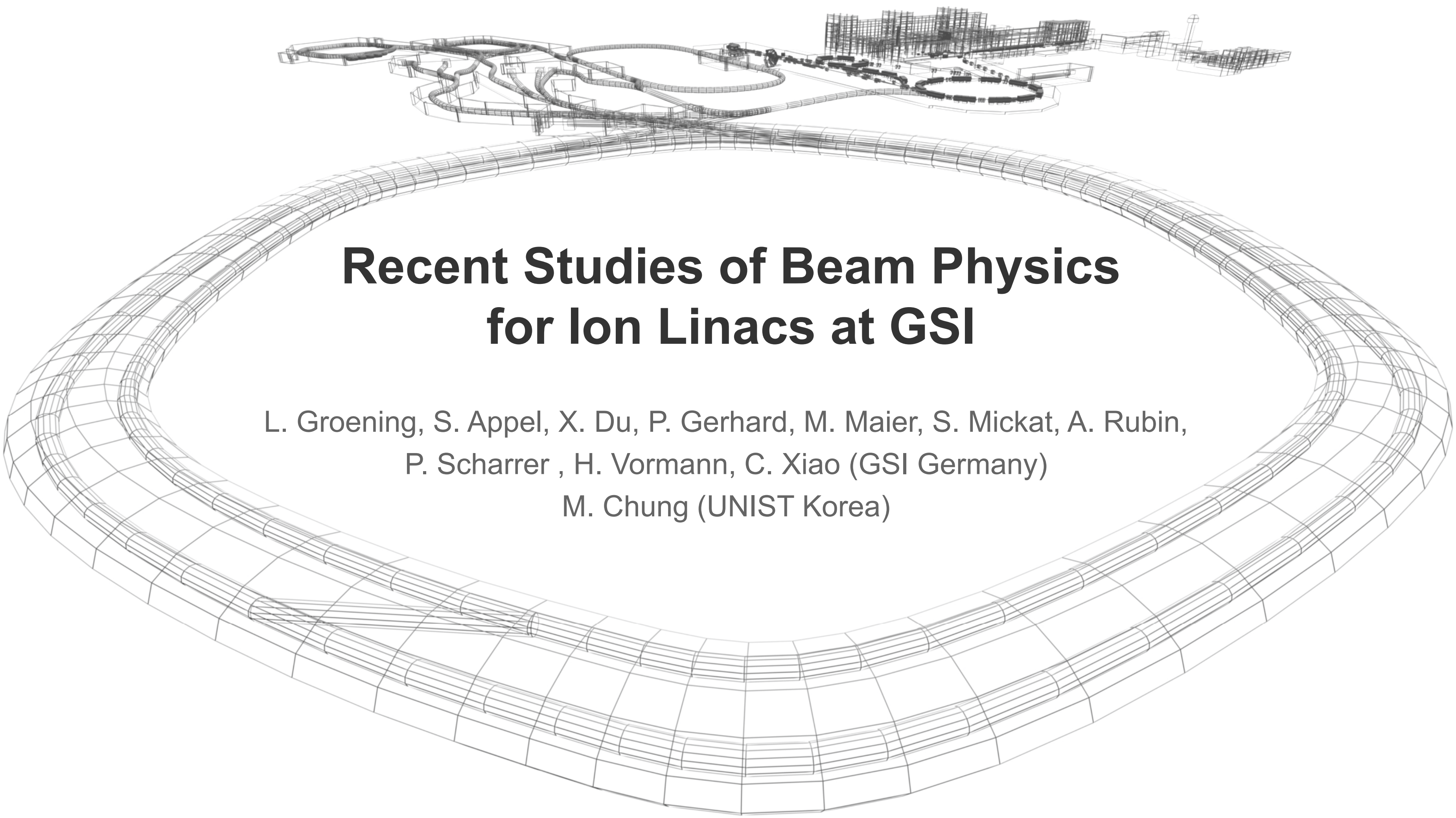

Recent Studies of Beam Physics for Ion Linacs at GSI

L. Groening, S. Appel, X. Du, P. Gerhard, M. Maier, S. Mickat, A. Rubin,
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M. Chung (UNIST Korea)

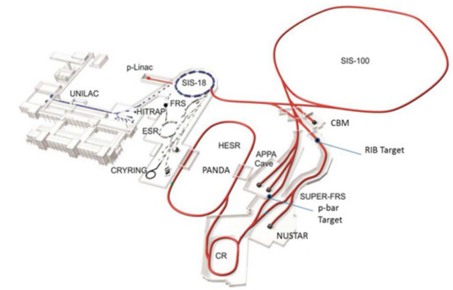
Outline

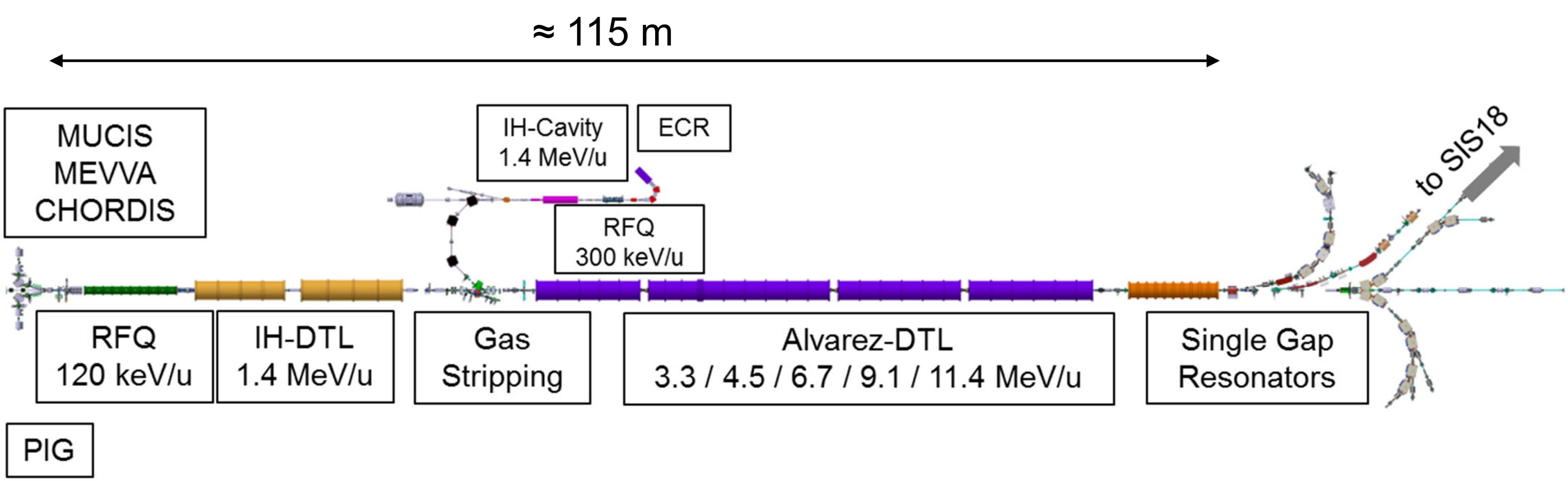
- UNILAC Introduction
- Increase of ion stripping efficiency
- Advances in longitudinal beam dyn. modeling of DTLs
- complete 4d transverse diagnostics
- Extension of Busch theorem to beams
- Generic algorithm for ring injection optimization

