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CHROMATICITY MEASUREMENT USING BEAM TRANSFER FUNCTION IN HIGH ENERGY SYNCHROTRONS

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Abstract

Control of chromaticity is often critical to mitigate collective instabilities in high energy synchrotrons, yet classical measurement methods are of limited use during high intensity operation. We explore the possibility to extract this information from beam transfer function measurements, with the development of a theoretical background that includes the impact of wakefields and by analysis of multi-particle tracking simulations. The investigations show promising results that could improve the operation of the HL-LHC by increasing stability margins.

INTRODUCTION

Operation with the lowest positive chromaticity is usually advised in high energy hadron synchrotrons (i.e. above transition) to both ensure the self-stabilisation of the rigid bunch mode while maximising the beam lifetime. In machines with a fast cycle, the chromaticity is often set empirically to the lowest value that does not result in instabilities. For machines with a long cycle, such as the Large Hadron Collider, this approach is time consuming. In addition, dynamic effects in superconducting magnets and lengthy beam optics operations may lead to strong variations of the chromaticity requiring dynamic corrections. In the LHC, the chromaticity is corrected through the cycle based on a measurement during a dedicated cycle featuring an energy modulation driven by the RF cavities. Such an energy modulation cannot be used with high intensity beams (i.e. multiple bunches) due to the large amount of beam losses generated. The operation with high intensity beams therefore relies on the corrections done with low intensity beams and the cycle-to-cycle reproducibility of the machine. The design chromaticity for the LHC is 2 units [1]. Figure 1 shows the octupole strength required to stabilise the beams, taking into consideration the residual noise that the beam experiences (dashed lines). We observe that the design chromaticity of 2 units is quite optimal to minimise the strength of the octupole required to stabilise the beams, while avoiding the potential adverse effects on the single particle dynamics that could occur at high chromaticity, e.g. around 20 units. However, this working point is close to the most critical configuration in terms of required octupole strength, i.e. around a chromaticity of 0. The cycle-to-cycle reproducibility is not enough to guarantee a sufficient stability of the chromaticity at the optimal working point. The operational working point was therefore set at 15 units [2], allowing for potential cycle-to-cycle variations in the order of ±5 units. Due to this uncertainty, the optimal range of chromaticities, here around 3 units, cannot be exploited. Enabling this opportunity could reduce the need for octupole strength by about a factor 2, thus offering significant operational margins, e.g. in terms of control of linear coupling or residual lattice nonlinearities, which need to be compensated with more strength in the arc octupoles to maintain Landau damping [5]. Alternatively, this might also allow for a reduction of the collimator gaps, thus improving the cleaning efficiency.

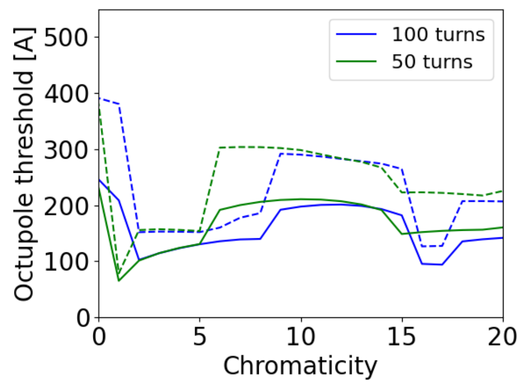

Figure 1: Octupole current required for the stabilisation of the beam in the LHC in the configuration described in Table 1. The solid lines correspond to the octupole threshold obtained neglecting the impact of residual sources of noise on the beam (e.g. [3]), while the dashed lines take into account the destabilising effect of noise [4]. The blue and green curves are labelled with the corresponding damping time of the active feedback.

We observe in Fig. 1 that the width of this range of chromaticities featuring minimal octupole requirement depends on the gain of the transverse damper. In addition, the non-Gaussian modifications of the longitudinal distribution caused by the active blowup during the ramp [6] may significantly narrow down the range of optimal chromaticities [7]. To exploit this optimal working point, we therefore seek a chromaticity measurement technique that features a low level of beam losses, even during high intensity operation, and an accuracy in the order of 1 unit. The Beam Transfer Function (BTF) appears as a good candidate, since it contains information about the chromaticity [8] and it can be measured with a transverse excitation affecting a single bunch among a fully filled machine thanks to fast kickers and pickups. After a brief reminder of important theoretical aspects of the BTF and a description of our numerical model, we shall describe different ways to extract the chromaticity from the BTF measurement, identifying their strengths and shortcomings based on their application to simulation data.

