X-RAY SMITH-PURCELL RADIATION FOR NON-INVASIVE SUBMICRON DIAGNOSTICS OF ELECTRON BEAMS HAVING TeV ENERGY*

A.A. Tishchenko†, D.Yu. Sergeeva, National Research Nuclear University MEPhI, Moscow, Russia

Abstract

We present the general theory of X-ray Smith-Purcell radiation from ultrarelativistic beams proceeding from our earlier results. The theory covers also the case of oblique incidence of the beam to the target, which leads to the conical effect in spatial distribution of Smith-Purcell radiation and allows one to count the divergence of the beam; also, the analytical description of the incoherent form-factor of the beam is given.

INTRODUCTION

Non-invasive diagnostics is one of the topical problems for the facilities like DESY, SLAC and the future ones like CLIC, ILC, etc. All modern diagnostics schemes are based on optical radiation which restricts limitation on the beam length and diagnostic resolution due to the diffraction (Rayleigh) limit. We suggest X-ray Smith-Purcell radiation as an instrument operating with smaller wavelengths and hence much more suiting for non-invasive submicron beam diagnostics.

Diffraction radiation (DR) arises when a charged particle moves near a target. The Coulomb field of the particle polarized the target material and the polarization currents arise, which leads to the radiation generation. Smith-Purcell radiation (SPR) is a special case of DR occurring when the target is a periodic structure.

Both these types of radiation are usually called the polarization radiation because the source of the radiation in these schemes is the target material rather than the charged particles itself [1].

As it follows from the definitions given above, when both DR and SPR are generated, there is no direct interaction between the particle and the target. It means that DR and SPR can be a base for noninvasive diagnostics. And, generally speaking, the information obtained can be both about the beam and about the target [2, 3].

We construct the theory of Smith-Purcell radiation at frequencies

$$\omega >> \omega_p, \qquad (1)$$

where ω_p is the plasma frequency, which usually has values about $20-30\,eV$. In this frequency region the response of dielectric or metal to the external field is similar, because behavior of the conductivity electrons and electrons bounded in atoms coincides in the electromagnetic field acting at frequencies higher than the atomic ones. Consequently, the properties of target material are defined by the function $\varepsilon(\omega)$, which has the form:

$$\varepsilon(\omega) = 1 - \omega_p^2 / \omega^2 . \qquad (2)$$

DR in the high-frequency limit $\omega >> \omega_p$, was investigated in the papers [4] for non-relativistic particles, and in [5] for gratings, i.e. X-ray SPR. Non-relativistic particles, however, emits DR only for impact-parameters of the order of the wavelength, which is not the case for X-ray domain. The analytics in the paper of M.J. Moran [5], on the other hand, was based on the theoretical description given in [6] by M.L. Ter-Mikhaelyan, who developed the theory an infinitely thin perfectly conducting target. This makes these results inapplicable for frequencies larger than optical ones. After that, the basis of the X-ray DR theory was given in [7] and for X-ray SPR in [8] for single-particle radiation.

PROPERTIES OF RADIATION

Let us consider the generation of SPR from the beam consisting of N_e electrons. The target is a grating of N elements (slabs, strips, grooves, etc) with the period d. The slab sizes in x, y, z directions are a, infinite, half-infinite, correspondingly, see Fig. 1. The beam is supposed to move at a constant distance h from the target surface. The particle velocity is $\mathbf{v} = v(\cos\alpha, \sin\alpha, 0)$.

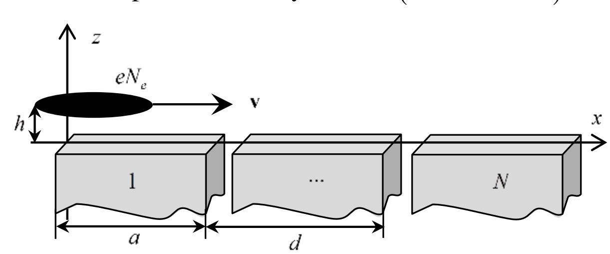

Figure 1: Generation of the Smith-Purcell radiation from a beam of electrons.

The main characteristic of the radiation we will operate with is the spectral-angular distribution of the radiation, which can be written in form

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†Tishchenko@mephi.ru

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$$\frac{d^2W(\mathbf{n},\omega)}{d\Omega d\omega}=\frac{d^2W_1(\mathbf{n},\omega)}{d\Omega d\omega}4\sin^2\left(\frac{a\varphi}{2}\right)\frac{\sin^2(dN\varphi/2)}{\sin^2(d\varphi/2)}F, \quad (3)$$

where $d^2W_1/d\omega d\Omega$ is the distribution of radiation energy from a single particle, F is the form-factor of the bunch,

$$\varphi=\frac{\omega}{c}\frac{1}{\beta_x}\left(1-n_x\beta_x-n_y\beta_y\right), \quad (4)$$

$\boldsymbol{\beta}=\mathbf{v}/c$; n_x and n_y are the components of the wave vector of radiation in vacuum $\mathbf{k}=\mathbf{n}\,\omega/c$.