UNIversal Linear Accelerator UNILAC

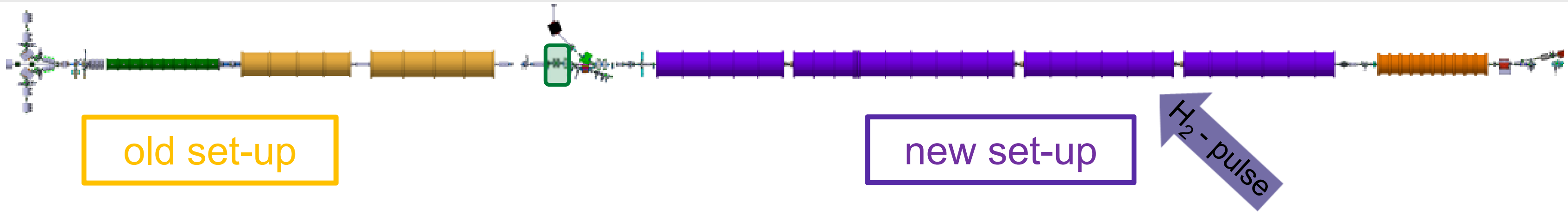
design parameters after upgrade

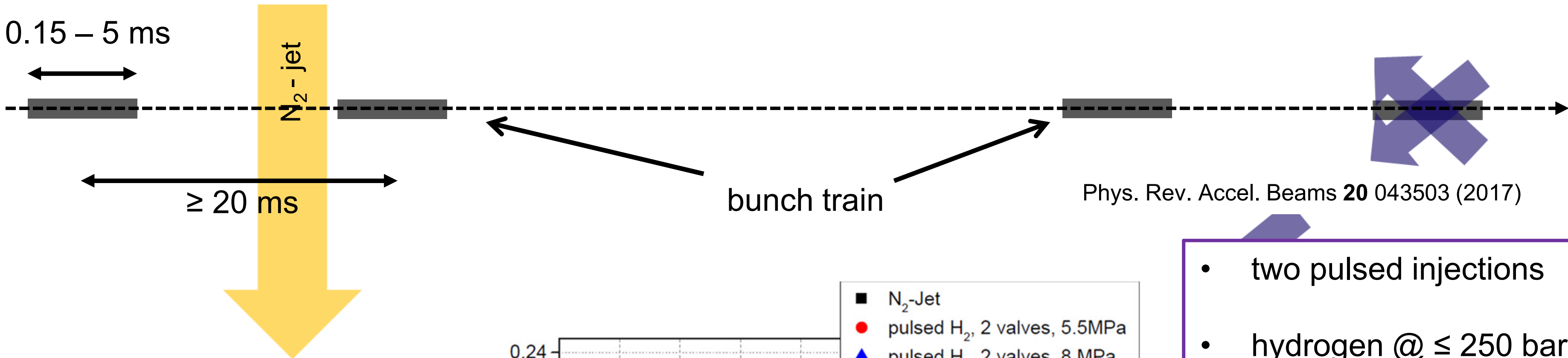
ion A/q	≤ 8.5, i.e. $^{238}U^{28+}$	
beam current (pulse) * A/q	1.76 (0.5% duty cycle)	emA
input beam energy	2.2	keV/u
output beam energy	3.0 - 11.7	MeV/u
normalized total output emittance, horizontal / vertical	0.8 / 2.5	mm mrad
beam pulse duration	≤ 1000	µs
beam repetition rate	≤ 10	Hz
operating frequency	36.136 / 108.408	MHz
length	≈ 115	m





Increase of stripping efficiency





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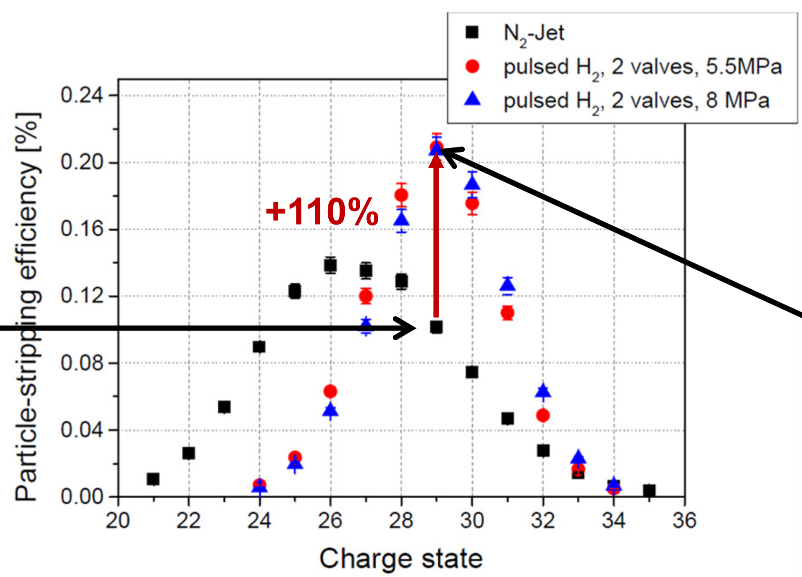
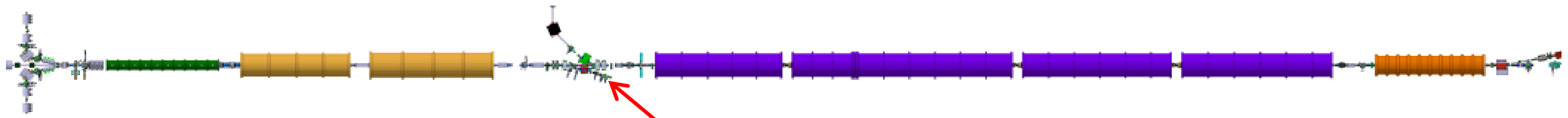
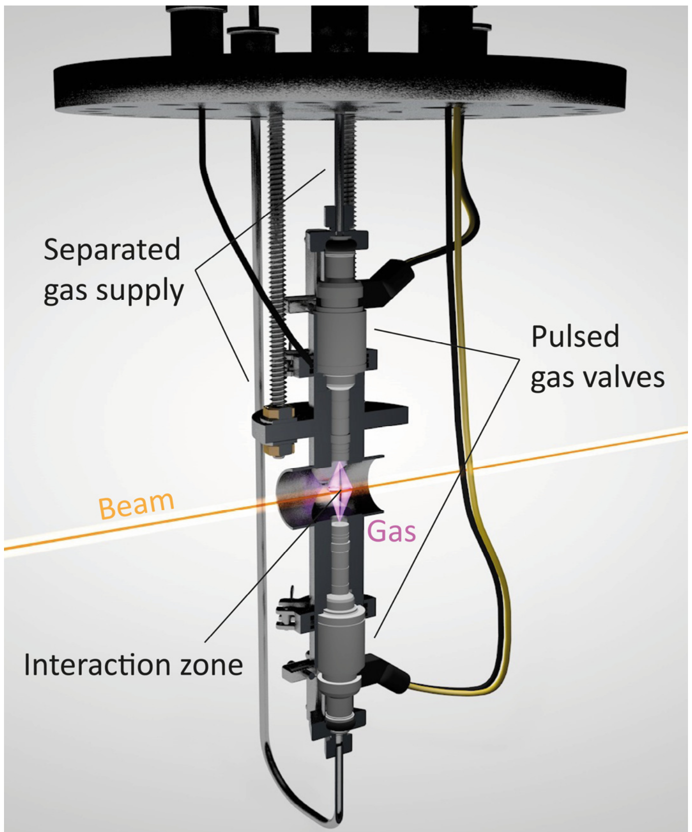
- one single continuous jet
- nitrogen @ 4 bar
- stripper parameters are constant

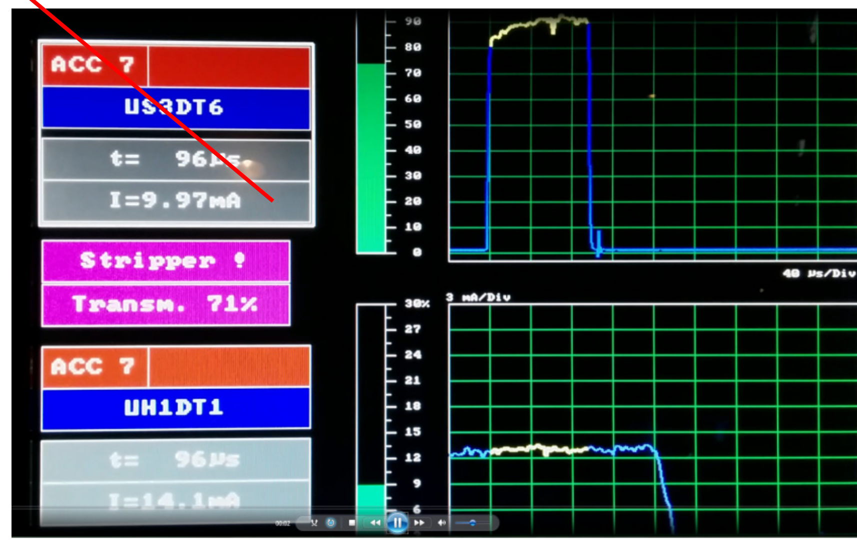


- two pulsed injections
- hydrogen @ ≤ 250 bar
- higher density @ beam, but lower averaged gas load at pumps
- variable pressure, duration, and rep.rate
- pulse-to-pulse variation of stripper parameters

