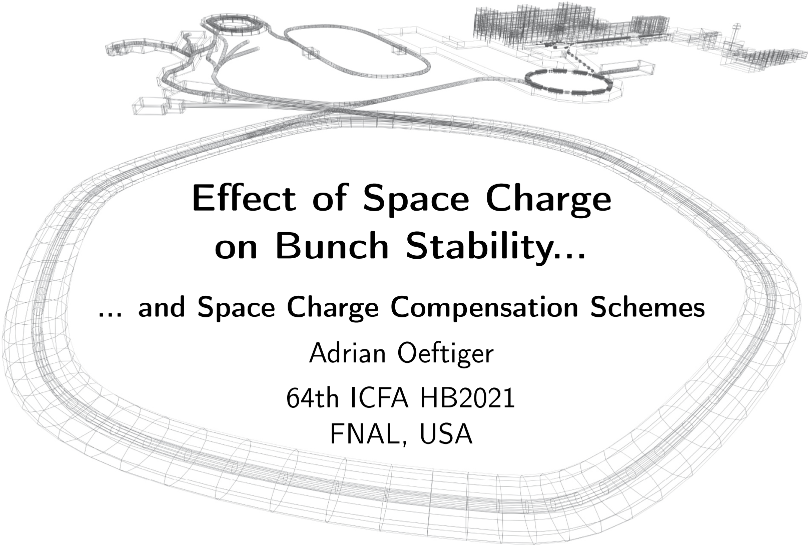

Effect of Space Charge on Bunch Stability...

... and Space Charge Compensation Schemes

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64th ICFA HB2021
FNAL, USA

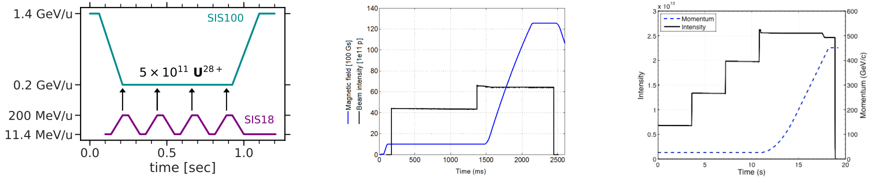
Abstract

In order to advance high-intensity synchrotrons, understanding the long-term dynamics of stored bunches at strong space charge conditions is a crucial ingredient. In many cases, the primary intensity limitation is given by the interplay of space charge and betatron resonances.

This talk discusses the recently published insights on how the apparent contradiction between theoretical studies ("coherent advantage") and high-intensity machine operation + dedicated beam experiments may be resolved: besides Landau damping of nonlinear parametric resonances, also incoherent resonance mechanisms and synchrotron motion play a crucial role. In particular this contribution addresses the long standing question of the validity of predictions by space charge simulations based on fast approximative (frozen) models. The talk concludes with results for the FAIR synchrotron SIS100, which demonstrate the matching predictions for resonance-free working points from long-term simulations using high-resolution self-consistent and frozen space charge models. The outlook phrases open questions with respect to space charge compensation schemes.

Synchrotrons operating close to space charge limit:

- **long duration**: model up to seconds of storage time (accumulation)
- **bunched beam**: large space charge tune footprints
- **complex dynamics** due to synchrotron motion
- **goal**: understand and alleviate detrimental impact of space charge induced crossing of betatron resonances



(a) FAIR SIS18-SIS100 cycling (b) CERN PS cycle (pre LS2) (c) CERN SPS cycle (pre LS2)

Figure: Synchrotron examples in strong space charge regime

The Apparent Contradiction

Ideally aim for self-consistent space charge simulations (traditionally PIC)

- numerical challenges for modelling seconds of storage time (non-symplecticity, noise in PIC models)

  $\longrightarrow$ **theory perspective**
    - often for 2D coasting or short-term 3D conditions
    - prediction of "coherent advantage"

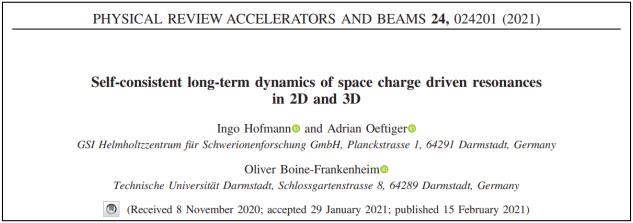
  $\longrightarrow$ **machine perspective**
    - absence of clear signature of coherent dynamics
    - simulation studies accompanying beam experiments typically resort to "frozen" (Gaussian) field map models

    $\rightsquigarrow$ only incoherent resonance dynamics covered!
    Uncertainty about coherent resonance aspects!

