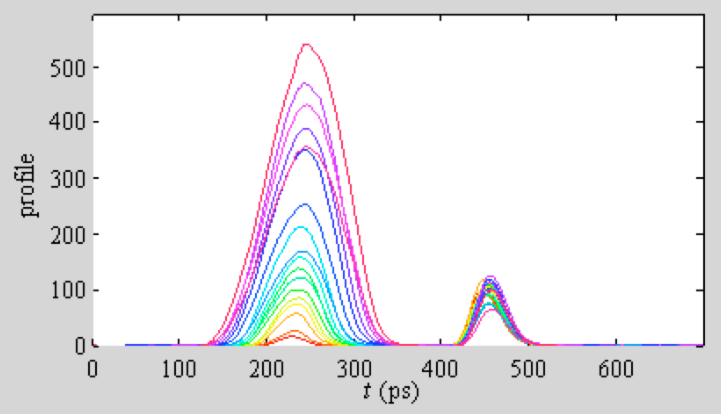
Coherent Energy Loss

Current-dependent shift of synchronous phase

Reference bunch technique

$$\Delta\phi_s = \frac{I_b k_\parallel}{f_0 V_{\rm rf} \cos\phi_s}$$





Instrumentation:

- streak camera
- dissector tube
- RF system diagnostics

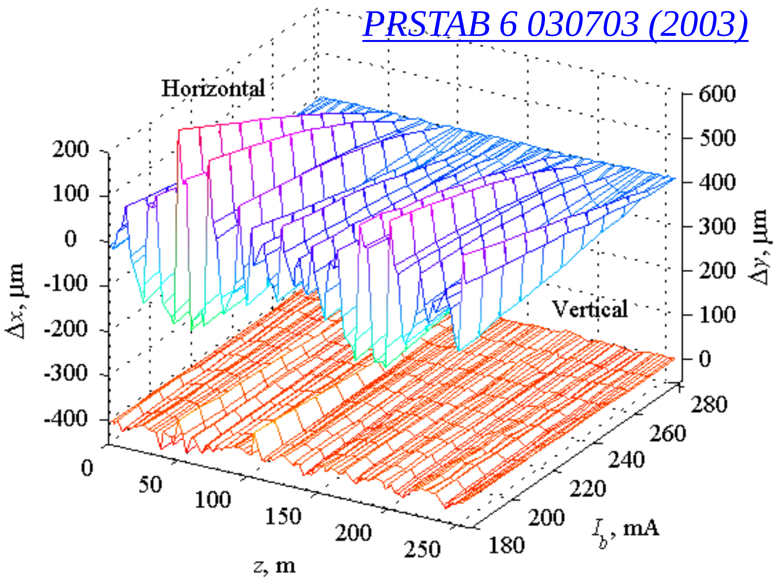
Current-dependent dispersive orbit distortion

[J.P.Koutchouk, CERN LEP- TH/89-2, 1989]

$$\Delta x(z) \approx D(z)\frac{\Delta I_b}{f_0(E/e)}\int_{z_0}^{z} k'_\parallel(\zeta)\mathrm{d}\zeta$$

$$k_\parallel = \frac{f_0}{\Delta I_b}\frac{E}{e}\frac{\Delta x(z)}{D(z)}$$

[PRSTAB 6 030703 (2003)]



Instrumentation:

- beam position monitors

Beam-based Impedance Model

Resistive-wall impedance

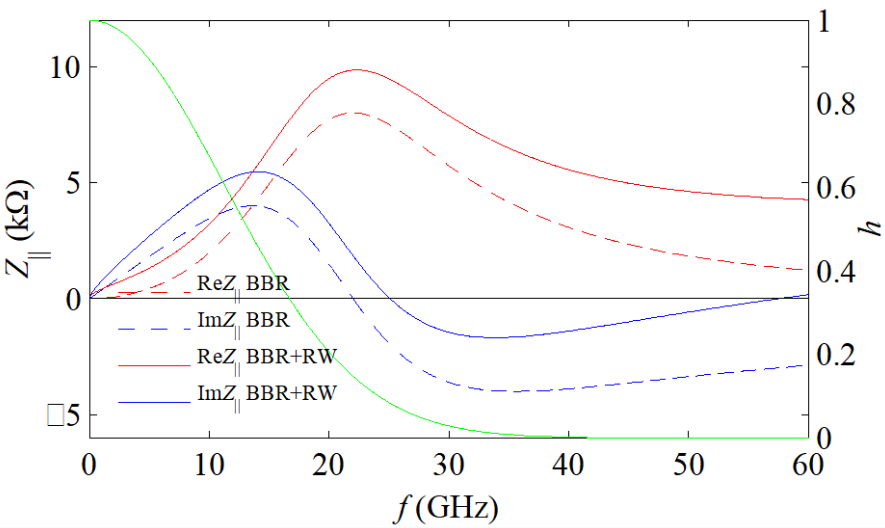
$$\frac{Z_{\parallel}^{rw}}{L} = G_0(b/a)\frac{1 - i\,\mathrm{sgn}\,\omega}{2\pi b}\sqrt{\frac{Z_0\rho|\omega|}{2c}}$$

