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A LINEARIZED VLASOV METHOD FOR THE STUDY OF TRANSVERSE e-CLOUD INSTABILITIES

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Abstract

Using a Vlasov approach, e-cloud driven instabilities can be modeled to study beam stability on time scales that conventional Particle In Cell simulation methods cannot access. The Vlasov approach uses a linear description of e-cloud forces that accounts for both the betatron tune modulation along the bunch and the dipolar kicks from the e-cloud. Forces from e-clouds formed in quadrupole magnets as well as dipole magnets have been expressed in this formalism. In addition, the Vlasov approach can take into account the effect of chromaticity. To benchmark the Vlasov approach, it was compared with macroparticle simulations using the same linear description of e-cloud forces. The results showed good agreement between the Vlasov approach and macroparticle simulations for strong e-clouds, with both approaches showing a stabilizing effect from positive chromaticity. This stabilizing effect is consistent with observations from the LHC.

INTRODUCTION

Electron clouds, also known as e-cloud, in circular accelerator beam chambers can induce coupled and single-bunch instabilities [1, 2]. While the former can usually be controlled using standard transverse feedback systems, mitigating single-bunch instabilities, caused by fast intra-bunch motion, requires wide-band feedback systems, which are typically out of reach in accelerators operating with short bunches [3]. Alternatively, these instabilities can often be mitigated by large chromaticity and/or octupole magnets to introduce amplitude detuning [4].

The study of e-cloud instabilities heavily relies on numerical simulations. Conventional simulations, which employ macroparticle tracking and the Particle-In-Cell method for e-cloud-beam interaction, are computationally intensive, which makes evaluation of slow instabilities challenging [5]. Alternatively, the Vlasov equation, describing multi-particle systems under conservative forces, can be used to simulate beam instabilities due to collective effects, particularly through the linearized Vlasov equations derived from perturbation theory [6, 7].

The Vlasov equation is applicable when simulating instabilities driven by beam-coupling impedances [8]. Efforts have also been made to extend this approach to simulate e-cloud-driven instabilities, where e-cloud forces are represented as impedances. However, both dipolar forces and the betatron tune modulation along the longitudinal coordinate are required to describe the e-cloud forces in the Vlasov approach [9, 10].

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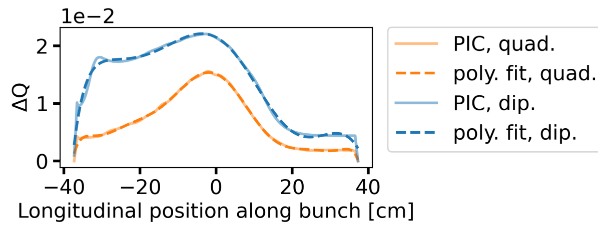

Figure 1: The betatron tune introduced by the e-cloud in the LHC quadrupoles, orange, and the LHC dipoles, blue, at high e-cloud conditions.

e-CLOUD IN VLASOV

Linearized Description of e-Cloud Forces

Quadrupolar forces When a bunch passes through an e-cloud, it attracts electrons, increasing the electron density at the beam's location, which alters the betatron tune along the bunch. This detuning can be characterized with single-pass PIC simulations. Fig. 1 shows the resulting tune modulation for the parameters in Table 1.

Table 1: The table presented here lists the simulation parameters that exert the strongest influence on the characteristics of the e-cloud-driven instability.

Parameter	Value
SEY	2.0
Bunch Intensity	1.2e+11
interaction points per turn	8
element with e-cloud	quadrupoles or dipoles
beam energy	450 GeV (injection)

To model the betatron tune modulation caused by the e-cloud, a polynomial is used:

$$\Delta Q(z) = \sum_{n=0}^{N_p} A_n z^n \ . \tag{1}$$

The results in Fig. 1 demonstrate that a realistic model can be obtained by truncating the sum at $N_p = 10$. There is the flexibility to incorporate detuning from e-cloud in quadrupoles, dipoles, or both into the simulations.