Increase of stripping efficiency







>10 emA of U^{29+} achieved behind stripper

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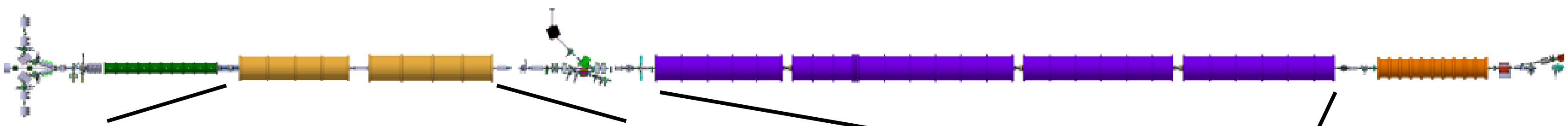
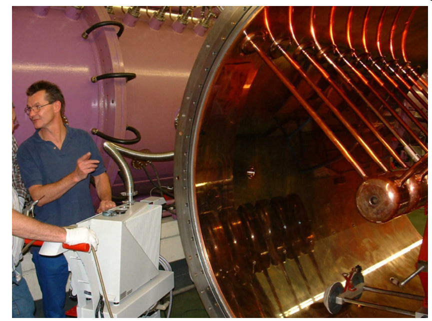
Longitudinal modeling of DTLs

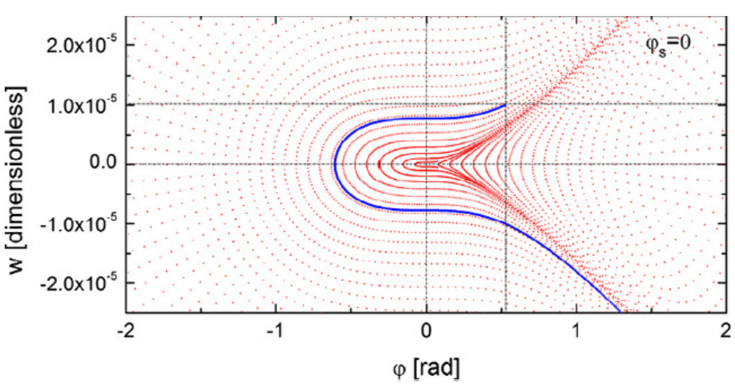
UNILAC has two types of DTLs with different long. beam dynamics

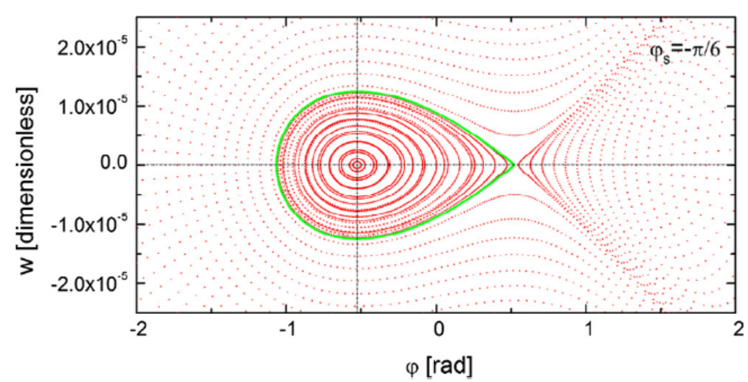




IH-cavities with KONUS
(combined 0-degrees structure)

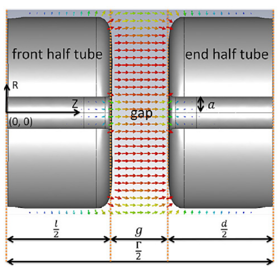
Alvarez-cavities with synchronous particle





Longitudinal modeling of DTLs

- KONUS: no stable bucket → very sensitive to rf–voltage & phase at rf–gaps
- using pre–defined phases and/or TTF–factors may deliver less precise results (z–codes)
- using too simple approximations for E–field inside gap may deliver less precise results
- GSI developed a MATHCAD routine using:
  - gap field calculation from BEAMPATH using Fourier–Bessel series



$$E_z(z,r,t) = -\cos(\omega t + \psi_0)\sum_{m=1}^{M} E_m I_0(\mu_m r)\sin\left(\frac{2\pi m z}{\Gamma}\right),$$

$$E_r(z,r,t) = \cos(\omega t + \psi_0)\sum_{m=1}^{M} \frac{2\pi m E_m}{\mu_m \Gamma} I_1(\mu_m r)\cos\left(\frac{2\pi m z}{\Gamma}\right),$$

$$B_\theta(z,r,t) = \sin(\omega t + \psi_0)\sum_{m=1}^{M} \frac{2\pi E_m}{\mu_m \lambda c} I_1(\mu_m r)\sin\left(\frac{2\pi m z}{\Gamma}\right),$$

and

$$\mu_m = \frac{2\pi}{\lambda}\sqrt{\left(\frac{m\lambda}{\Gamma}\right)^2 - 1}, \quad \Gamma = l + 2g + d, \quad E_0 = \frac{U}{g},$$

$$E_m = \frac{4U}{I_0(\mu_m a)\Gamma}\,\frac{\pi m(l+g)}{\Gamma}\,\frac{\sin\left[\frac{\pi m(l+g)}{\Gamma}\right]}{\frac{\pi m(l+g)}{\Gamma}}\,\frac{\sin\left(\frac{\pi m g}{\Gamma}\right)}{\frac{\pi m g}{\Gamma}}$$

  - longitudinal tracking based on adaptive step size in dt (and hence dz) with Bulirsch–Stoer method

$$Z(t,z) := \begin{bmatrix} z \\ \frac{dz}{dt} \end{bmatrix} = \begin{bmatrix} z \\ \beta c \end{bmatrix} \qquad DZ(t,z) := \frac{dZ(t,z)}{dt} = \begin{bmatrix} \beta c \\ \dfrac{q}{\frac{m_0}{\sqrt{1-\beta^2}}} E_z(z)\cos(\omega t + \psi_0) \end{bmatrix}$$

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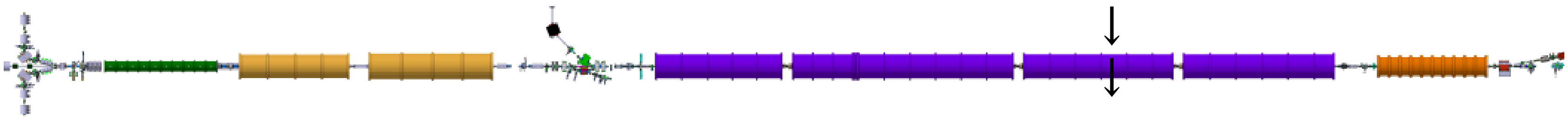
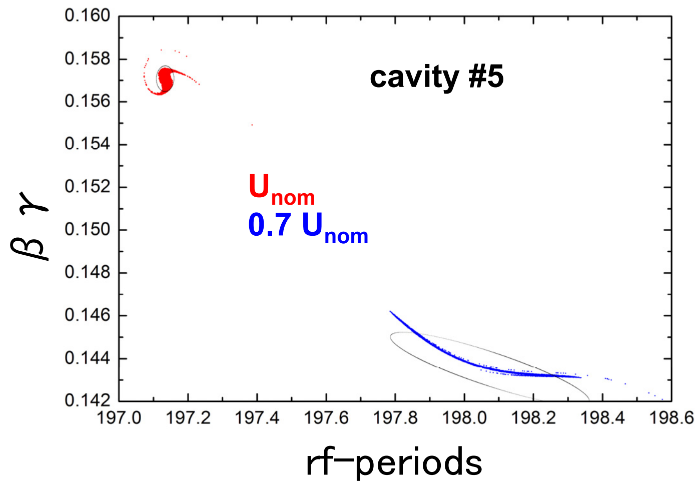
Longitudinal modeling of DTLs

- new tracking method allows for understanding the “intermediate energy puzzle“ at the UNILAC that rose 1983:



  - last two Alvarez-cavities deliver beam with 100% transmission and very low dp/p, if they are operated at just 80 / 70 % of their nominal voltage
  - energy gains are reduced by 66 / 68 % w.r.t. value at nominal voltage
  - beam delivered to users since 1983 to their full satisfaction



work in progress ...