G_0 is a form-factor for elliptical or rectangular cross-section

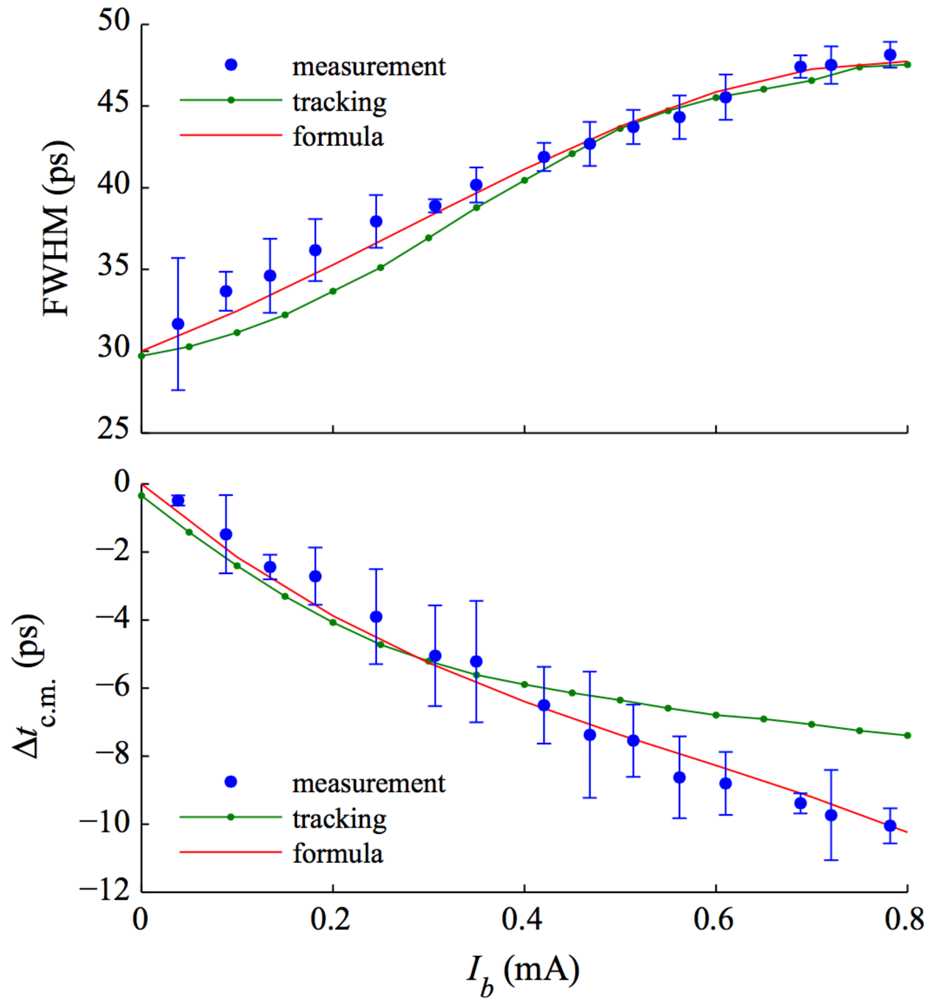
Broad-band resonator

$$Z_{\parallel}^{\mathrm{bbr}} = \frac{R_s}{1 + iQ(\omega/\omega_r - \omega_r/\omega)}$$

$$Z_{\parallel} = Z_{\parallel}^{\mathrm{bbr}} + Z_{\parallel}^{\mathrm{rw}}$$



PRSTAB 18 064401 (2015)



Energy Spread – Microwave Instability

Measurement & simulation

NAPAC-2016 TUPOB51

$$\sigma_x^2 = \beta_x \varepsilon_x + (\eta_x \delta)^2$$

$$\delta = \sigma_E / E$$

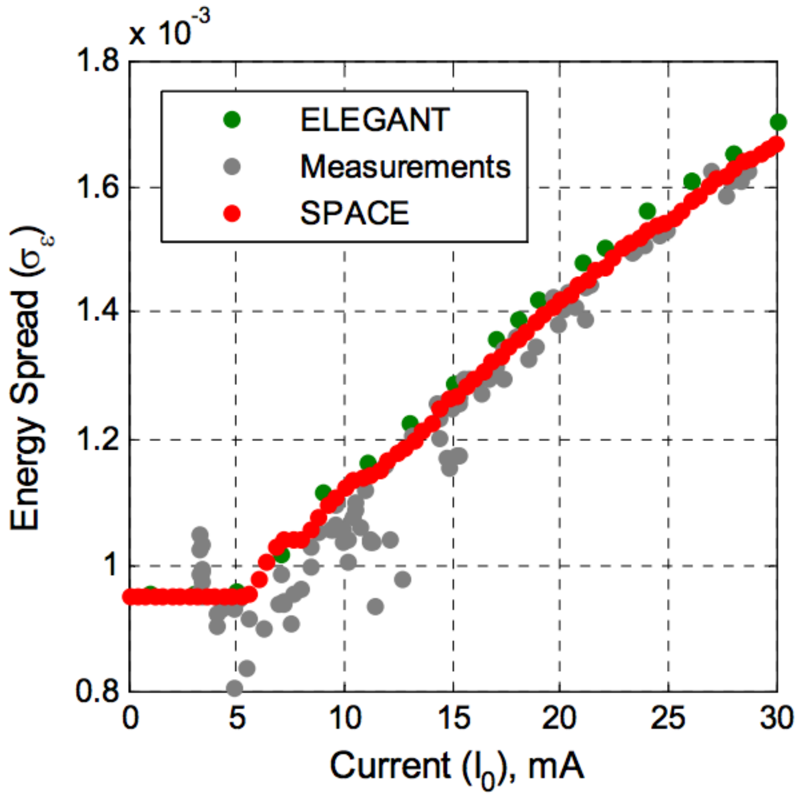
Software:

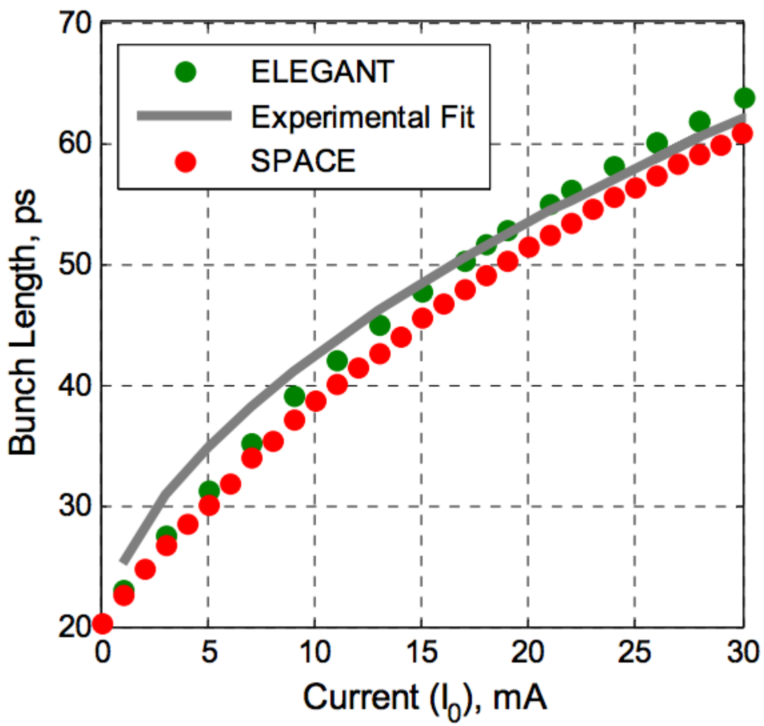
SPACE code: Vlasov-Fokker-Planck equation solver