Dipolar forces To model the dipolar forces from e-cloud, a set of sinusoidal bunch distortions are selected,

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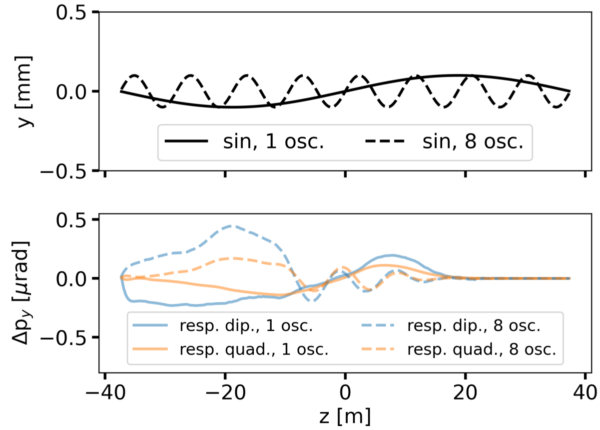

Figure 2: Two sinusoid bunch distortions (top) of varying number of oscillations along the bunch and the corresponding response functions (bottom) generated by e-cloud in dipoles, blue, and in quadrupoles, orange.

$h_n(z)$, which satisfy the orthogonality condition:

$$\int h_n(z)h_{n'}(z)\mathrm{d}z = H_n^2\delta_{n,n'} \,, \tag{2}$$

where H_n is the norm of $h_n(z)$ and $\delta_{n,n'}$ is the Kronecker delta. The transverse centroid as a function of the position along the bunch, $\bar{x}(z)$, can be described by a linear combination of $h_n(z)$ functions:

$$\bar{x}(z) = \sum_{n=0}^{\infty} a_n h_n(z), \text{ where } a_n = \frac{1}{H_n^2}\int \bar{x}(z)h_n(z)\mathrm{d}z \,. \tag{3}$$

The response functions, $k_n(z)$, represent the e-cloud kick along the bunch resulting from a distortion of the bunch distribution equal to $h_n(z)$. From simulations, it is possible to verify that there is linear behavior, such that the kick $\Delta x'$ of an arbitrary distribution $\bar{x}(z)$ is given by:

$$\Delta x' = \sum_{n=0}^{\infty} a_n k_n(z) \,, \tag{4}$$

where the coefficients a_n are given by Eq. (3).

The responses, $k_n(z)$, can be obtained by conducting short, single-pass, PIC simulations. Figure 2 illustrates an example of two sinusoidal bunch distortions (top) with varying numbers of oscillations along the bunch and their corresponding response functions (bottom) after passing through the e-cloud in dipoles, (blue) and quadrupoles (orange).

Vlasov Method

The Vlasov equation governs the time evolution of a particle distribution function $\psi(y, y', z, \delta; t)$, with no particle creation or destruction, and interactions solely through electromagnetic fields. One can introduce a perturbation $\Delta\psi$ and linearize the Vlasov equation to first order, which with an added detuning term dependent on longitudinal phase space, $\Delta Q(r, \phi)$, gives [7, 10]:

$$\frac{\partial \Delta\Psi}{\partial t} - \omega_0(Q_{y0} + \Delta Q(r,\phi))\frac{\partial \Delta\Psi}{\partial \theta_y} + \omega_s\frac{\partial \Delta\Psi}{\partial \phi} = -\frac{\eta g_0(r)}{\omega_s m_0 \gamma}\frac{df_0}{dJ_y}\sqrt{\frac{2J_yR}{Q_{y0}}}\sin\theta_y F_y^{coh}(z,t) \,. \tag{5}$$

Here, ω_0 is the revolution angular frequency, Q_{y0} is the unperturbed betatron tune, ω_s is the synchrotron angular frequency, η is the slippage factor, m_0 is the particle rest mass, γ is the relativistic factor, and R is the accelerator radius. Transverse dipolar forces from the e-cloud are in $F_y^{coh}(z,t)$, while the resulting detuning and chromaticity is in the term $\Delta Q(r,\phi)$, which depends on longitudinal phase space coordinates. The polar coordinates in the longitudinal (r, ϕ) and transverse (J_y, θ_y) phase space, where J_y is the vertical action, are defined as follows:

$$\begin{cases} z = & r\cos\phi \\ \delta = & \frac{\omega_s}{v\eta} r\sin\phi \end{cases} \qquad \begin{cases} y = & \sqrt{\frac{2J_yR}{Q_{y0}}}\cos\theta_y \\ y' = & \sqrt{\frac{2J_yQ_{y0}}{R}}\sin\theta_y \,. \end{cases} \tag{6}$$