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Table 1: LHC Machine and Beam Parameters

Parameter	**Value**
Energy [TeV]	7
Bunch intensity [10^{11}]	1.8
Norm. trans. emittance [μm]	2
β^* [m]	1
ATS Telescopic index	1
Frac. horizontal tune	0.31
Frac. vertical tune	0.32
R.m.s bunch length [cm]	8
RF voltage [MV]	12
Synchrotron tune	0.0018
Wakefields model	[9]

BTF Theory

The BTF is defined as the ratio of the beam oscillation amplitude at a given frequency Ω to the external excitation amplitude at that frequency. Following [10], we may write it for a given chromaticity Q' as:

$$\mathrm{BTF}(\Omega, Q') = \sum_l \mathcal{T}_l(\Omega) w_l(Q'), \tag{1}$$

where we could separate the expression in two integrals that depend on the tune spread and on the chromaticity, respectively. We have

$$\mathcal{T}_l(\Omega) \equiv 2\pi^2 \int_0^\infty \int_0^\infty \frac{J_y \dfrac{\partial f_0}{\partial J_y}}{\Omega - \omega_0 Q(J_x, J_y) - l\omega_s} \, \mathrm{d}J_x \, \mathrm{d}J_y \,, \tag{2}$$

with ω_0 the revolution frequency, ω_s the synchrotron frequency, $Q(J_x, J_y)$ the betatron tune as a function of the transverse actions and l the azimuthal mode number. The unperturbed transverse distribution is given by f_0. The weights of the azimuthal modes are given by

$$w_l(Q') \equiv 2\pi \int_0^\infty J_l\left(\frac{Q' r_z}{Q_s \beta_z}\right)^2 g_0(r_z) \frac{r_z \, \mathrm{d}r_z}{\beta_z}, \tag{3}$$

with Q_s the synchrotron tune and β_z the longitudinal β function given by the product of the r.m.s. bunch length and the r.m.s relative momentum spread. $J_l(\cdot)$ is the Bessel function of the first kind. The longitudinal distribution $g_0(r_z)$ is expressed as a function of the longitudinal amplitude of oscillation, i.e. $z = r_z \cos(\Phi)$.

In the presence of wakefields, the BTF takes a similar form [10]:

$$\mathrm{BTF}_{\mathrm{wake}}(\Omega, Q') = \sum_m \mathcal{T}^{\mathrm{wake}}_{l_m}(\Omega) \tilde{\eta}^*_m \eta_m, \tag{4}$$

with a summation over the modes m with a corresponding azimuthal number l_m. We have defined

$$\mathcal{T}^{\mathrm{wake}}_{l_m}(\Omega) \equiv \frac{\mathcal{T}_{l_m}(\Omega)}{1 + 2\Delta\Omega_m \mathcal{T}_{l_m}(\Omega)}, \tag{5}$$

with $\Delta\Omega_m$ the complex tune shift of mode m. The corresponding weight depends on the shape of the longitudinal eigenfunction $m_m(J_z, \phi)$:

$$\eta_m \equiv \int_0^{2\pi} \mathrm{d}\phi \int_0^\infty \mathrm{d}J_z \, m_m(J_z, \phi) g_0(J_z, \phi), \tag{6}$$

with J_z and ϕ the longitudinal action angle variables.

Numerical Model

To evaluate the performance of the various approaches, we numerically generate a set of measurements using macroparticles simulations with the code `COMBI` [11]. We start by initialising the coordinates of 10^6 macroparticles randomly distributed in 6D phase space. The tracking is then performed through a loop over 10^5 turns each consisting of the following actions executed sequentially:

- Rotation in transverse and longitudinal phase space based on the respective tunes, including linear detuning with the relative momentum deviation (chromaticity) and linear detuning with the transverse actions. Linear motion is assumed in the longitudinal plane.
- Change of transverse momentum of each of the particles due to the wakefields based on the transverse positions of the particles ahead of it (longitudinal slice model, with 100 slices) and the wakefields' model.
- A change of momentum is applied to all particles in the beam. The value of the kick is changed every turn based on a random number generator and is recorded into a file.
- The average position of the particles in the beam is recorded into a file.