- many contributions on this topic over the years (see [1] for review)
  - parametric and coherent resonance theory:
    H. Okamoto, K. Kojima, C. Li, D. Jeon, I. Hofmann, ...
  - frozen modelling of space charge:
    G. Franchetti, V. Kapin, Y. Alexahin, F. Schmidt, H. Bartosik, ...

$\Longrightarrow$ basic agreement: maximum incoherent tune shift too conservative!

Motivation behind this Study


PHYSICAL REVIEW ACCELERATORS AND BEAMS **24,** 024201 (2021)

Self-consistent long-term dynamics of space charge driven resonances in 2D and 3D

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(Received 8 November 2020; accepted 29 January 2021; published 15 February 2021)

Figure: Ref. [1]

- understand apparent absence of coherent dynamics
  ⟶ **I. Landau damping**
- leverage high performance computing for accurate modelling
  ⟶ **II. long-term behaviour of 3D bunches**
- validate fast and accurate prediction models to identify *resonance-free* working points
  ⟶ **III. frozen space charge models**

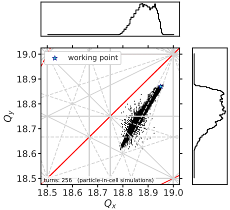
⟹ push the "space charge limit"!

I. Landau Damping of Parametric Coherent Resonances

Incoherent vs. Coherent

incoherent perspective

⇝ tunes of all particles in bunch distribution

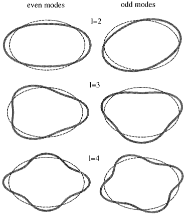


incoherent resonance condition:

$$m k_{xy} = h \cdot 360^\circ$$

coherent perspective

⇝ tunes of eigenmodes of transverse distribution moments



courtesy I. Hofmann [2]

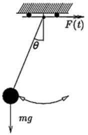
coherent resonance condition:

$$m \underbrace{\left(k_0 - C_m \Delta k_{KV}\right)}_{\text{mode tune}} = h \cdot 360^\circ$$

Parametric Resonance

non-parametric resonance

Externally driven harmonic oscillator:



(Ref. [3])

governed by e.o.m. of type

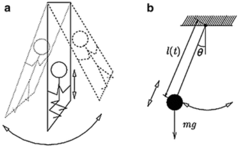
$$x'' + \kappa_0^2 x = F(s)$$

$\longrightarrow$ amplitude: **linear** growth

$\Longrightarrow$ beam dynamics example:
integer (dipole error) resonance

parametric resonance

Parametric harmonic oscillator:



governed by e.o.m. of type

$$x'' + \kappa_0^2 x = -\kappa_0^2 F(s) x$$

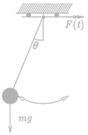
$\longrightarrow$ amplitude: **exponential** growth

$\Longrightarrow$ beam dynamics example:
180° stop band in FODO cell
(Mathieu instability)

Parametric Resonance

non-parametric resonance

Externally driven harmonic oscillator:



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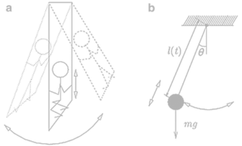
parametric resonance

Parametric harmonic oscillator:



Attention!

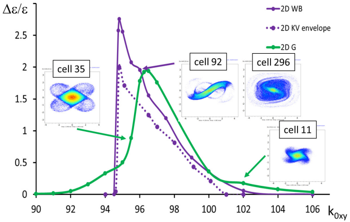
resonance frequency halved!