Here, v is the velocity. In addition, the unperturbed bunch distribution ψ_0 has been factorized as:

$$\psi_0 = \frac{\eta v}{\omega_0} f_0(J_y) g_0(r) \,. \tag{7}$$

By introducing an ansatz for $\Delta\psi$ of the form $e^{j\Omega t}\Delta\psi(J_x, \theta_x, J_y, \theta_y, r, \phi)$, where the time dependence is contained in the complex exponential $e^{j\Omega t}$, the linearized Vlasov equation (5) can be reduced to an eigenvalue problem. The instability growth rate, $-\mathrm{Im}(\Omega)$, and the mode frequency shift, $(\frac{\mathrm{Re}(\Omega)}{\omega_0} - Q_0)/Q_s$, can then be obtained from the eigenvalues. For a detailed derivation, see Ref. [11].

BENCHMARKS WITH MACROPARTICLE SIMULATIONS

Simulations using PyHEADTAIL as a macroparticle tracking, and the e-cloud forces in the Vlasov formalism, were used to benchmark the Vlasov approach. The linearized Vlasov equation, now treated as an eigenvalue problem, yields a set of possible modes, each with a complex angular frequency Ω. In Fig. 3, one vertical line of colored dots represents the real part of the solution of the Vlasov problem taking into account e-cloud formed in both quadrupoles and dipoles. The mode frequency shift from macroparticle simulations, shown by the black dots, is calculated by performing a spectral analysis on the tracking data. The top plot in Fig. 3 shows the mode frequency shift as a function of chromaticity for simulations including only diploar forces from e-cloud. The mode frequency shift obtained from the Vlasov simulations for the strongest modes corresponds very well to the one obtained from macroparticle simulations.

When detuning along the bunch is considered, the Vlasov modes shown in the bottom graph of Fig. 3 are obtained. These modes exhibit a chromaticity dependence similar to the modes in the top graph. However, additional weaker modes appear between the Q_s lines.

The imaginary part of each complex frequency Ω provides the mode's growth rate. The resulting growth rates of the Vlasov modes (blue) as a function of chromaticity can be seen in Fig. 4. The top graph displays results from simulations excluding the detuning term from e-cloud and

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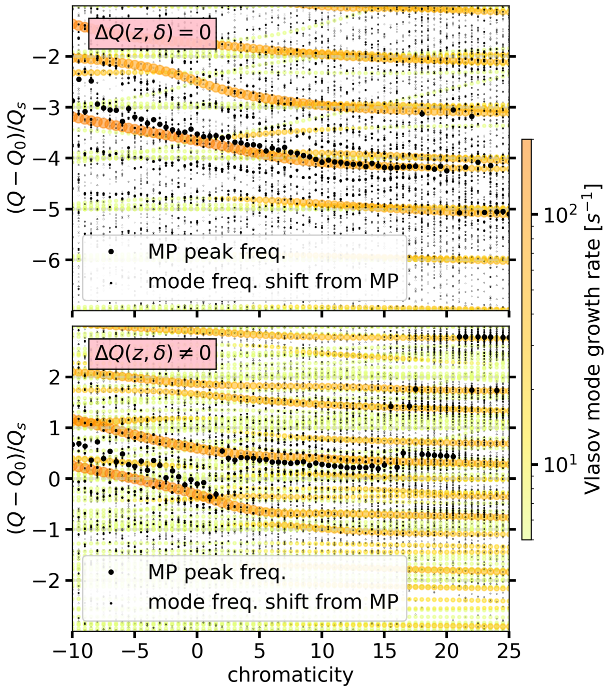