4d transverse diagnostics: Coupling & eigen-emittances

- linear (4d), Hamiltonian beam line elements preserve :
  - 4d rms emittance $E_{4d}^2 = det \begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix}$
  - the two eigen–emittances

$$\varepsilon_1 = \frac{1}{2}\sqrt{-tr[(CJ)^2] + \sqrt{tr^2[(CJ)^2] - 16det(C)}}$$

$$\varepsilon_2 = \frac{1}{2}\sqrt{-tr[(CJ)^2] - \sqrt{tr^2[(CJ)^2] - 16det(C)}}$$

$$E_{4d} = \varepsilon_1 \cdot \varepsilon_2$$

$$C = \begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix} \qquad J := \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

4d transverse diagnostics: Coupling & eigen-emittances

- if, and only if there is no x↔y correlation, i.e., $C = \begin{bmatrix} <xx> & <xx'> & 0 & 0 \\ <x'x> & <x'x'> & 0 & 0 \\ 0 & 0 & <yy> & <yy'> \\ 0 & 0 & <y'y> & <y'y'> \end{bmatrix}$
  - rms emittances = eigen–emittances
- if there is any coupling
  - rms emittances ≠ eigen–emittances
  - coupling parameter t $t = \frac{\varepsilon_x \varepsilon_y}{\varepsilon_1 \varepsilon_2} - 1 \geq 0$

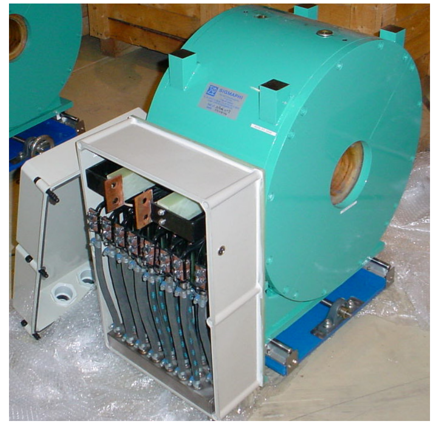
- term „eigen–emittance“ is quite unknown, since generally beams are considered as uncoupled
- measured beam moments are reliable, only if they deliver reasonable eigen–emittances !

4d transverse diagnostics:
Motivation through coupling elements

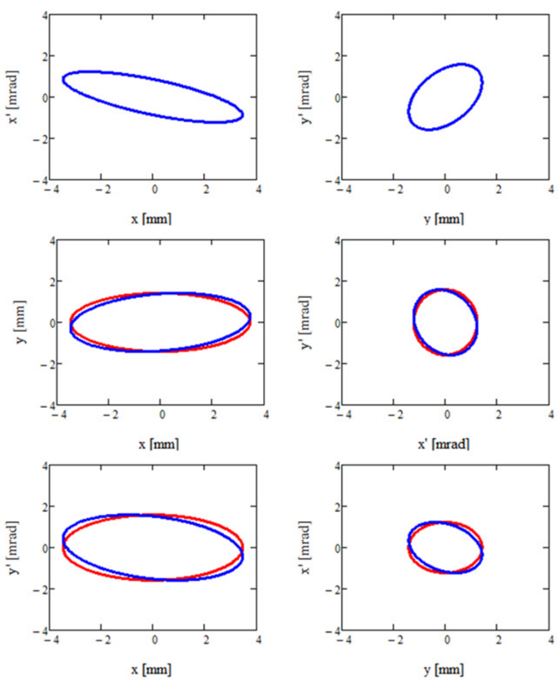
initial beam:

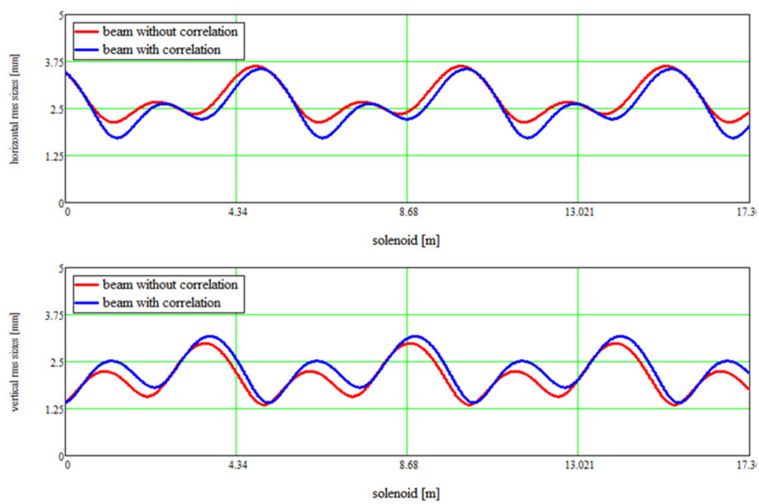
$$C_1=\begin{bmatrix} 12.00 & -3.00 & 0.00 & 0.00 \\ -3.00 & 1.50 & 0.00 & 0.00 \\ 0.00 & 0.00 & 2.00 & 1.00 \\ 0.00 & 0.00 & 1.00 & 2.50 \end{bmatrix}$$ uncorrelated

$$C_2=\begin{bmatrix} 12.00 & -3.00 & 1 & -1.5 \\ -3.00 & 1.50 & -0.5 & -0.35 \\ 1.00 & -0.50 & 2.00 & 1.00 \\ -1.50 & -0.35 & 1.00 & 2.50 \end{bmatrix}$$ correlated

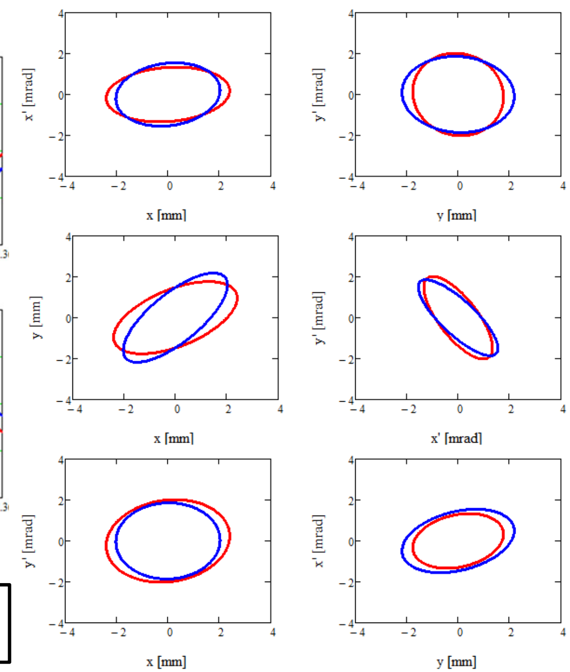


$$M_{sol}=\begin{bmatrix} C^2 & \frac{SC}{K} & SC & \frac{S^2}{K} \\ -KSC & C^2 & -KS^2 & CS \\ -SC & -\frac{S^2}{K} & C^2 & \frac{SC}{K} \\ KS^2 & -SC & -KSC & C^2 \end{bmatrix}$$