The BTF is obtained as the ratio of the cross power spectral density of the positions and the kicks to the power spectral density of the kicks. They are computed with Welch's algorithm [12].

BTF WITHOUT WAKEFIELDS

Sideband Amplitude Ratio

By assuming that the amplitude of a given sideband is affected only by its corresponding term l in Eq. (1), which means that the betatron tune spread is much smaller than the synchrotron tune, we may write the ratio of the maximum amplitude of the first lower or upper sideband $l = \pm 1$ to the one of the main mode $l = 0$:

$$R_a = \frac{\mathcal{T}_{\pm 1}(\Omega \pm \Omega_s) w_{\pm 1}(Q')}{\mathcal{T}_0(\Omega) w_0(Q')} = \frac{w_{\pm 1}(Q')}{w_0(Q')}. \tag{7}$$

Equation (2) was used to obtain the final expression. The ratio of the amplitude of the first sideband is therefore a powerful tool to assess the chromaticity for low intensity beams

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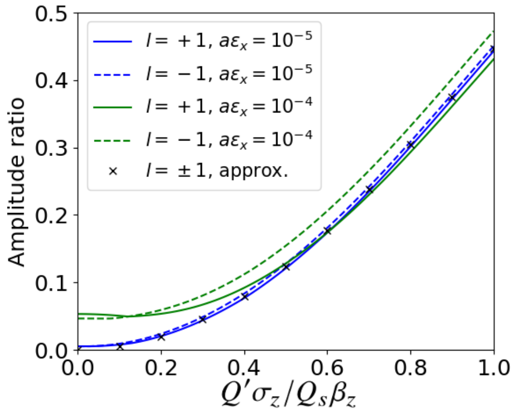
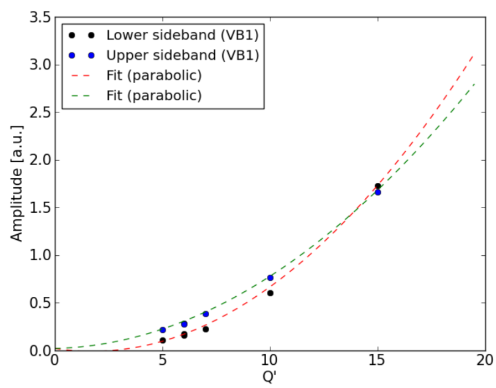


(a) COMBI



(b) Experimental observation. Courtesy of [13]

Figure 2: Amplitude ratio obtained analytically (upper plot, black crosses) and numerically for different sidebands l and detuning. The datasets are labelled by their r.m.s. detuning driven by the direct term a. ϵ_x is the geometric transverse emittance. The lower plot shows the result of an experiment at the LHC (dots). Parabolic fits were tried whereas the theory rather suggests Eq. (7).

(i.e. neglecting wakefields). Indeed, by solving Eq. (3) for a Gaussian longitudinal distribution, we have [8]:

$$w_l^G(Q') = \mathrm{e}^{-\left(\frac{Q'\sigma_z}{Q_s\beta_z}\right)^2} I_l\left(\left(\frac{Q'\sigma_z}{Q_s\beta_z}\right)^2\right), \qquad (8)$$

with σ_z the r.m.s. bunch length and $I_l(\cdot)$ the modified Bessel functions of the first kind. As shown in Fig. 2, the predicted behaviour is well verified in numerical simulations without wakefields as well as experimentally at the LHC. With a large betatron tune spread (green curves in Fig. 2a) deviations are observed. While the hypothesis used to obtain the simplified expression for the amplitude ratio is no longer valid, the expression for the BTF (Eq. (1)) remains valid. In such configurations where the signals of the sidebands are mixed, it becomes more convenient to use a nonlinear fit approach.