Figure 3: The mode frequency shift vs. chromaticity for simulations with (bottom) and without (top) the detuning term, $\Delta Q(r,\phi)$, from e-cloud. The calculated Vlasov modes are plotted as green-yellow-orange and the spectra from the macroparticle tracking is plotted as black dots.

the bottom graph displays results including this term. The growth rate of macroparticle simulations (orange), determined through exponential fits, follow the behavior of the worst Vlasov mode for chromaticities smaller than 15 in both cases. However, at high chromaticities in the bottom graph, macroparticle simulations exhibit a stabilizing effect not seen in Vlasov modes. One possible explanation for this observation is that such a stabilizing mechanism is not correctly captured by the employed first-order perturbation approach.

FIRST COMPARISON AGAINST MEASUREMENTS

Instability measurements were conducted at the Large Hadron Collider in conditions with strong e-cloud, after a large fraction of the ring had been exposed to air. Trains of 24 bunches were injected with different values of vertical chromaticity and the bunch by bunch positions was measured turn by turn.

The instability growth rates were obtained from the measurement data and the results are plotted in Fig. 5, where blue corresponds to measurements from beam 1 and red corresponds to measurements from beam 2. The growth rate at chromaticity 5 is around 40 s^{-1}, and then decreases with chromaticity. Macroparticle simulations using the Vlasov e-cloud formalism of forces, plotted as indigo, exhibit a similar dependence on chromaticity. However, there is better agreement with simulations if one assumes that a larger fraction of the e-cloud is in quadrupoles, as plotted as teal.

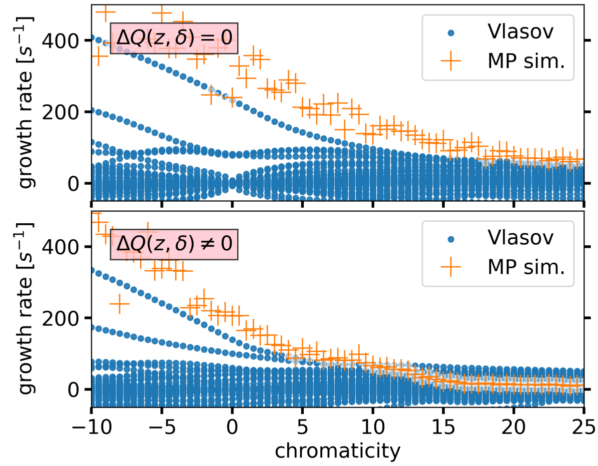

Figure 4: Instability growth rate from macro particle simulations (orange) and from calculated Vlasov modes (blue) as a function of chromaticity for only dipolar force (top) and with detuning from e-cloud (bottom).

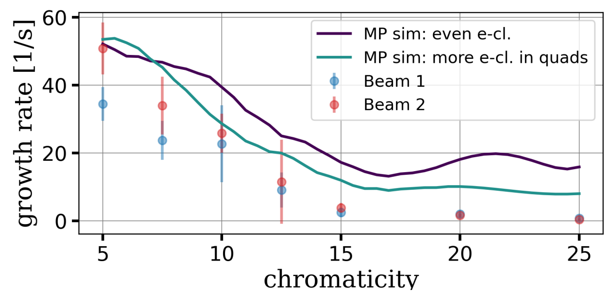

Figure 5: The instability growth rates from measurement of beam 1 (blue) and beam 2 (red) for several values of chromaticity. The simulated growth rates assuming even e-cloud formation in dipole and quadrupole magnets (indigo) and assuming sightly more e-cloud in quadrupoles (teal).

CONCLUSIONS

e-cloud resulting to both dipolar forces and a detuning along the bunch have been expressed in a dedicated Vlasov formalism for e-cloud forming in dipole as well as quadrupole magnets. The Vlasov modes display similar behavior to macroparticle simulations using the same linearized description of the forces. However, at high chromaticity there is a stabilizing mechanism in the macroparticle simulations not captured in the linearized Vlasov approach. In general, simulations predict a stabilizing effect of chromaticity, which is observed also in measurements.

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