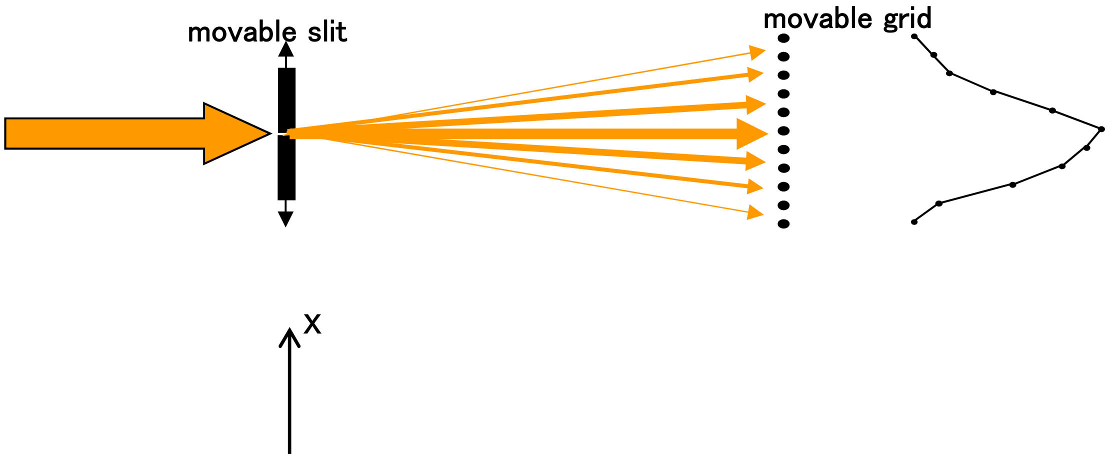


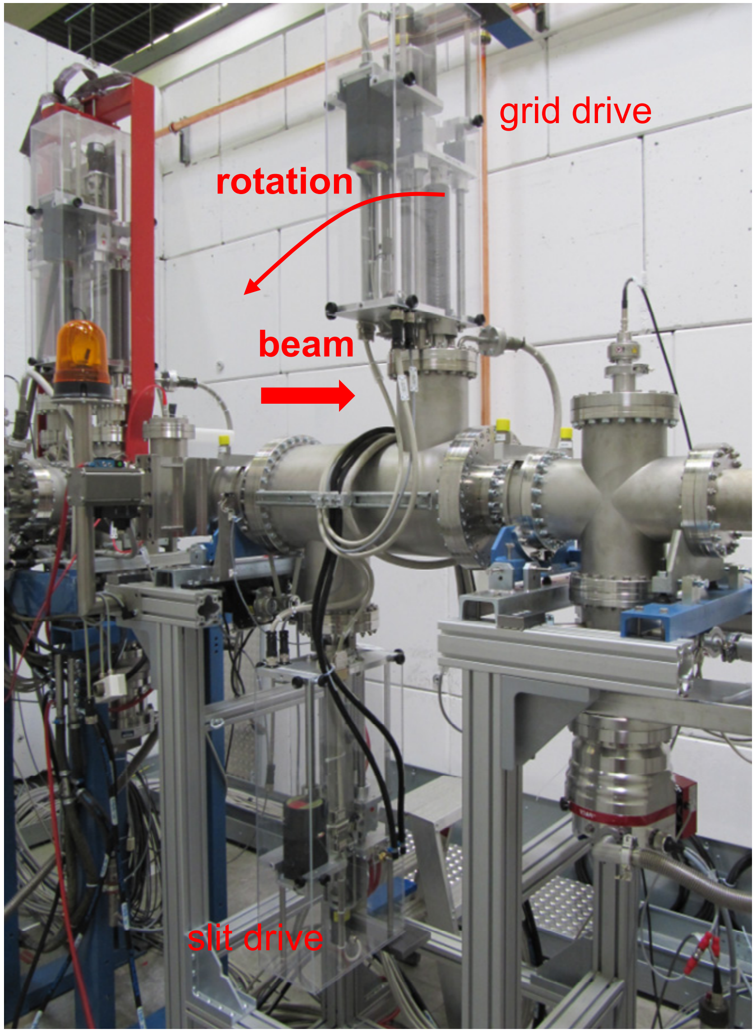
final beam:



red and blue envelopes differ

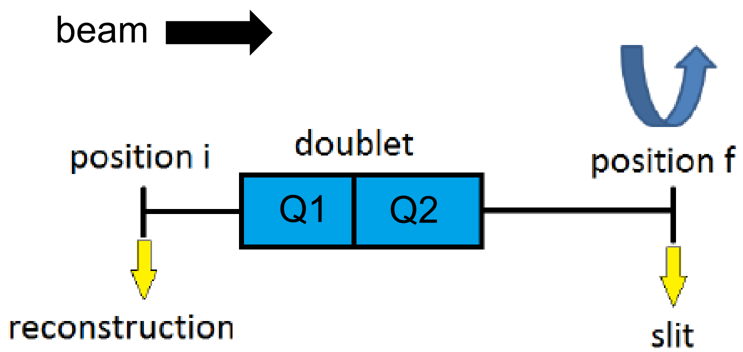
4d transverse diagnostics: Concept





- slit & grid are mounted inside a rotatable chamber
- rotation gives access to <xy>, <xy'>, <x'y>, <x'y'>
- chamber does not rotate during measurement

4d transverse diagnostics: Concept

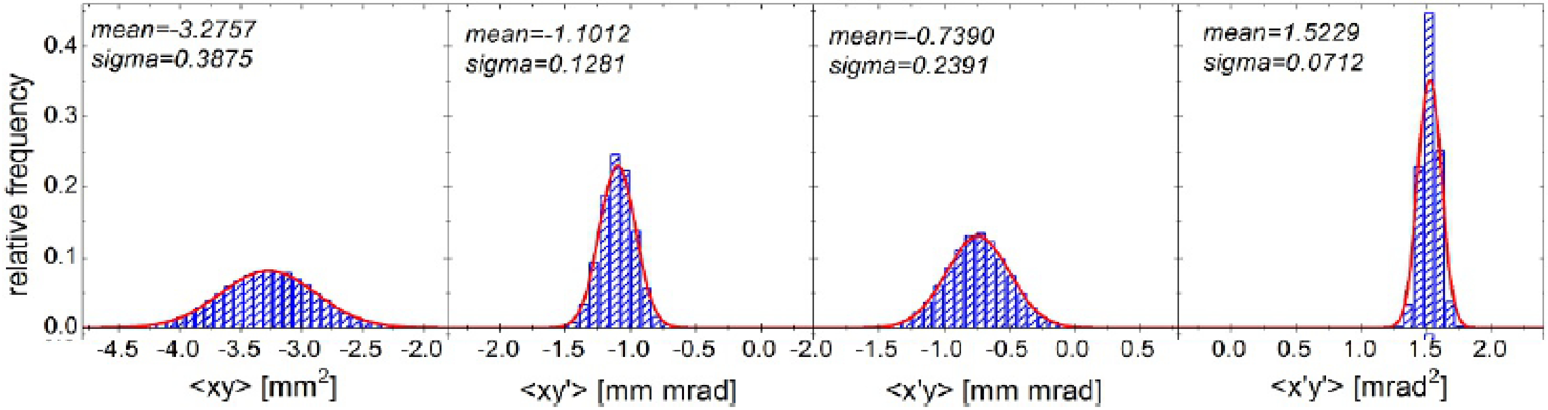


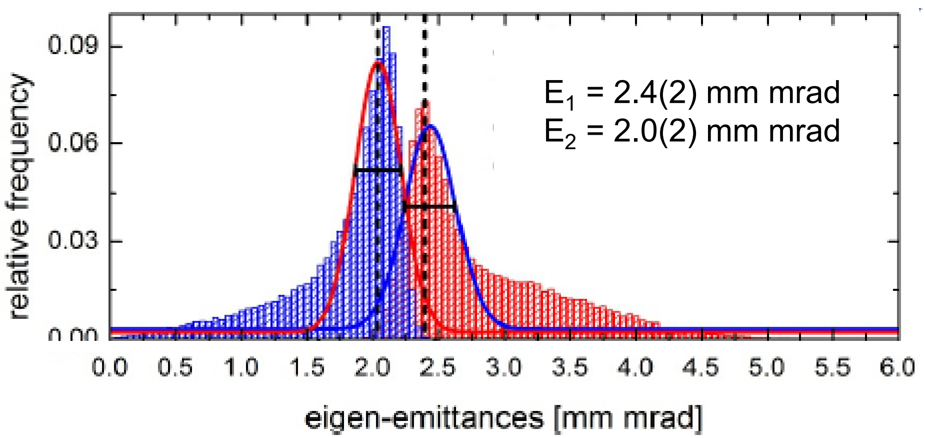
- beam moments to be determined at „reconstruction point“
- „reconstruction point“ and ROSE are separated by adjustable non-coupling elements
- slit & grid of ROSE can be rotated simultaneously