Testing of the ratio of sines in Eq. (3) for maxima it is easy to obtain the Smith-Purcell dispersion relation:

$$\frac{d}{\lambda}\frac{1}{\beta_x}\left(1-n_x\beta_x-n_y\beta_y\right)=m, \quad m=1,2.... \quad (5)$$

The form-factor has the form [9]

$$F=N_eF_{inc}+N_e\left(N_e-1\right)F_{coh}, \quad (6)$$

where its parts, i.e. incoherent form-factor F_{inc} and coherent one F_{coh}, contain information about such beam parameters as sizes, form, energy, emittance.

The chosen coordinate system and the angles of radiation are shown in Fig. 2. The unit wave-vector $\mathbf{n}$ is defined as:

$$\begin{aligned} n_x &= \sin\theta\cos\phi, \\ n_y &= \cos\theta, \\ n_z &= \sin\theta\sin\phi. \end{aligned} \quad (7)$$

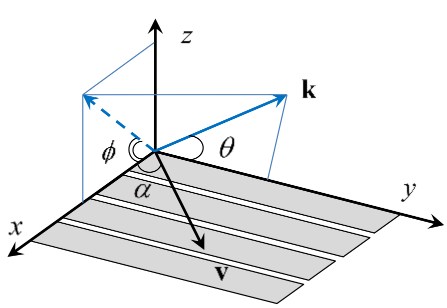

Figure 2: Chosen coordinate system and the angles of radiation.

The distribution of X-ray Smith-Purcell radiation energy from a single particle has the form [8]:

$$\frac{d^2W_1(\mathbf{n},\omega)}{d\Omega d\omega}=\frac{e^2}{c}\left(\frac{\varepsilon(\omega)-1}{4\pi\beta_x\varphi}\right)^2 e^{-2\rho h}\frac{\omega^4}{c^4}P, \quad (8)$$

where e is the charge of an electron,

$$P=\frac{\left|\left[\mathbf{n}',\left[\mathbf{n}',\mathbf{A}\rho^{-1}-i\mathbf{e}_z\right]\right]\right|^2}{\left|\rho-i(\omega/c)\sqrt{\varepsilon(\omega)-1+n_z^2}\right|^2}, \quad (9)$$

$$\varphi=\frac{\omega}{c}\frac{1}{\beta_x}\left(1-n_x\beta_x-n_y\beta_y\right), \quad (10)$$

$$\rho=\frac{\omega}{c\beta\gamma}\sqrt{1+\gamma^2\beta_x^{-2}\left(n_y\beta^2-\beta_y\right)^2}, \quad (11)$$

$$\mathbf{A}=\frac{\omega}{\beta_x c}\left(1-n_y\beta_y-\beta_x^2, n_y\beta_x-\beta_x\beta_y, 0\right), \quad (12)$$

and $\mathbf{n}'=\varepsilon^{-1/2}(\omega)\left(n_x,n_y,\sqrt{\varepsilon(\omega)-1+n_z^2}\right)$ is the unit wave-vector of the radiation in the media.

The radiation is maximal when the argument of exponent in Eq. (8) is minimal, i.e. when ρ is minimal:

$$n_y=\frac{\beta_y}{\beta^2}, \quad (13)$$

It means that the radiation is distributed over a conical surface with the main axis along y direction, see Fig. 3. The analytical description for SPR was given in [8], and the similar effect can also be seen in diffraction radiation [10].

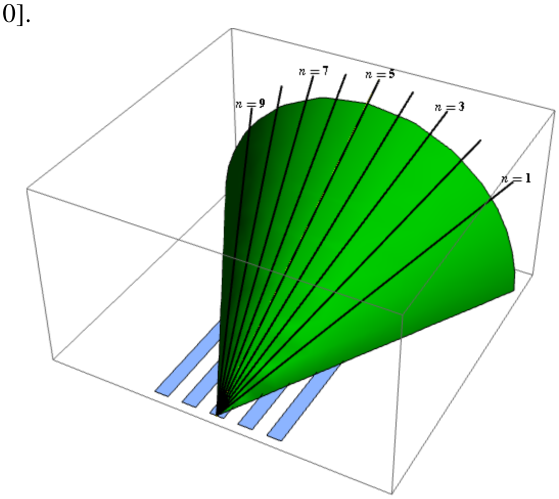

Figure 3: Conical distribution of Smith-Purcell radiation. The black lines correspond to the Smith-Purcell peaks.

The coherent and incoherent form-factors for uniform distribution and for Gaussian distribution for $\alpha=0$ were obtained and explored theoretically in [2, 3, 11]

For the beam moving at oblique angle α to the periodicity direction the form-factors can be calculated as:

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$$F_{inc} = \int_V d^3 r \left| e^{-i\mathbf{rQ}} \right|^2 f(\mathbf{r}),$$
$$F_{coh} = \left| \int_V d^3 r e^{-i\mathbf{rQ}} f(\mathbf{r}) \right|^2, \quad (14)$$

where the integrals are taken over the bunch volume V, the distribution function of the particles in the bunch $f(\mathbf{r})$ is the written in the system where the bunch is at rest,

$$\mathbf{Q} = \left(q_x \cos\alpha + q_y \sin\alpha, -q_x \sin\alpha + q_y \cos\alpha, q_z \right), \quad (15)$$

$$\mathbf{q} = \left(\frac{\omega - k_y v_y}{v_x}, k_y, -i\rho \right). \quad (16)$$