PRAB 19, 024401 (2016)

Instrumentation:

- pin-hole X-ray camera
- synchrotron light monitor





The threshold is not visible in bunch lengthening

Transverse Broad-band Impedance

What Can We Measure?

Single-bunch effects are dependent on integral parameters combining the impedance $Z_\perp$ and the bunch power spectrum h:

Effective impedance

$$(Z_\perp)_{\text{eff}} = \frac{\int_{-\infty}^{\infty} Z_\perp(\omega) h(\omega - \omega_\xi) d\omega}{\int_{-\infty}^{\infty} h(\omega - \omega_\xi) d\omega}$$

$\omega_\xi \equiv \frac{\xi \omega_0}{\alpha}$ is the chromatic frequency

Kick factor

$$k_\perp = \int_{-\infty}^{\infty} V_\perp(t) \lambda(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} Z_\perp(\omega) h(\omega) d\omega$$

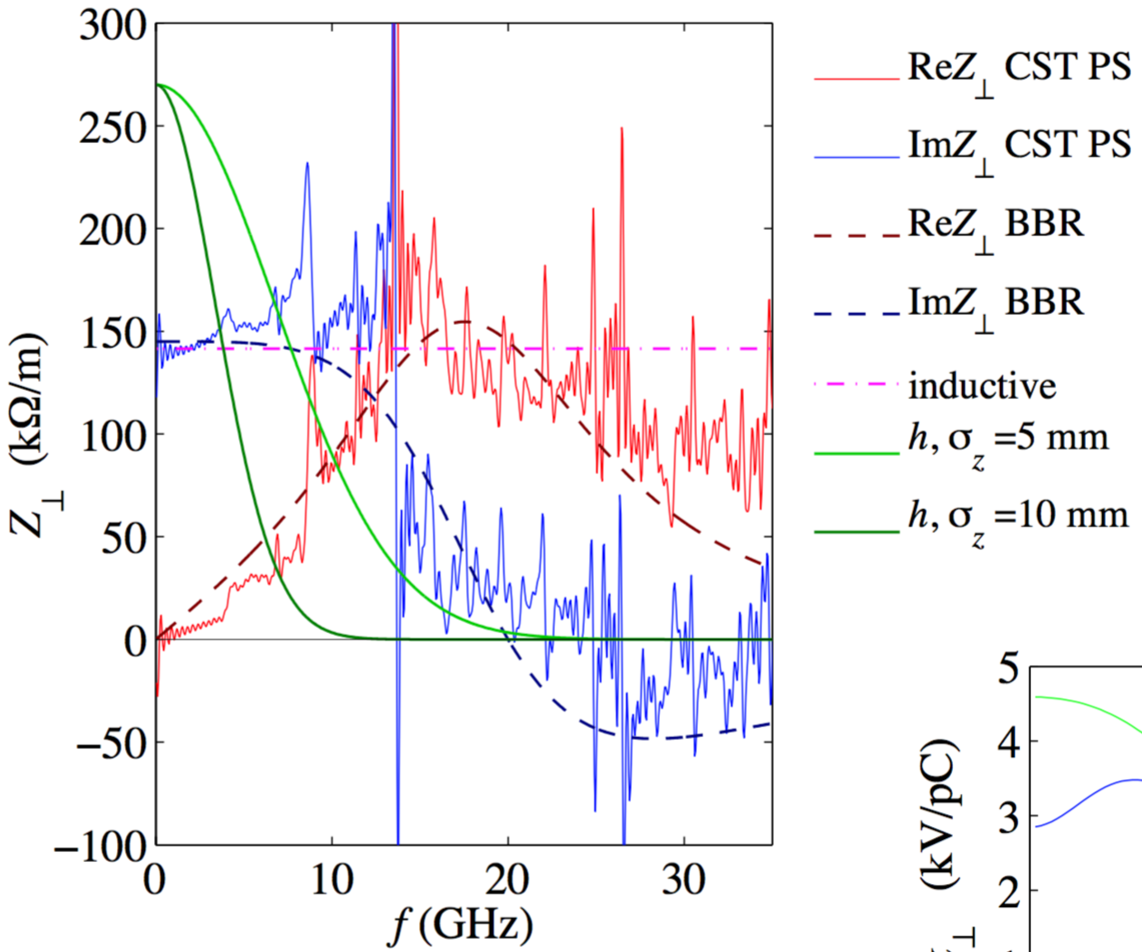
$h(\omega) = \tilde{\lambda}(\omega) \tilde{\lambda}^*(\omega)$ is the bunch power spectrum

$h(\omega) = e^{-\omega^2 \sigma_t^2}$ for Gaussian bunch

Measurable effects

- coherent betatron tune shift
- chromatic head-tail damping
- orbit distortion by local impedance