4d transverse diagnostics: Beam matrix measured

$$\begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix} = \begin{bmatrix} 8.57 & -4.34 & -3.28 & -1.10 \\ -4.34 & 3.35 & -0.74 & 1.52 \\ -3.28 & -0.74 & 11.20 & -3.05 \\ -1.10 & 1.52 & -3.05 & 1.87 \end{bmatrix}$$

in mm, mrad

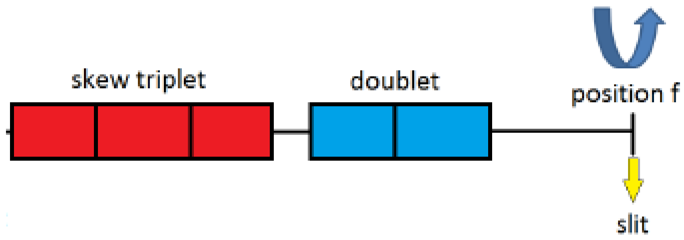




$$t = \frac{\varepsilon_x \varepsilon_y}{\varepsilon_1 \varepsilon_2} - 1 = 1.2(2)$$

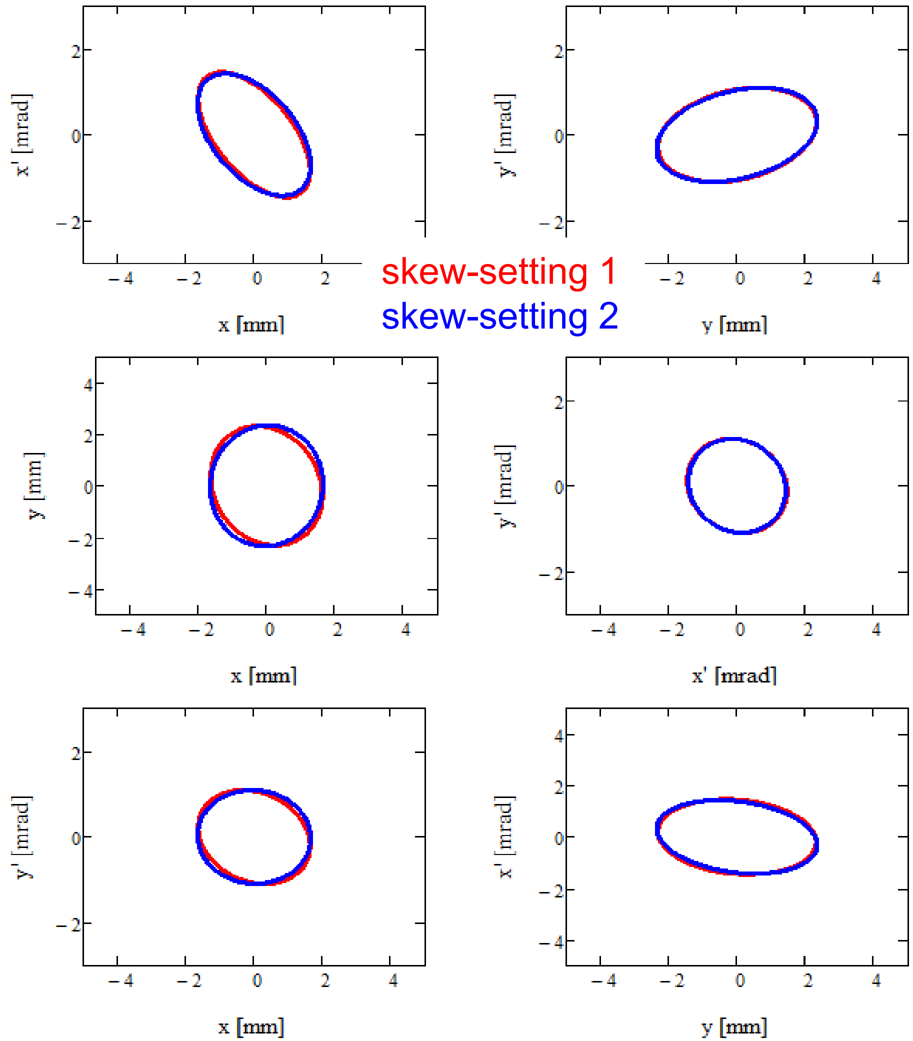
NIM A 820 14 (2016)
Phys. Rev. Accel. Beams **19** 072801 (2016)

4d transverse diagnostics: Check reliability



- measurements were done for two different skew triplet settings
- both measurements were back-transformed to entrance of skew triplet
- back-transformations delivered very similar results
- the two measurements are consistent
- ROSE seems to be reliable

at skew triplet entrance:



Busch theorem for beams: Single particle (H. Busch 1926)

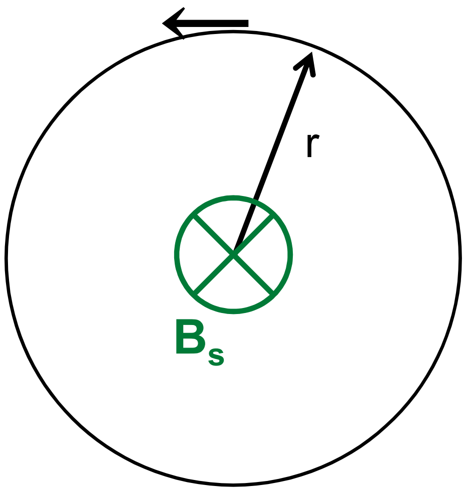
Busch theorem states

$$L_s = \left[\vec{r} \times (\vec{P} + eq\vec{A})\right] \cdot \vec{e}_s = const$$

using cylindrical coordinates :

$$x = r\cos\theta, \qquad y = r\sin\theta$$

$$\vec{P} = m \begin{bmatrix} \dot{x} \\ \dot{y} \\ \beta c \end{bmatrix}$$



$$L_s = mr^2\dot{\theta} + \frac{1}{2} eqB_s r^2 = const$$

$$\boxed{L_s = mr^2\dot{\theta} + \frac{eq}{2\pi}\Psi = const}$$

H. Busch, Z. Phys. **81** (5) 924 (1926)

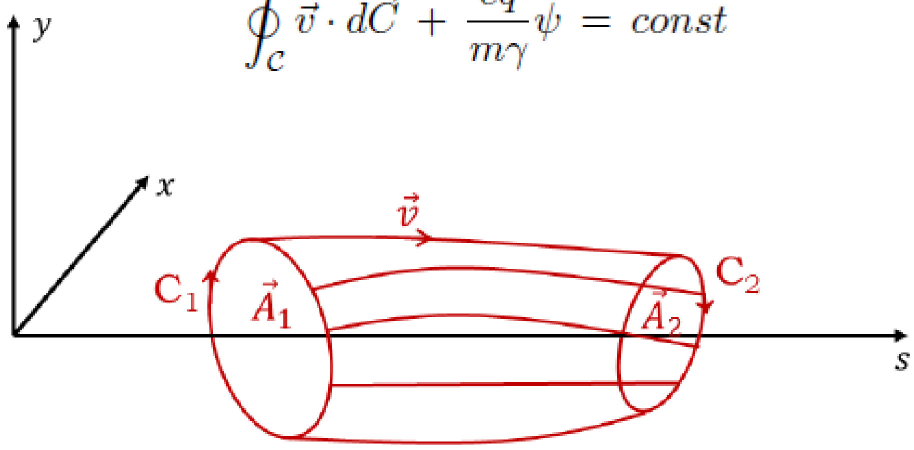
L_s = orbital angular momentum + flux through area of cyclotron motion = const