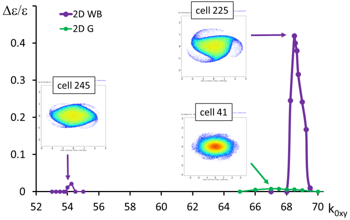
$\rightsquigarrow$ consequence for synchrotrons: driving harmonic $h \mapsto h/2$

$\Longrightarrow$ parametric resonances in tune diagram appear twice as dense

Parametric Coherent Resonances

Fig.: Gaussian distribution in FODO





$\Delta k_{KV} = 12°$

resonance condition

$$m\left(k_0 - C_m \Delta k_{KV}\right) = \frac{1}{2} 360°$$

Waterbag distribution:

- $m = 2$: envelope instability
  $\Longrightarrow$ 90° stop band
- $m = 3$: sextupole moment instability
  $\Longrightarrow$ 60° stop band
- $m = 4$: octupole moment instability
  $\Longrightarrow$ 45° stop band

$\Longrightarrow$ Gaussian distribution: no coherent response for nonlinear orders!

Landau Damping

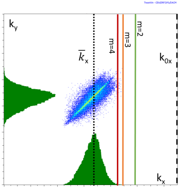


Fig.: waterbag

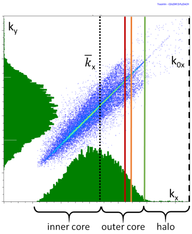


Fig.: Gaussian

Landau damping requires mode frequency inside incoherent spectrum $f_0(k_{xy})$ on descending flank:

$$\partial f_0 / \partial k_{xy} < 0$$

- waterbag distribution: $m \leq 4$ outside spectrum!
  - $\Longrightarrow$ nonlinear modes unstable
- Gaussian distribution: $m > 2$ inside spectrum
  - $\Longrightarrow$ nonlinear modes stabilised via Landau damping

Divide Gaussian spectrum into 3 parts:

- **inner core**: rising flank of f_0 until average k_{xy}
- **outer core**: descending flank of f_0 (until $m = 2$)
- **halo**: beyond coherent modes

II. Long-term Behaviour of 3D Bunches

From 2D to 3D

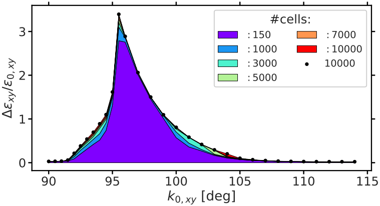


Fig.: 2D coasting beam

RF $\Longrightarrow$ on

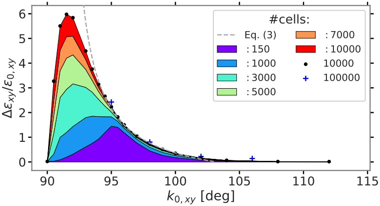


Fig.: 3D bunched beam ($k_{xy}/k_z = 300$)

Coasting beam:

- short-term coherent dynamics (150 FODO cells sufficient)
- long-term incoherent resonances (halo and outer core)

Bunched beam:

- short-term coherent dynamics shadowed in the long term
- synchrotron oscillation reduces growth of coherent peak

From 2D to 3D

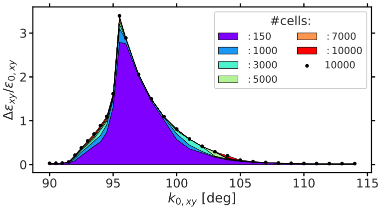


Fig.: 2D coasting beam

RF $\Longrightarrow$ on

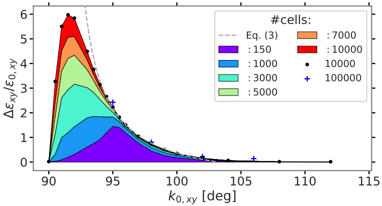


Fig.: 3D bunched beam ($k_{xy}/k_z = 300$)