For the cylindrical bunch of the length l and the radius r_0 with the uniform distribution of the particles, moving at the angle α to the target periodicity, one can find:

$$F_{inc} = 2\frac{I_1(2\rho r_0)}{2\rho r_0},$$
$$F_{coh} = 4\frac{\sin^2(\omega l/2v)}{(\omega l/2v)^2}\frac{I_1^2(r_0\omega/c\beta\gamma)}{(r_0\omega/c\beta\gamma)^2}. \quad (17)$$

As one can see from Eq. (17), the coherent form-factor does not depend on the observation angle and the angle between the beam velocity and the direction of periodicity. In the incoherent form-factor this dependence exits (in ρ), but it is rather weak, and in the maxima defined by Eq. (13) it disappears.

The distribution of the Smith-Purcell peaks generated by a cylindrical bunch is shown in Fig. 4. The graph was plotted taking into account Eq. (13).

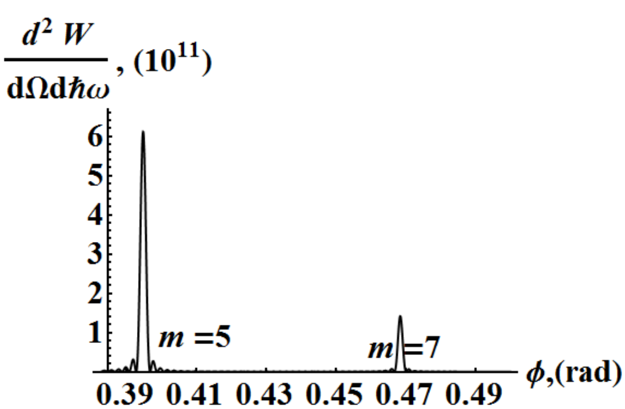

Figure 4: The distribution of X-Ray SPR over the conical surface at $\theta = \arccos\left(\beta^{-1}\sin\alpha\right)$. Here $\gamma = 2\cdot 10^6$ (electron energy $E = 1TeV$), $\hbar\omega_p = 26.1eV$ (beryllium), $d = 0.9\,\mu m$, $a = 0.45\,\mu m$, $\lambda = 12\,nm$, $h = 60\,\mu m$, $\alpha = 30^0$, $N = 20$, $N_e = 10^{10}$, $r_0 = 30\,\mu m$, $l = 60\,\mu m$.

If we suppose that the particles in the bunch moves at different angles to the general direction of the bunch, then Eq. (17) will contain the factor describing the velocity distribution, or even information about the divergence of the beam [12].

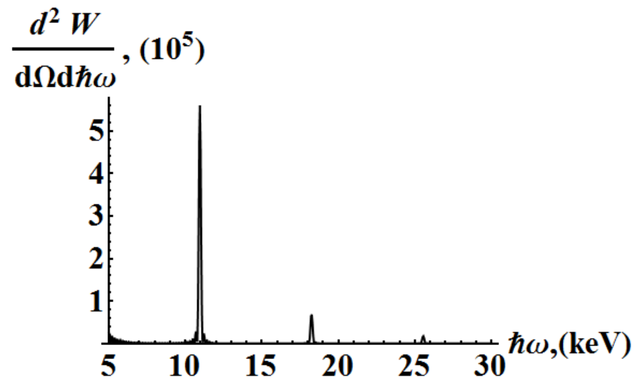

Figure 5: The distribution of X-Ray SPR over the conical surface at $\theta = \arccos\left(\beta^{-1}\sin\alpha\right)$, $\phi = 5^0$. Here $\gamma = 2\cdot 10^6$ (electron energy $E = 1TeV$), $\hbar\omega_p = 26.1eV$ (beryllium), $d = 0.1\,\mu m$, $a = 0.05\,\mu m$, $h = 1\,\mu m$, $\alpha = 30^0$, $N = 20$, $N_e = 10^{10}$, $r_0 = 1\,\mu m$, $l = 60\,\mu m$.

The Fig. 5 shows the distribution of the radiation intensity over radiated photons energies, and one can see the clear peaks of SPR, suitable for diagnostics.

CONCLUSION

Thus, X-ray SPR is emitted at the frequencies up to

$$\omega = \frac{\gamma c}{2h}, \quad (18)$$

which for the electron energies of the order of 1 TeV can reach the photon energies up to some 30-40 of KeV. Manufacturing the gratings with a period from some hundreds to tens of nanometres, we can obtain the SPR peaks under the angles from 3-4 to some tens of degrees, which is suitable for the experimental diagnostics schemes. The expression for spectral-angular distribution, generalizing the one obtained in [8], contains the information about both the target and beam parameters, including the length of the beam, which opens new possibility of submicron non-invasive diagnostics for the future electron colliders like CLIC, ILC, etc. In contrast with the optical range based diagnostics schemes, X-ray range makes it possible the non-invasive diagnostics with submicron and even nanometres accuracy.

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