Busch theorem for beams: Single particle trajectories

Busch theorem can be generalized to :

P. T. Kirstein, G. S. Kino, and W. E. Waters,
Space Charge Flow (McGraw-Hill, New York, 1967), p. 14

$$\oint_C \vec{v} \cdot d\vec{C} + \frac{eq}{m\gamma}\psi = const$$



C_i enclose possible single particle trajectories

C_i := circles with constant r_i $\rightarrow$ $\boxed{m\gamma r^2\dot{\theta} + \frac{eq}{2\pi}\Psi \; = \; const}$

Busch theorem for beams:
Reformulate preservation of eigen-emittances

- „generalized" eigen-emittance through replacing (x',y') by generalized momenta

$$C = \begin{bmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{bmatrix} \longrightarrow \tilde{C} = \begin{bmatrix} \langle x^2\rangle & \langle xp_x\rangle & \langle xy\rangle & \langle xp_y\rangle \\ \langle xp_x\rangle & \langle p_x^2\rangle & \langle yp_x\rangle & \langle p_xp_y\rangle \\ \langle xy\rangle & \langle yp_x\rangle & \langle y^2\rangle & \langle yp_y\rangle \\ \langle xp_y\rangle & \langle p_xp_y\rangle & \langle yp_y\rangle & \langle p_y^2\rangle \end{bmatrix}$$

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad \tilde{\varepsilon}_{1/2} = \frac{1}{2}\sqrt{-tr[(\tilde{C}J)^2] \pm \sqrt{tr^2[(\tilde{C}J)^2] - 16\,det(\tilde{C})}}$$

- if $\tilde{\varepsilon}_{1/2}$ are preserved, also $\tilde{\varepsilon}_1^2 + \tilde{\varepsilon}_2^2$ must be preserved

- state $\tilde{\varepsilon}_1{}^2 + \tilde{\varepsilon}_2{}^2 = const$ using generalized momenta

$$p_x := x' + \frac{A_x}{(B\rho)} = x' - \frac{yB_s}{2(B\rho)}$$
$$p_y := y' + \frac{A_y}{(B\rho)} = y' + \frac{xB_s}{2(B\rho)}$$

to obtain expression for useful „laboratory" eigen-emittances

Busch theorem for beams

$$\boxed{(\varepsilon_1-\varepsilon_2)^2}+\boxed{\left[\frac{AB_s}{(B\rho)}\right]}^2+2\frac{B_s}{(B\rho)}\boxed{[\langle y^2\rangle\langle xy'\rangle-\langle x^2\rangle\langle yx'\rangle+\langle xy\rangle(\langle xx'\rangle-\langle yy'\rangle)]}=const$$

- sum of three quantities forms an invariant
- difference of eigen-emittances, flux through beam area, ...
- what is the third ?
  - has dimension m^3
  - scales with beam rms area as for $y = ax$ it vanishes
  - vanishes for uncorrelated beams
  - invariant under rotation around beam axis
  - it turns out to be the beam's vorticity multiplied by the beam area

Busch theorem for beams

original Busch theorem for single particle :

$$\oint_C \vec{v} \cdot d\vec{C} + \frac{eq}{m\gamma}\psi = const$$

theorem extended to accelerated particle beams :

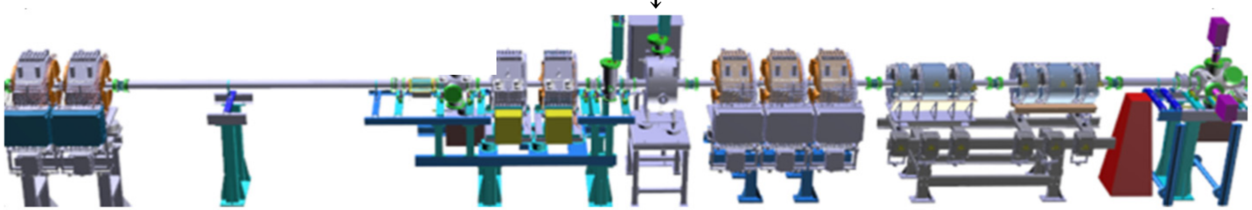
$$(\varepsilon_{n1} - \varepsilon_{n2})^2 + \frac{4eq\psi\,\beta\gamma}{mc\pi} \oint_C \vec{\vec{r}}' \cdot d\vec{C} + \left[\frac{eq\psi}{mc\pi}\right]^2 = const$$

- both expressions include flux and „vorticity“ $\vec{v} \cdot d\vec{C} \cong (\vec{\nabla} \times \vec{v})\, d\vec{A}$ (Stoke‘s law)
- the extended theorem additionally includes eigen-emittances
- theorem allows very fast modeling of setups for emittance gymnastics

Phys. Rev. Accel. Beams **20** 014201 (2018)

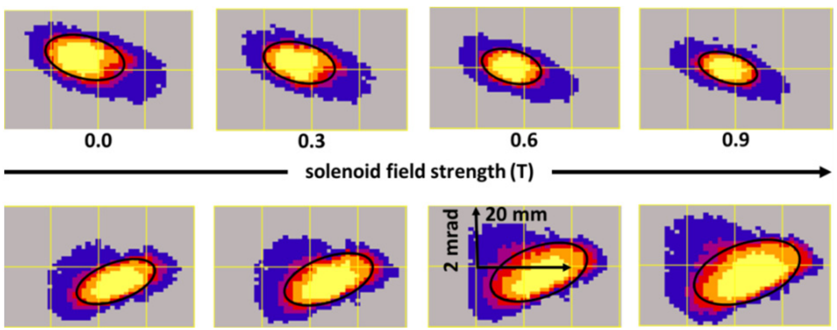
Busch theorem for beams:
Application to emittance shaping (EMTEX)

charge state stripping inside solenoid

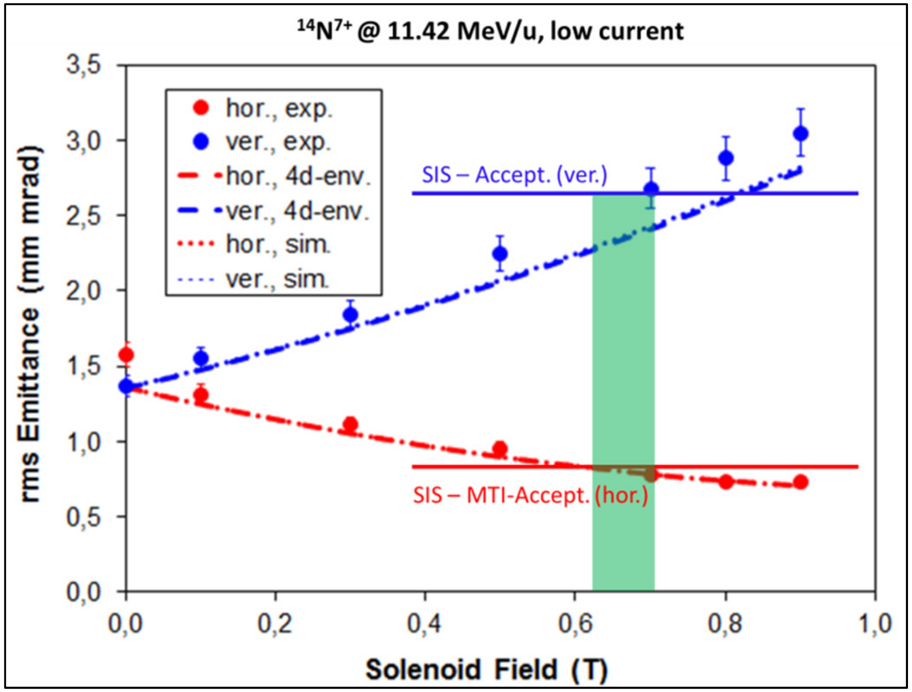


$$(\varepsilon_{x,7+} - \varepsilon_{y,7+})^2 = (\varepsilon_{x,3+} - \varepsilon_{y,3+})^2 + (A_f B_0)^2 \left[\frac{1}{(B\rho)_{7+}} - \frac{1}{(B\rho)_{3+}}\right]^2$$

