Characterization of high intensity beam in Linacs

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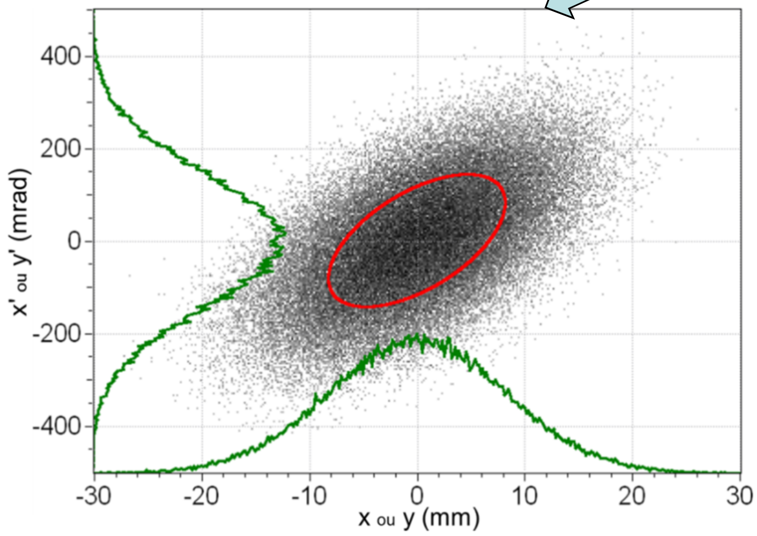
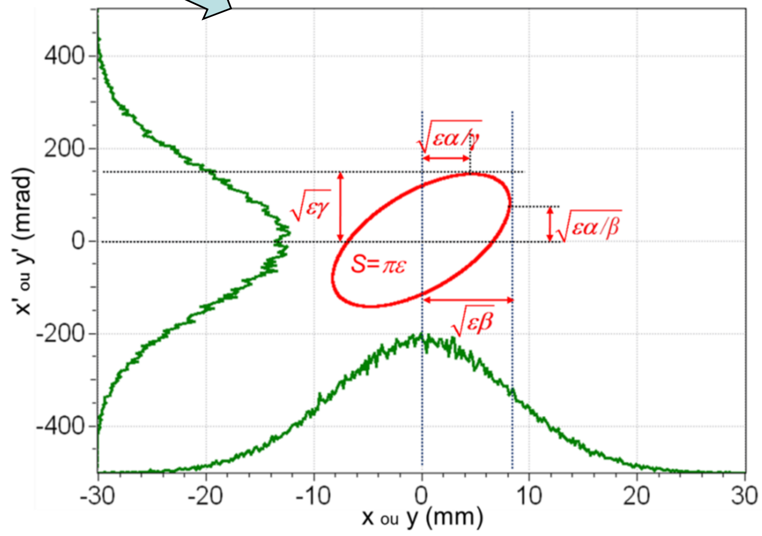
The two classical ways

Classically





Coordinates of macroparticles in 6D phase space
→ 10^{5-6} parameters
→ very big number → num. simulations