Transverse Impedance Models

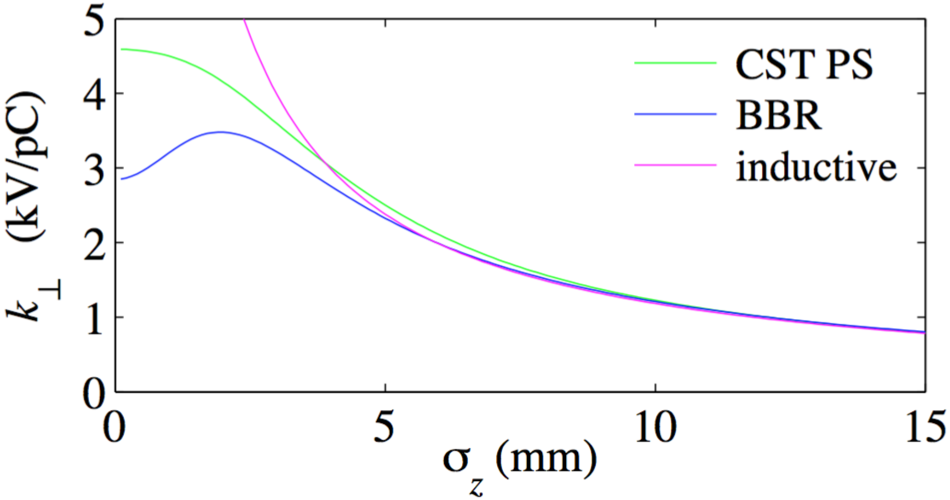


Broad-band resonator

$$Z_\perp^{\text{bbr}} = \frac{\omega_r}{\omega} \frac{R_s}{1 + iQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

Inductive model

$$Z_\perp = iR_s$$



Complex Frequency Shift

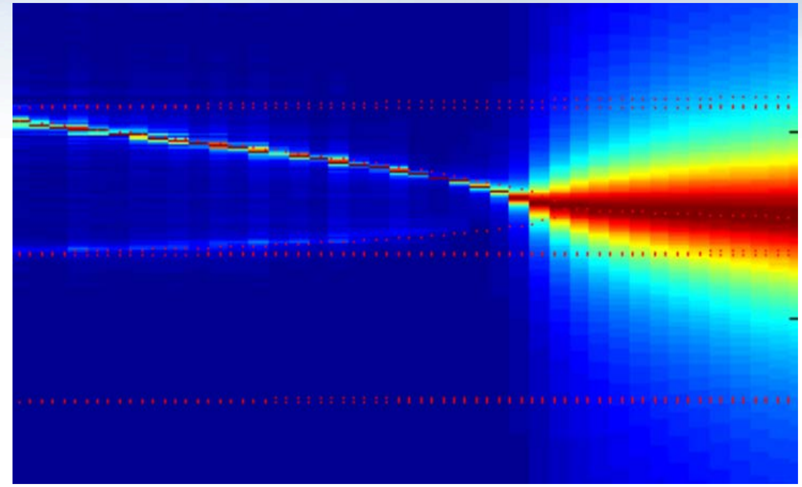
Eigenvalue problem for head-tail modes:

$$\det\left[\left(\frac{\Delta\Omega_\beta}{\omega_s}-l\right)\mathbf{I}-\mathbf{M}\right]=0$$

$$M_{kk'}=\frac{I_b}{2\omega_s\,E/e}\sum_i \beta_i \int_{-\infty}^{\infty} Z_{\perp i}(\omega)\,g_{lk}(\omega-\omega_\xi)\,g_{lk'}(\omega-\omega_\xi)\,d\omega$$

$$g_{lk}(\omega)=\frac{(-1)^{|l|}}{\sqrt{2\pi k!\left(|l|+k\right)!}}\left(\frac{\omega\sigma_t}{\sqrt{2}}\right)^{|l|+2k} e^{-\frac{\omega^2\sigma_t^2}{2}}$$

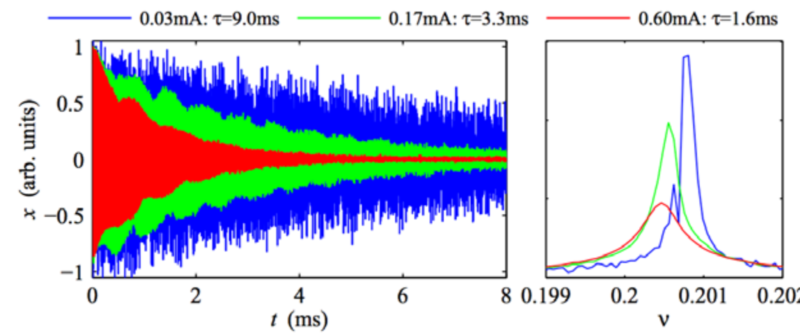
(for Gaussian bunch)

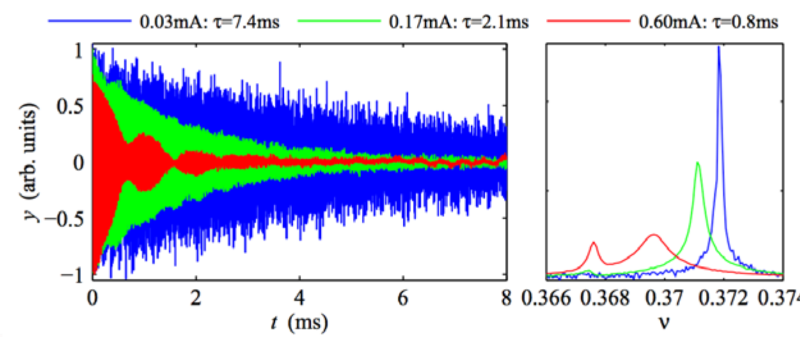


Linear approximation:

if $\Delta\nu_\beta << \nu_s$ $\qquad \Delta\nu_\beta=-\frac{I_b}{2\omega_0\,E/e}\sum_i \beta_i k_{\perp i}$

if $\frac{\xi\sigma_z}{\alpha R} << 1$ $\qquad \tau_\xi^{-1}=\frac{\xi\omega_0}{2\pi\alpha\,E/e} I_b \sum_i \beta_i \,\mathrm{Re}\, Z_{\perp i}$





Instrumentation:

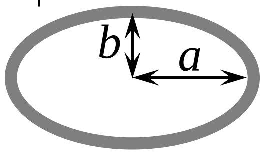
- beam position monitors

Beam-based Impedance Model

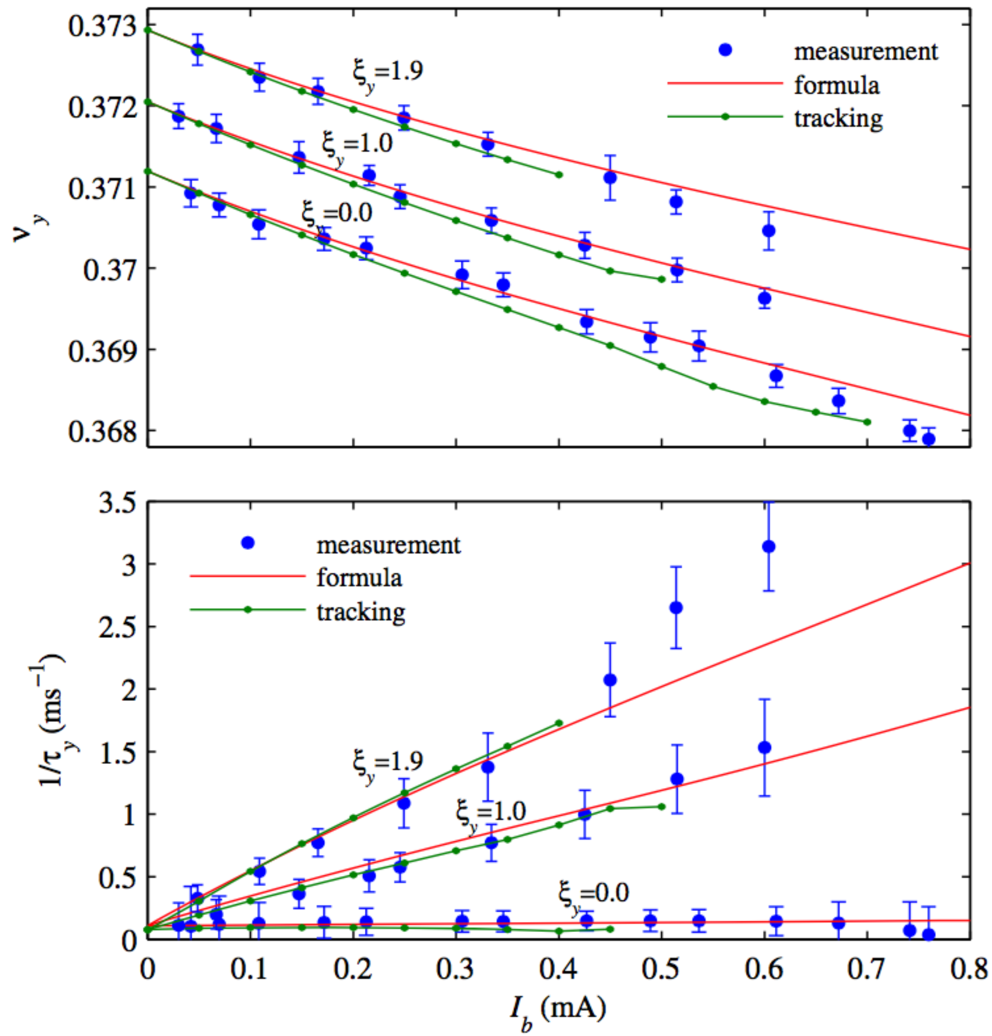
Resistive-wall impedance

$$\frac{Z_\perp^{rw}}{L} = G_{1x,y}(b/a)\frac{\operatorname{sgn}\omega + i}{\pi b^3}\sqrt{\frac{cZ_0\rho}{2|\omega|}}$$

$G_{x,y}$ is a form-factor for elliptical or rectangular cross-section



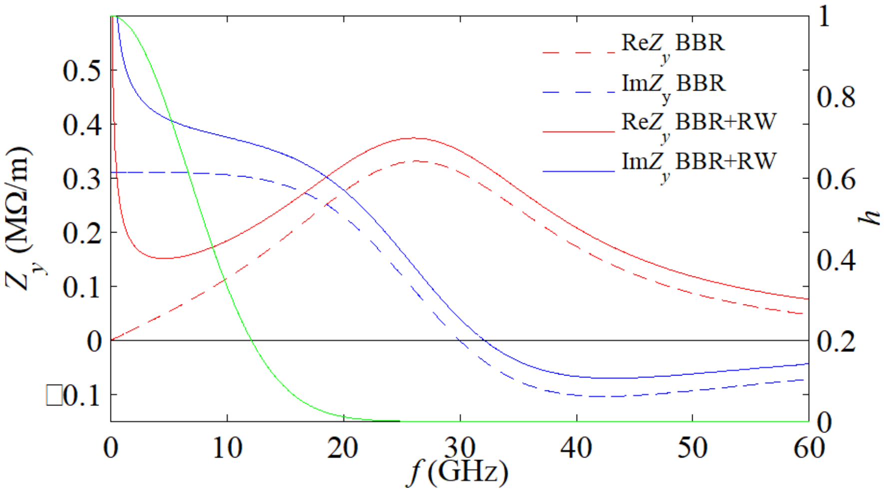
PRSTAB 18 064401 (2015)