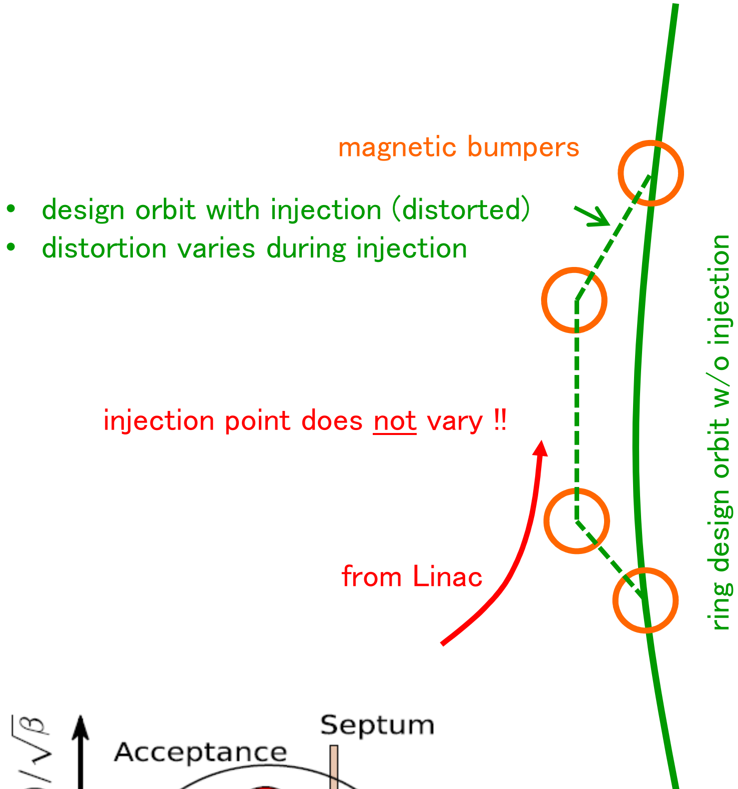
emittance shaping

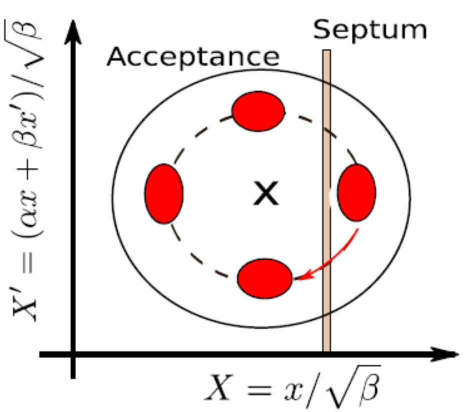


Phys. Rev. Lett. **113** 264802 (2014)



Optimization of multi-turn injection





<u>many injection parameters to be optimized</u>:

- position of incoming beam at septum — x_s, x'_s
- initial distortion from bumper — x_b
- bump decreasing rate — T
- # turns to be injected — n
- hor. tune — Q_x
- hor. emittance from linac — ε_x

target of optimization:

- maximum current after injection
- minimum integrated losses
- emittance (brilliance) from linac

Optimization of multi-turn injection: Generic algorithm

- create about 150 random sets of parameters (parents = generation #1)

x_s	x'_s	x_b	T	n	Q_x	ε_x	parent 1
			⋮				
x_s	x'_s	x_b	T	n	Q_x	ε_x	parent 150

- check all parents for their fitness w.r.t. target of optimization

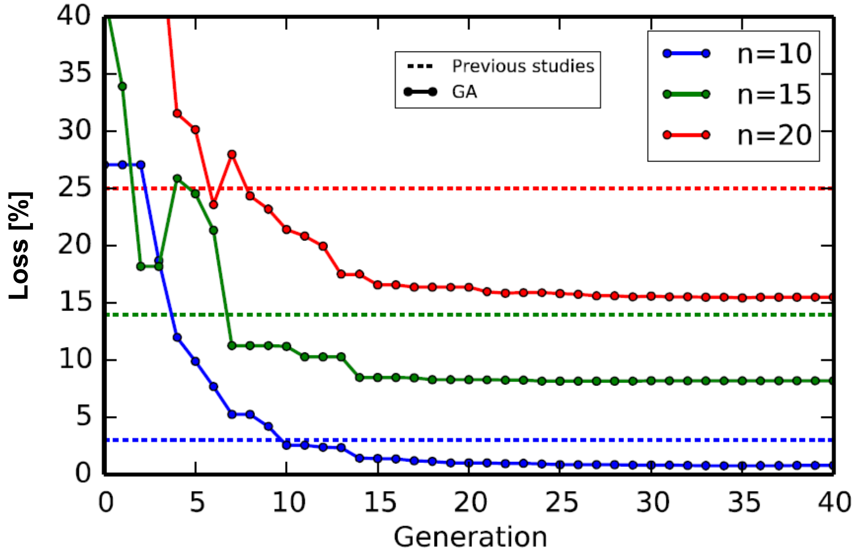
- pick best 100 parents and create about 50 children (gen. #2) from:
  - mutation of one parent (vary each parameter by some percent)
  - combination of parameters from two parents

- check all 50 children + 100 parents for their fitness

- pick best 100 from this pool and create 50 children (gen. #3) from:
  - mutation of one parent (vary each parameter by some percent)
  - combination of parameters from two parents

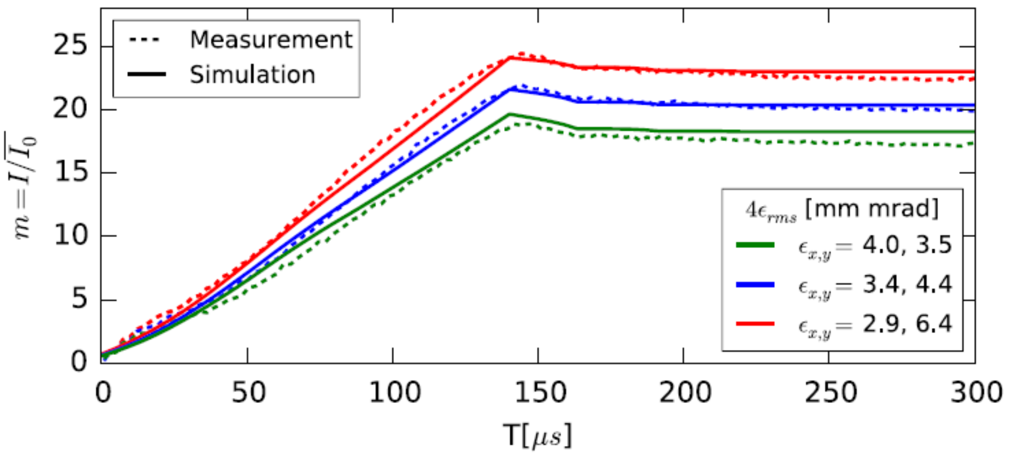
- check all 50 children + 100 parents for their fitness ...

Optimization of multi-turn injection: Generic algorithm



algorithm reduces losses by a factor of 2

Nucl. Instrum. & Meth. **A** 852, p.73 (2017)



Nucl. Instrum. & Meth. **A** 866, p.36 (2017)

Summary

- Ion stripping efficiency increases by 50% using pulsed H_2 jet
- Advanced longitudinal beam dyn. modeling of DTLs explains UNILAC 's "intermediate-energy puzzle"
- Demonstration of complete 4d transverse diagnostics at ions with E $\gtrsim$ 150 keV/u
- Fast modeling of emittance gymnastics by extending Busch theorem to beams
- Generic algorithm increases efficiency of multi-turn injection into rings