

# TRANSVERSE COHERENT INSTABILITY STUDIES FOR THE HIGH-ENERGY PART OF THE MUON COLLIDER COMPLEX\*

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## Abstract

The International Muon Collider Collaboration (IMCC) is studying a 3 TeV center-of-mass muon collider ring as well as a possible next stage at 10 TeV. Current studies focus mainly on the 10 TeV collider and its injector complex. Muons being 200 times heavier than electrons, limitations from synchrotron radiation are mostly suppressed, but the muon decay drives the accelerator chain design. After the muon and anti-muon bunches are produced and 6D cooled, a series of Linacs, recirculating Linacs and Rapid Cycling Synchrotrons (RCS) quickly accelerate the bunches before the collider ring.

A large number of RF cavities are required in the RCS to ensure that over 90 % of the muons survive in each ring. The effects of cavities higher-order modes on transverse coherent stability have been investigated in detail, including a bunch offset in the cavities, along with possible mitigation measures.

In the collider ring, the decay of high-energy muons is a challenge for heat load management and radiation shielding. A tungsten liner will protect the superconducting magnet from decay products. Impedance and related beam stability have been investigated to identify the minimum vacuum chamber radius and transverse damper properties required for stable beams.

## INTRODUCTION

A muon collider could reach high luminosity and multi-TeV center-of-mass (c.o.m) energy collisions with leptons. The muon mass being 200 times larger mass than the electron, limitations from synchrotron radiation encountered with electron-positron colliders are suppressed [1]. The goal of the International Muon Collider Collaboration [2, 3] and the MuCol program [4] is to study a 3 TeV c.o.m  $\mu^+ - \mu^-$  collider, with the option of a following 10 TeV c.o.m stage.

Maximum instantaneous luminosities of  $1.8 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$  for the 3 TeV collider and  $20 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$  for the 10 TeV collider are targeted. Reaching these high luminosities requires to minimize the decay of both the muon and the anti-muon bunch. With a 10 km circumference for the 10 TeV collider, the revolution

period of a beam traveling at the speed of light is  $33 \mu\text{s}$ , much longer than the muon lifetime at rest of  $\tau_0 = 2.2 \mu\text{s}$ . However, thanks to relativistic time dilation, the muon lifetime in the laboratory frame  $\tau$  increases with the Lorentz factor  $\gamma$  as  $\tau = \gamma\tau_0$ . In the 10 TeV collider, with  $\gamma = 47323$ , the muon lifetime reaches  $\tau = 104 \text{ ms}$ .

The US MAP project [5] developed a muon/anti-muon production and acceleration concept, schematized in Fig. 1. Muons are first produced by hitting a target with a high power proton beam. Then both longitudinal and transverse emittances are reduced through a ionization cooling stage. Afterwards an acceleration stage provides a fast energy increase to the muon beams to quickly increase their  $\gamma$ .

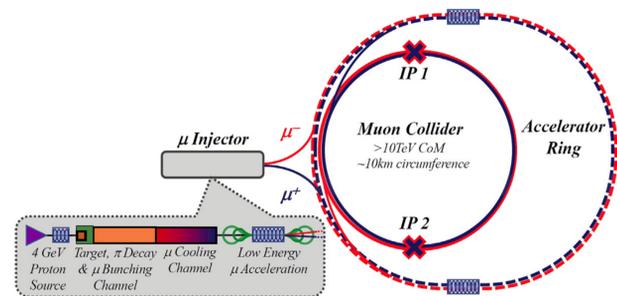


Figure 1: Proposed layout of a muon collider and its muon production and acceleration stages. Picture from Ref. [6].

Emittance preservation through the acceleration stages are fundamental to obtain the target luminosity in the absence of synchrotron radiation damping mechanism. Transverse beam coupling impedance models were developed for the first Rapid Cycling Synchrotron and the 10 TeV collider, and are then used to simulate coherent beam stability effects in these two accelerators.