Broad-band resonator

$$Z_\perp^{\text{bbr}} = \frac{\omega_r}{\omega}\frac{R_s}{1+iQ(\omega/\omega_r - \omega_r/\omega)}$$

$$Z_\perp = Z_\perp^{\text{bbr}} + Z_\perp^{\text{rw}}$$



Measurement of Local Impedance

$$\Delta x' = \frac{q}{E/e} k_\perp x$$ ***wakefield kick***

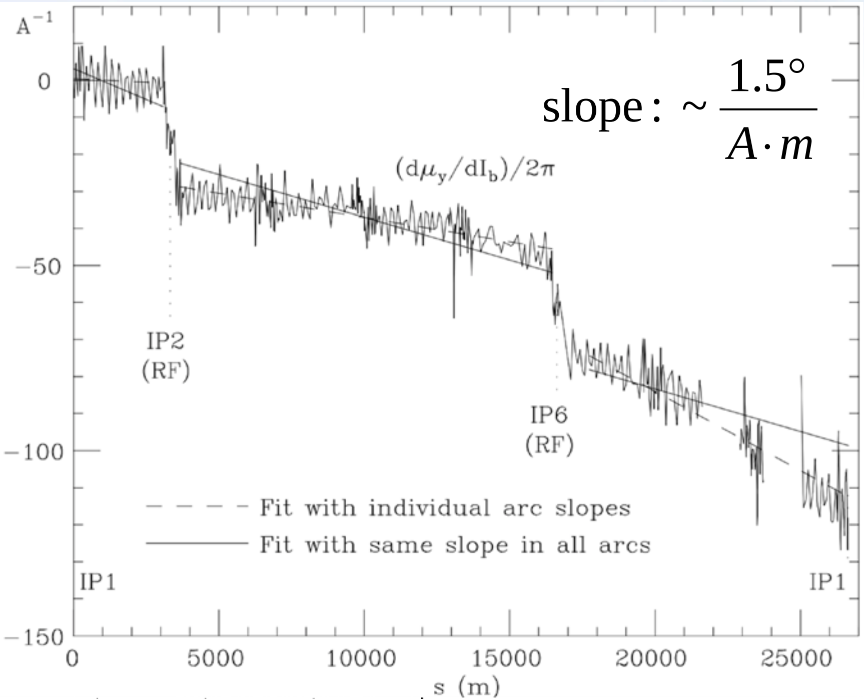
Current-dependent shift of betatron phase

$$\Delta\mu_\beta(s) = -\frac{I_p}{8\pi\,(E/e)\,C}\int_0^z \beta(s)\,\mathrm{Im}\,Z_\perp(s)\,ds$$

Instrumentation:

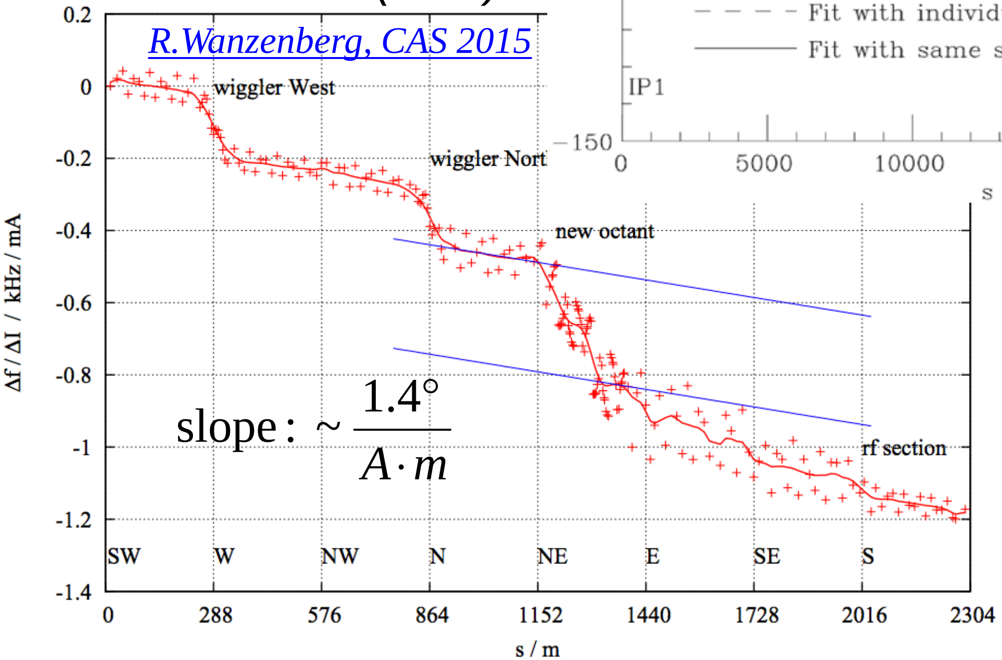
- beam position monitors

LEP (1995) [PAC-1995 Proc. pp 570-572]



PETRA-III (2015)

[R.Wanzenberg, CAS 2015]



Measurement of Local Impedance

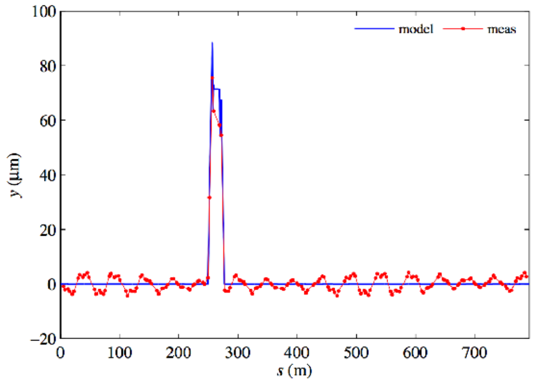
Orbit bump method

DIPAC-1999 PT19,
NIM A 525 (2004) 433–438

$$\Delta x' = \frac{q}{E/e} k_{\perp} x$$ ***wakefield kick***

Current-dependent orbit distortion

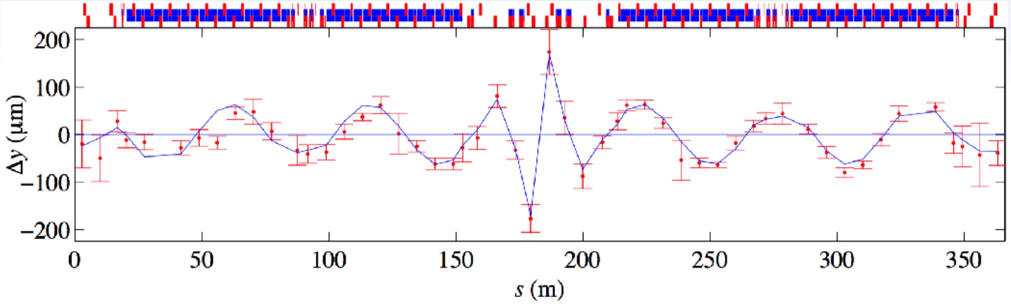
$$\Delta x(s) = \frac{\Delta q_b}{E/e} k_{\perp} x_0 \frac{\sqrt{\beta(s_0)\beta(s)}}{2\sin \pi \nu_\beta} \times$$
$$\times \cos\left(\left|\mu(s) - \mu(s_0)\right| - \pi \nu_\beta\right)$$