Nonlinear Fit

In the following we attempt to fit the phase of the BTF obtained with COMBI using Eq. (1). The chromaticity and direct linear detuning coefficient were used as Degrees Of Freedom (DOF). To avoid unnecessary DOF, the indirect detuning coefficient was set to -0.7 times the direct one, as expected with the LHC octupoles [14]. The Nelder-Mead algorithm [15] was used to minimise the sum of the absolute differences between the BTF phase from the analytical model and from the numerical data evaluated at 327 frequencies equally spaced in a range that includes the signal of sidebands -2 to 2, inclusive.

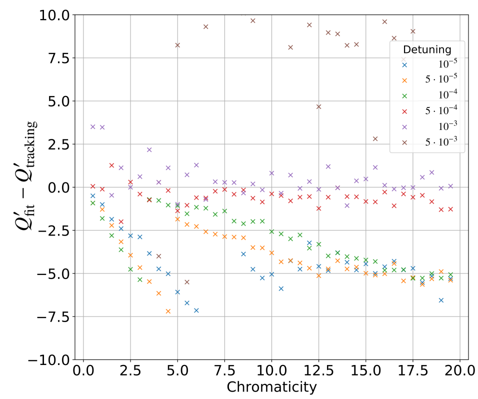


Figure 3: Error on the chromaticity of the fit of simulated tracking data with the analytical formula neglecting wake fields (Eq. (1)). The datasets are labelled with the corresponding r.m.s. tune spread driven by the direct term $a\epsilon_x$.

Figure 3 shows the accuracy of the fit for different chromaticities and detunings. Most fits fail for configurations with a low amplitude detuning ($< 10^{-4}$). In such configurations, the BTF is rather sharp at the sidebands and the resolution of the BTF obtained numerically is limited by the number of turns in the tracking simulations. For an intermediate range of detunings ($< 10^{-3}$ which is comparable to Q_s), the accuracy of the fitted chromaticity remains around the unit. Such good fits are illustrated in a configuration where the detuning is significantly smaller than the synchrotron tune (Fig 4a) and comparable to the synchrotron tune (Fig. 4b). The fits tend to fail for large values of detuning, as the signals of the different sidebands overlap due to their broad extend in frequency. A configuration at the limit of the capability of the non-linear fit is shown in Fig. 4c.

The range of detunings for which the nonlinear fit method is accurate seems perfectly suited for an application to the LHC at flat top before collision, for which the r.m.s. detuning is usually smaller than Q_s. Nevertheless, the goal is to obtain a measurement technique that is compatible with high intensity operation. It is therefore crucial to consider the effect of the wakefields.

BTF WITH WAKEFIELDS

Nonlinear Fit

In the presence of wakefields, the BTF is distorted depending on the beam intensity. Such a distortion of the phase is

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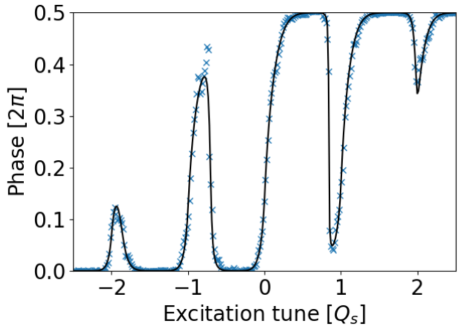
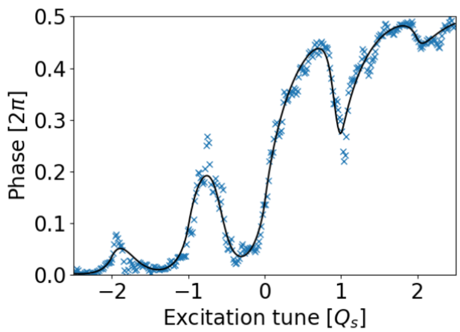
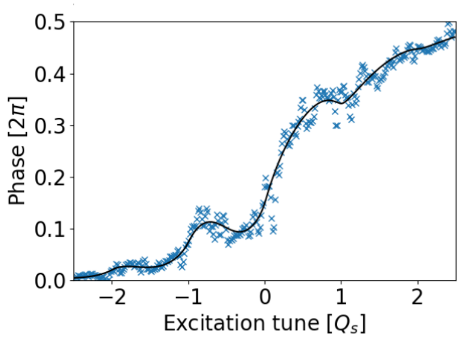