## IMPEDANCE AND STABILITY STUDIES FOR THE RCS

The last acceleration stage before the 3 TeV collider ring comprises three Rapid Cycling Synchrotrons that increase the muon and anti-muon beam energy from  $\sim 60 \text{ GeV}$  to  $1.5 \text{ TeV}$ . For the 10 TeV collider, a fourth RCS would further accelerate the muon beams from  $1.5 \text{ TeV}$  to  $5 \text{ TeV}$ .

In each of these RCS, the targeted muon survival rate is at least 90 %. For the first RCS of the chain (RCS 1), whose parameters are summarized in Table 1, an energy increase of  $14.8 \text{ GeV}$  per turn is required, which in turn requires a total of  $20.9 \text{ GV}$  RF voltage for the ring [7].

\* Work funded by the Swiss Accelerator Research and Technology program (CHART) and by the European Union (EU). Views and opinions expressed are however those of the authors supported only and do not necessarily reflect those of the EU or European Research Executive Agency (REA). Neither the EU nor the REA can be held responsible for them.

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It is proposed to use 1.3 GHz TESLA-type superconducting RF cavities, originally designed for linear electron-positron collider, which are  $\sim 1$  m long and can reach up to  $50 \text{ MV m}^{-1}$  gradient [8, 9]. Longitudinal beam dynamics studies showed that these cavities could fit the RCS needs [7]. Assuming a cavity gradient of  $30 \text{ MV m}^{-1}$ ,  $\sim 700$  cavities would be needed for RCS 1.

Table 1: Machine Parameters for RCS 1

Circumference $C_0$	5990 m
Injection energy	63 GeV
Extraction energy	313 GeV
Energy gain per turn	14.8 GeV
Number of turns	17
Acceleration time	0.34 ms
Bunch intensity	$2.54 \times 10^{12}$
Bunch length at injection $1\sigma_z$	13.1 mm
Synchronous phase	$45^\circ$
Total RF voltage	20.9 GV
Synchrotron tune $Q_s$	1.52 (at injection)
Transverse normalized emittance	$25 \mu\text{m rad}$
Number of RF stations $n_{\text{RF}}$	32

High-order modes (HOM) generated by the cavities are the main concern for transverse beam stability. At first, a general stability criterion was developed to quickly estimate the impact on transverse coherent beam stability of a single HOM in the RCS 1 [10]. Afterwards a transverse impedance model was created using the HOM table from the Low-Loss type TESLA cavity described in Ref. [11]. Figure 2 shows the resulting real part of the transverse dipolar impedance.

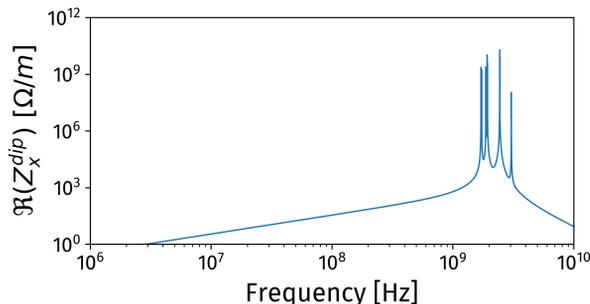


Figure 2: Transverse dipolar impedance (real part) versus frequency. 696 cavities are assumed in the model, and all transverse HOMs from the Low-Loss TESLA type cavity are included.

Tracking simulations were performed with the macroparticle code PyHEADTAIL [12]. Because of the high acceleration gradient and therefore RF voltage, the synchrotron tune  $Q_s$  in the RCS 1 is very large ( $Q_s = 1.52$  at injection energy). To meet the longitudinal phase space stability criteria  $Q_s < 1/\pi$  [13], the RF cavities are distributed over 32 equally spaced stations [7]. The transverse and longitudinal tracking and the wakefield effect are therefore divided in 32 segments in tracking simulations.

Simulations investigated the potential operation with an uncorrected chromaticity to avoid the use of chromatic sex-

tupole magnets and therefore save space in the lattice [10]. A factor of 2 on the total transverse impedance must be applied to reach the instability threshold. Then with a natural, negative, chromaticity of  $Q' = -19$ , the bunch would become unstable over the 17 turns of acceleration, leading to transverse emittance growth in the order of 20%. Further simulations are currently ongoing to investigate the effect of an initial transverse offset of the muon bunch that could be created by the injection system. Mitigation measures such as the use of one or multiple transverse damper units along the RCS 1 ring [14] are studied as well.