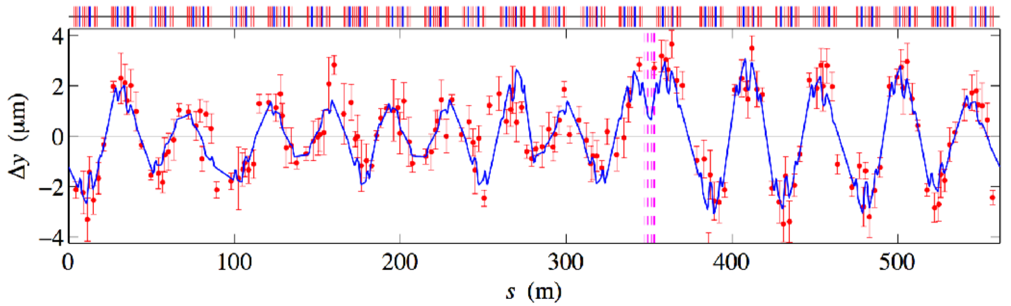
Instrumentation:

- beam position monitors

VEPP-4M: $\Delta y \sim 100\mu m$ EPAC-1998 THP09F

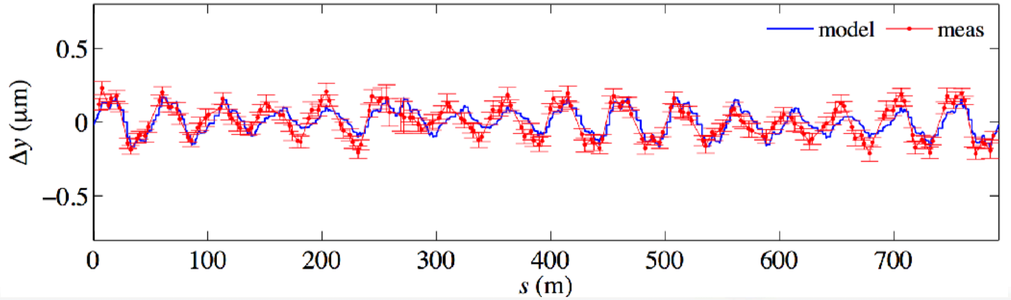


Diamond: $\Delta y \sim 2\mu m$ PRSTAB 17, 074402 (2014)



NSLS-II: $\Delta y \sim 0.2\mu m$ NIM A A 871 (2017) 59–62

New technique: AC local bump



Measurement of Local Impedance

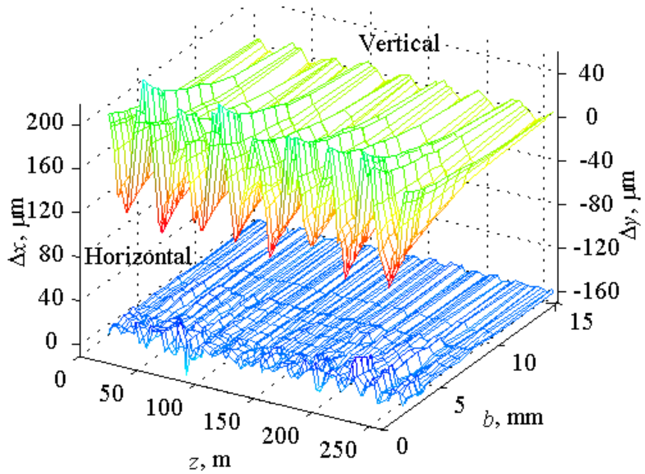
Movable elements (variable impedance)

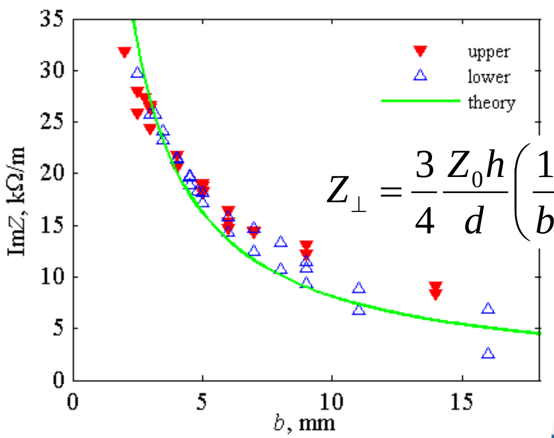
Instrumentation:

- beam position monitors

ELETTRA scraper: orbit distortion

[PRSTAB 6 030703 (2003)]

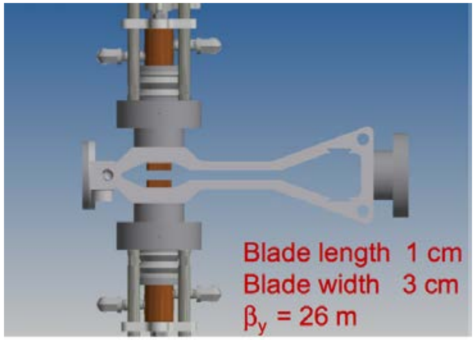


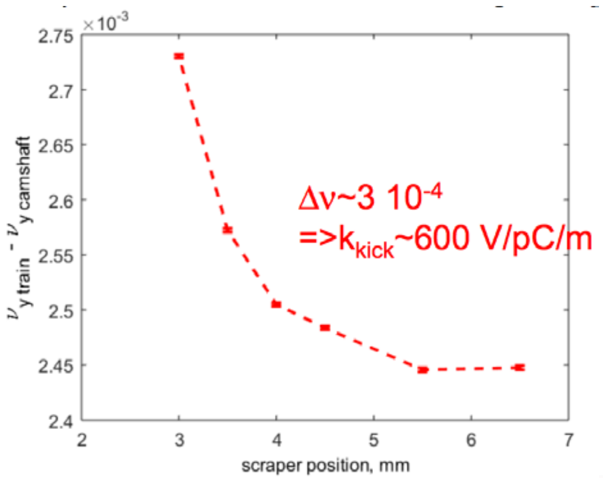


$$Z_\perp = \frac{3}{4}\frac{Z_0 h}{d}\left(\frac{1}{b}-\frac{1}{d}\right)$$

NSLS-II scraper: tune shift, reference bunch technique

[IBIC2016 TUCL02]