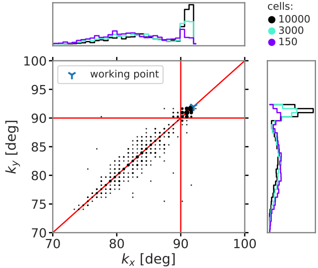
Asymptotic rms emittance growth in bunched beams:

$$\left(\frac{\Delta\epsilon}{\epsilon}\right)_{asympt.} \approx \frac{\Delta k_{KV}}{k_0 - 90^\circ} - 1$$

$\Longrightarrow$ compression of tune footprint into interval between resonant tune and bare tune: $[90^\circ, k_0]$

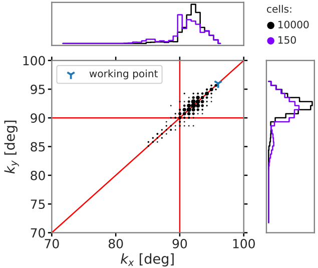
Tune Footprints & Dynamics

halo case, $k_0 = 92°$



- particles slowly drift across resonant tune
- tune spread shrinks (and rms emittance grows) until all particles above 90°

outer core case, $k_0 = 96°$



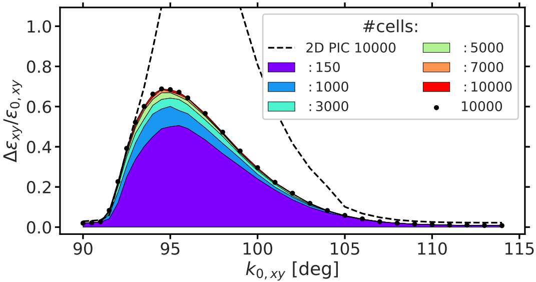
- initial envelope instability carries 80% of particles rapidly above 90°
- afterwards continuous drift across resonant tune

III. Frozen Space Charge Models

Frozen Space Charge

Model space charge with fixed frozen field map of Gaussian distribution:

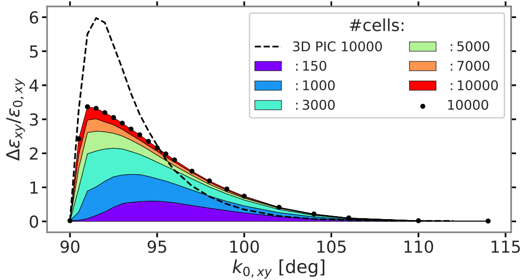
- 2D results mainly missing coherent dynamics (envelope instability)
  - ⟶ heavy underestimation of rms emittance growth (apart from halo)



Frozen Space Charge

Model space charge with fixed frozen field map of Gaussian distribution:

- 2D results mainly missing coherent dynamics (envelope instability)
  - $\longrightarrow$ heavy underestimation of rms emittance growth (apart from halo)
- 3D results mainly missing change of distribution (rms, profile)
  - $\longrightarrow$ underestimation in halo, overestimation in core
  - $\Longrightarrow$ frozen model useful for conservative prediction of resonance-free tunes



Summing Up...

Summary

Long-term evolution of 3D Gaussian bunches subject to space charge:

Observations

- parametric coherent resonances:
  - Landau damped for nonlinear orders $m > 2$
  - $m = 2$ envelope instability remains (90° stop band)
- coherent resonances:
  - short-term effects (fast saturation)
  - coherent stop band *embedded within* incoherent stop band
- space charge driven 90° stop band:
  - asymptotic behaviour $\left(\frac{\Delta\epsilon}{\epsilon}\right)_{asympt.} \approx \frac{\Delta k_{KV}}{k_0 - 90^\circ} - 1$

Conclusions

$\Longrightarrow$ resonance-free tune areas *bounded by incoherent* resonance stop bands

$\Longrightarrow$ long-term frozen space charge simulations *validated* for identification of resonance-free tune areas (pessimistic prediction!)