## IMPEDANCE AND STABILITY STUDIES FOR THE 10 TeV COLLIDER RING

Once in the collider ring, the two muon and anti-muon bunches are stored at a fixed energy for several milliseconds. Over time, they will decay to electrons and positrons, respectively. To avoid excessive radiation damage and heating of the magnets superconducting coils, these decay products must be intercepted. A radial scheme of the 10 TeV collider magnets, schematized in Fig. 3, foresees to use a tungsten shield to intercept these decay products and remove the excess heat [15, 16]. Based on considerations on beam optics, vacuum system, cryogenic system and beam stability, the innermost radius of the shield has been tentatively fixed to 23.5 mm [16–18]. The resistive-wall impedance created by this shielding was therefore investigated to check that the proposed radius is compatible with transverse coherent stability.

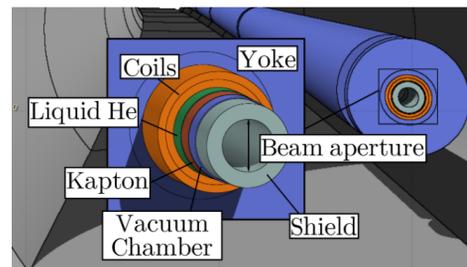


Figure 3: Radial scheme proposal for the collider magnets. The shield's function is to intercept muon decay products and protect the superconducting coils. Picture courtesy D. Calzolari [15].

The code ImpedanceWake2D [19] is used to compute the impedance model of chambers made of tungsten at 300 K (resistivity  $\rho_{\text{W},300\text{K}} = 55.6 \text{ n}\Omega\text{m}$ ) with a copper coating applied on the innermost radius (resistivity  $\rho_{\text{Cu},300\text{K}} = 17.9 \text{ n}\Omega\text{m}$ ). The chamber is for now assumed to have the same length as the complete collider  $C_0 = 10 \text{ km}$ . A parametric scan is performed on both the chamber radius (10 mm to 40 mm) and the copper coating thickness (0.1  $\mu\text{m}$  to 100  $\mu\text{m}$ ). This will allow to check with beam stability simulations the minimum radius achievable and determine the coating thickness needed to further reduce resistive-wall instabilities.

Figure 4 shows the real part of the dipolar beam coupling impedance obtained for a chamber radius  $r = 20 \text{ mm}$  and different copper coating thickness. The upper curve cor-

responds to a chamber without copper coating, the lower curve to a chamber with an infinite copper thickness. As expected chambers with thicker copper coating, such as the green line corresponding to a 100  $\mu\text{m}$  coating, follow the pure copper curve (blue line) over a larger frequency range and have therefore a smaller impedance overall than the uncoated tungsten chamber (orange curve).

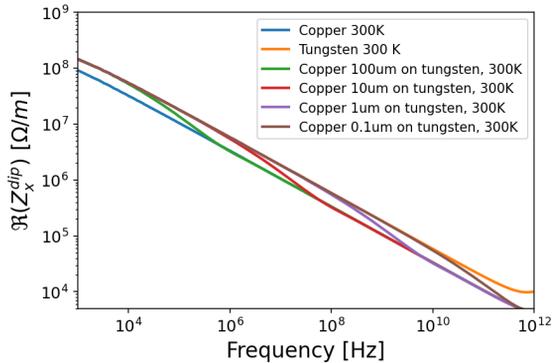


Figure 4: Transverse dipolar impedance (real part) versus frequency for a 10 km long chamber, with inner radius  $r = 20$  mm.

Stability simulations using the impedance models described beforehand are performed with the macroparticle tracking code PyHEADTAIL [12]. Impedance models are first Fourier-transformed to wakefields, then a single muon beam is tracked through the transverse linear map, the longitudinal map, the wakefields and the transverse feedback system. For the longitudinal map, all RF cavities are assumed to be lumped in one location, as the synchrotron tune is much smaller than  $1/\pi$  [13]. The strength of the transverse feedback is scanned in the 2-turn to 500-turn gain range, and a case without feedback is also simulated. The beam is assumed to have no initial transverse offset from the injection system. Beam parameters used for the simulations are reported in Table 2.