Transverse Narrow-band Impedance

Multi-bunch instability

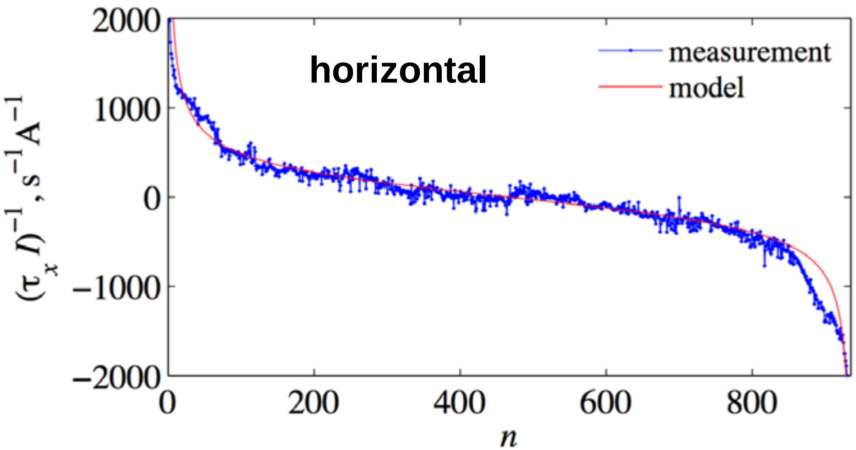
- resistive-wall impedance
- resonance modes

rising/damping time
Diamond Light Source [IPAC2016 TUPOR013]

$$\tau_n^{-1} = \mathrm{Im}\,\Delta\Omega_n$$

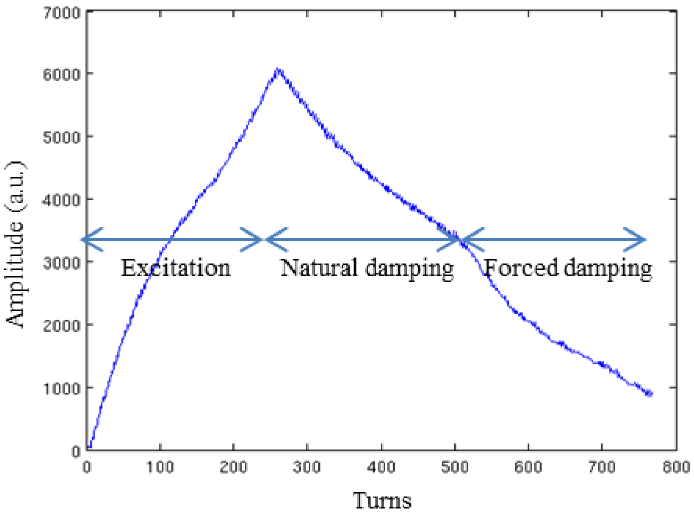
$$\Delta\Omega_n = -\frac{i}{4\pi}\frac{\omega_0 \overline{\beta}}{E/e} I \sum_{p=-\infty}^{\infty} Z_\perp(\omega_{pn}) h(\omega_{pn})$$

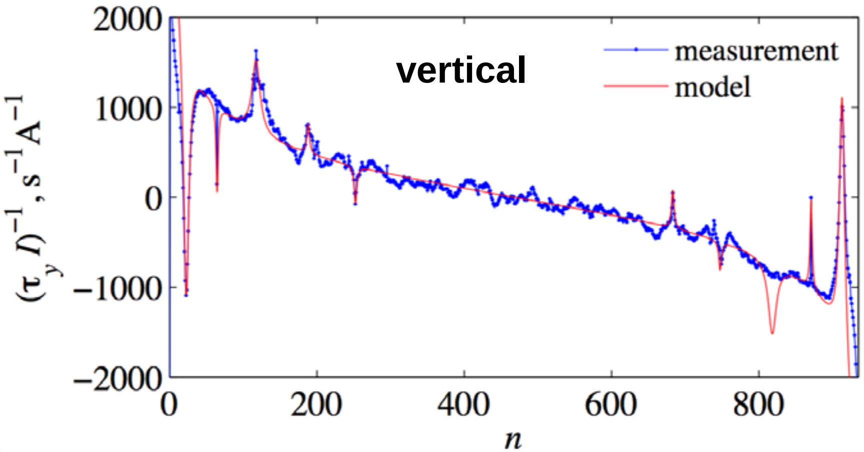
$$\omega_{pn} = (pN_b + n + \nu_\beta)\omega_0 \qquad h(\omega) = e^{-\omega^2\sigma^2}$$



Instrumentation:

- bunch-by-bunch feedback system





Summary

Impedance	Measurable effects	Instrumentation
longitudinal broad-band impedance	• bunch lengthening • synchronous phase shift • dispersive orbit distortion • energy spread growth (microwave instability)	• streak camera • dissector tube • RF system diagnostics • beam position monitors • pin-hole X-ray camera • synchrotron light monitor
transverse broad-band impedance	• coherent betatron tune shift • chromatic head-tail damping • orbit distortion (bump method)	• beam position monitors • pinger
transverse narrow-band impedance	• mode rising/damping times of transverse coupled-bunch instability	• bunch-by-bunch feedback system

Thank you for your attention!