$\Longrightarrow$ space charge limit: $\Delta Q_{KV} = 0.25 \Longleftrightarrow \Delta Q_{SC,Gauss} = 0.5$

Application to FAIR SIS100

Full-scale simulations[1] for SIS100 (20 000 turns = 1 680 000 cells):

- detailed SIS100 optics model with nonlinear field errors (up to $(a,b)_7$)
- heavy-ion beam with maximum space charge tune shift $\Delta Q_y = -0.3$

$\Longrightarrow$ PIC and frozen models match in predicted resonance-free tune areas

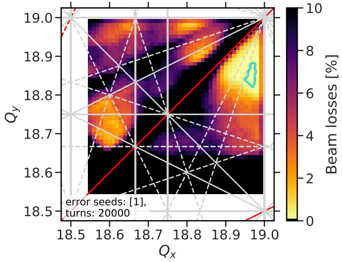


Figure: fixed frozen model, $\mathcal{O}(10\,\text{min/tune})$

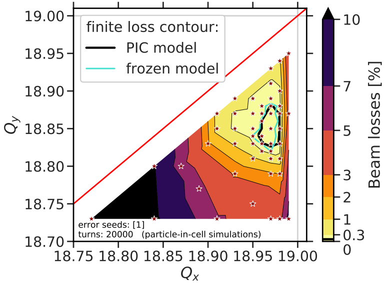


Figure: self-consistent PIC, $\mathcal{O}(4\,\text{d/tune})$

[1] cf. plenary talk by O. Boine-Frankenheim tomorrow

Conclusion & Outlook

Take home messages:

- need not worry about parametric coherent resonances as long as tune footprint remains unmodified (Landau damping)
- incoherent resonances can appear in outer core (no rms emittance growth in inner core, i.e. beyond Δk_{KV})
- frozen (3D Gaussian) space charge models are a viable tool for fast prediction of resonance-free working points!

Thoughts on recently suggested space charge compensation methods:

- shrinking the transverse tune footprint might unleash (otherwise Landau damped) nonlinear parametric coherent modes
  - ⇝ nonlinear electron lens profiles could be problematic in realistic synchrotron layouts

Conclusion & Outlook

Take home messages:

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- frozen (3D Gaussian) space charge models are a viable tool for fast

Thank you for your attention!

Thoughts on recently suggested space charge compensation methods:

- shrinking the transverse tune footprint might unleash (otherwise Landau damped) nonlinear parametric coherent modes
  - ⇝ nonlinear electron lens profiles could be problematic in realistic synchrotron layouts

References I

[1] Ingo Hofmann, Adrian Oeftiger, and Oliver Boine-Frankenheim. "Self-consistent long-term dynamics of space charge driven resonances in 2D and 3D". In: *Phys. Rev. Accel. Beams* 24 (2 Feb. 2021), p. 024201. DOI: 10.1103/PhysRevAccelBeams.24.024201. URL: https://link.aps.org/doi/10.1103/PhysRevAccelBeams.24.024201.

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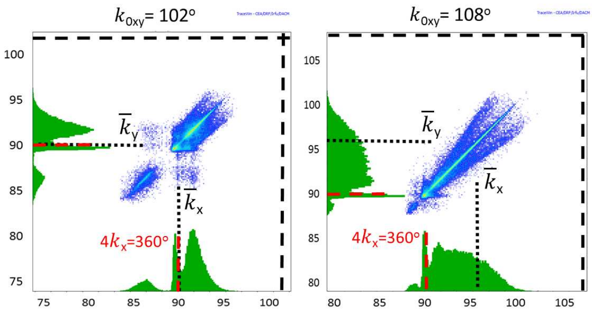
[3] Alan Champneys. "Dynamics of Parametric Excitation". In: *Encyclopedia of Complexity and Systems Science*. Ed. by Robert A. Meyers. New York, NY: Springer New York, 2009, pp. 1–31. ISBN: 978-3-642-27737-5. DOI: 10.1007/978-3-642-27737-5_144-3. URL: https://doi.org/10.1007/978-3-642-27737-5_144-3.



- left case: transition inner to outer core
  - ⟶ finite rms emittance growth, drifting of particles across resonance
- right case: inner core
  - ⟶ clear peak in incoherent spectrum on resonance tune, negligible rms emittance growth, no drifting