The muon decay will cause a reduction of the bunch intensity over the storage time in the collider ring. With  $\gamma = 47323$  at 5 TeV, the muon lifetime in the laboratory frame increases to  $\tau = 104$  ms, equivalent to 3121 turns in the 10 km long ring. A PyHEADTAIL module was developed to include this effect in simulations.

The results of coherent beam stability simulations are summarized in Fig. 5. Assuming that the emittance growth should remain below 10% after 3000 turns (which corresponds roughly to the muon lifetime), the plot shows the minimum chamber radius required to meet this criterion, as a function of the transverse damper strength.

For coatings thicker than 1  $\mu\text{m}$  (purple, red and green dashed curves in Fig. 5, all superimposed), results coincide with those for a chamber made of pure copper. With a thinner coating such as 0.1  $\mu\text{m}$  (brown curve) the required minimum radius increases and becomes intermediate with those that would be required for an uncoated tungsten shielding. With a 50-turn damping time, the minimum chamber radius required with a coating thickness above 1  $\mu\text{m}$  is 13 mm, well

Table 2: Machine Parameters for the Collider

Circumference $C_0$	10 km
Beam energy	5 TeV
Bunch intensity	$1.8 \times 10^{12}$ muons
Bunch length $l\sigma_z$	1.5 mm
Transverse norm. emittance	25 $\mu\text{m}$ rad
Momentum compaction factor $\alpha_p$	$-2.0 \times 10^{-6}$
Synchrotron tune $Q_s$	$2.33 \times 10^{-3}$
Average Twiss $\beta_x / \beta_y$	85 m / 51 m
Chromaticity $Q'_x / Q'_y$	0 / 0
Number of macroparticles	20000
Number of turns simulated	5000
Number of bunches simulated	1
Transverse damper gain	2 to 500 turns + no damper

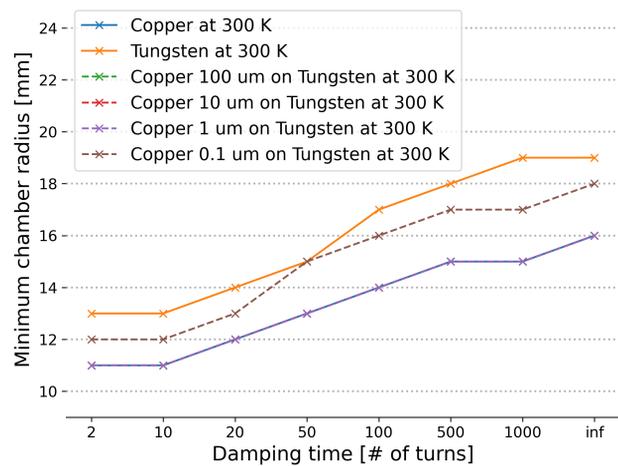


Figure 5: Minimum chamber radius required to keep the emittance growth below 10% after 3000 turns, for the different copper coating thickness considered and pure copper or pure tungsten (no copper coating).

below the 23.5 mm radius currently considered for the innermost radius of the tungsten shield.

## CONCLUSION AND OUTLOOK

Transverse coherent effects in the Rapid Cycling Synchrotron and the 10 TeV collider ring of the muon collider are currently being investigated. In the RCS, the effects of HOMs created by the large number of RF cavities have been considered, in combination with chromaticity or initial injection offset. For the collider ring, the inner radius of the magnet shielding required to preserve transverse emittance over one muon beam lifetime is compatible with the current design constraints from beam optics that require a 23.5 mm radius. With a copper coating thickness of 1  $\mu\text{m}$  at least, this radius can reach 13 mm if a 50-turn transverse damper is implemented as well. Further studies on the effect an initial transverse offset of the injected beam and of longitudinal parameters on transverse coherent stability, such as operation with zero momentum compaction and no RF voltage, are being undertaken.

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