(a) $a\epsilon_x = 10^{-4}$



(b) $a\epsilon_x = 3 \cdot 10^{-4}$



(c) $a\epsilon_x = 6 \cdot 10^{-4}$

Figure 4: Examples of BTF phases obtained numerically (blue crosses) and the corresponding fit (black lines) for a chromaticity of 15 units and different detunings. The excitation tune is shifted to the betatron tune and expressed in units of the synchrotron tune.

shown in Fig. 5. This distortion affects the sidebands, such that the method based on the amplitude ratio is no longer applicable. In addition, a significant frequency shift of the mode can be observed (here of the mode 0). The nonlinear fit based on Eq. (1), i.e. neglecting the presence of wakefields, cannot adjust to such shifts. The impact on the fit accuracy is shown in Fig. 6a. As previously, only configurations with a low enough tune spread yield a good accuracy, at least with a low intensity. For those good cases, a bias growing with the intensity is observed which can be attributed to the increasing strength of the wakefields.

In order to remove this bias, we rather use Eq. (4) as fitting

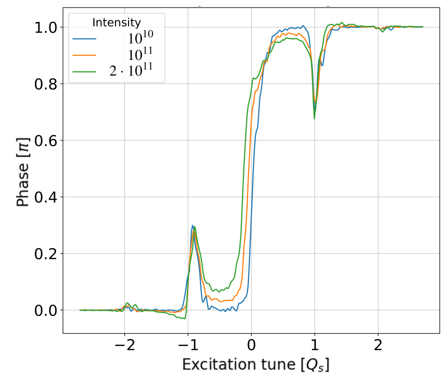


Figure 5: Examples of BTF phases obtained numerically for a chromaticity of 6 units, $a\epsilon_x = 10^{-4}$ and including wakefields. The excitation tune is shifted to the betatron tune and expressed in units of the synchrotron frequency.

function, adding now one complex tune shift per azimuthal mode number as DOF. The complexity of the fit is thus greatly increased as the number of DOF changed from 2 to 12. As a result, it was observed that Newton based algorithms tended to fail in this configuration, whereas they worked as efficiently as the Nelder-Mead approach for the simpler fit without wakefields. The accuracy of the fit in Fig. 6b shows that the bias observed earlier was indeed removed by this approach, at the expense of a low precision (shown by the larger error bars).

Another attempt was made with a simplified fitting function, still using Eq. (4) but assuming that only the mode 0 is affected by the wakefields, i.e. $\Delta Q_l = 0$ for $l \neq 0$. The number of DOF is thus decreased to 4. The idea of this simplified approach is that the main distortion observed in Fig. 5, i.e. the shift of mode 0, can still be fitted, whereas the more subtle impact on the sidebands is neglected. The accuracy of the fit shown in Fig. 6c shows again a suppression of the bias, yet the precision was not significantly improved.

NEURAL NETWORK

Finally, we attempt to reconstruct the chromaticity from a BTF using a neural network. In order to train the network a large dataset was generated using `COMBI`, with 7 intensities ranging from 10^{10} to $2 \cdot 10^{12}$, 39 chromaticities ranging from 0.5 to 20, 10 values of r.m.s. detuning arranged logarithmically from 10^{-6} to 10^{-2}. For each simulation, 10 samples were generated by adding random noise to the simulated positions before computing the BTF. The dataset is therefore constituted of 27300 BTF samples. 80% of the samples were used for training, while the remaining 20% were used to measure the accuracy.

Fully connected multilayer neural networks with different depth were tested using Keras [16]. It was found that a four layers network performs well. The input layer is constituted of 327 nodes, corresponding to the BTF phases in a range of frequencies including the sidebands -2 to 2. The two hidden layers each decrease in size by a factor 2 (163 and

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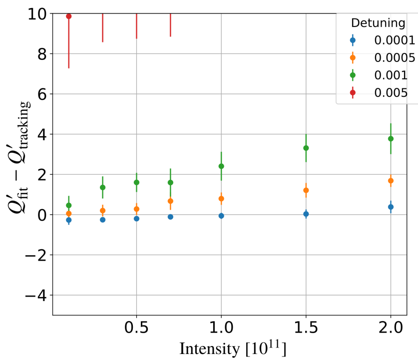
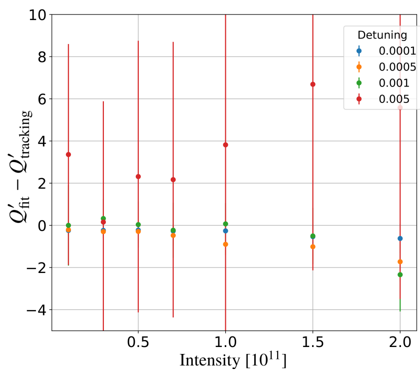
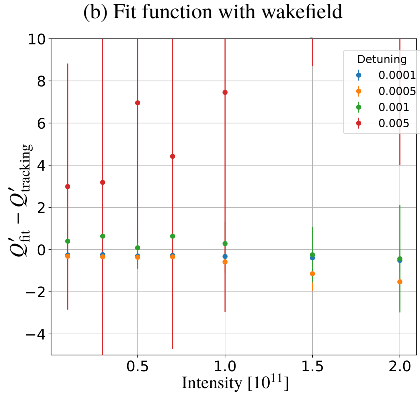


(a) Fit function without wakefield



(b) Fit function with wakefield



(c) Fit function featuring a simplified wakefields model

Figure 6: Error on the fitted chromaticity as a function of the beam intensity for different fitting functions. The average and r.m.s. were computed across 39 chromaticities ranging from 0.5 to 20. Each dataset is labelled with the r.m.s. tune spread driven by the direct term $a\epsilon_x$.

81 nodes respectively). The output layer is constituted of 3 nodes corresponding to the chromaticity, the direct detuning coefficient and the beam intensity, which were normalised between 0 and 1. The training is done with 100 epochs in batches of 75 samples using the ADAM method [17] with the mean absolute error as loss function. The nodes are activated with a Rectified Linear Unit (ReLU) function.

The accuracy of the chromaticity obtained by the neural network on the testing set is reported in Fig. 7. The accuracy is well below unity in the whole range of parameters on which the network was trained. This method therefore outperforms all the others in terms of accuracy and dynamic range. The need to train the network could become a significant drawback in case the training on simulation data is not sufficient for the network to perform sufficiently well on experimental data.

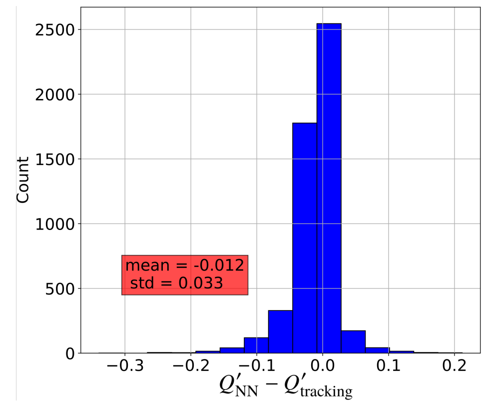


Figure 7: Error on the chromaticity obtained with the neural network.

CONCLUSION

The pros and cons of the various methods attempted to extract the chromaticity from a BTF can be summarised as follows

- **The amplitude ratio** is by far the simplest method and allows for an accurate measurement of the chromaticity in configurations with a betatron tune spread much lower than the synchrotron tune and in absence of wakefields.
- **Nonlinear fit neglecting wakefields** allows for a better accuracy in configurations featuring a tune spread comparable to the sychrotron tune, yet it still fails for larger tune spread. The presence of wakefields induces a bias in the reconstructed chromaticity.
- **Nonlinear fitting including wakefields** removes the bias induced by the wakefields. Yet the additional DOF tend to deteriorate the resolution.
- **Neural networks** clearly outperforms the other methods in terms of accuracy and dynamic range. The main drawback of this approach lies in the need for training, experimental tests are needed to assess the possibility to use a neural network trained on simulation data.

Both the nonlinear fit including wakefields and the neural networks have the potential to be used at the LHC and HL-LHC to reduce the need for octupole strength, thus motivating experimental tests.

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