Concentration ellipse
RMS ε, α, β (,γ)
→ 3 global parameters

The two classical ways

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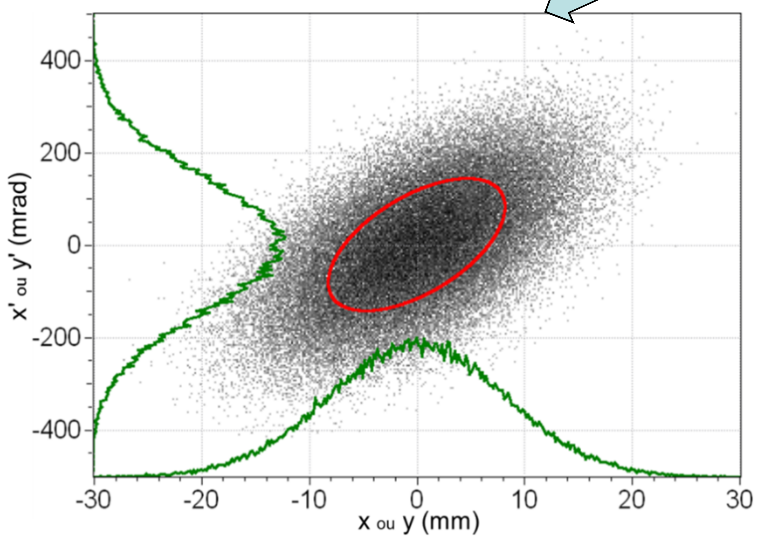
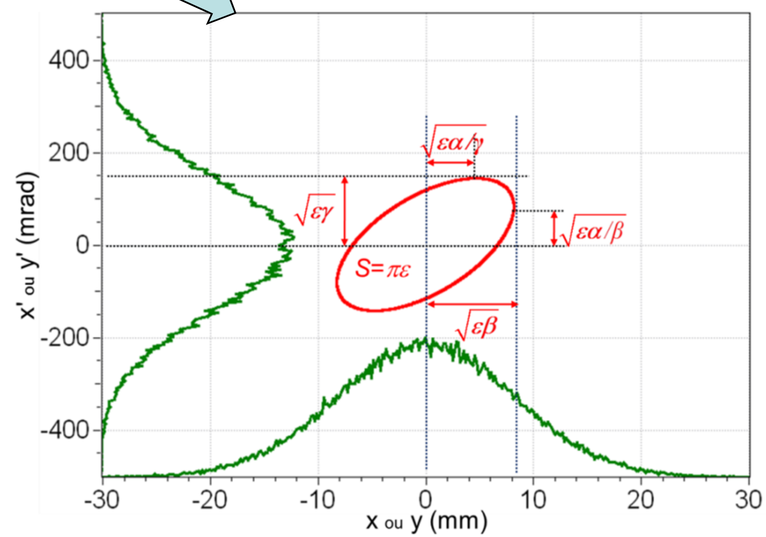




Coordinates of macroparticles in 6D phase space
→ $10^{5\text{-}6}$ parameters
→ very big number → num. simulations

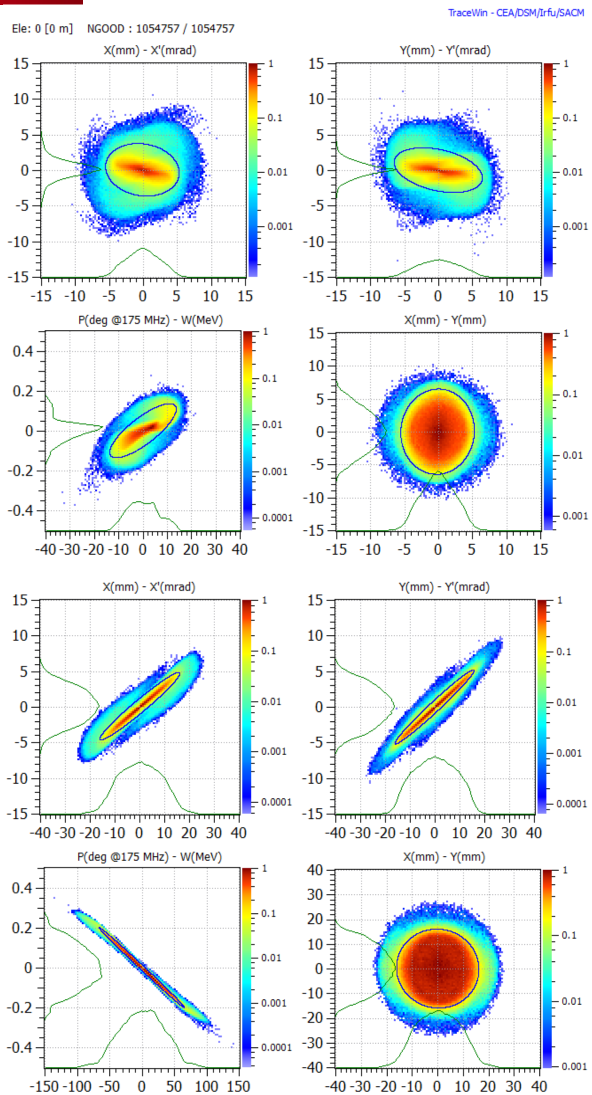
Concentration ellipse
RMS ε, α, β (,γ)
→ 3 global parameters

High intensity

More particles
Higher space charge and power
→ 10^9 particles → 6 10^9 parameters
→ huge number

RMS ε, α, β (,γ)
Not enough !!
Distribution dependent

Beam transport: distribution dependent (1)

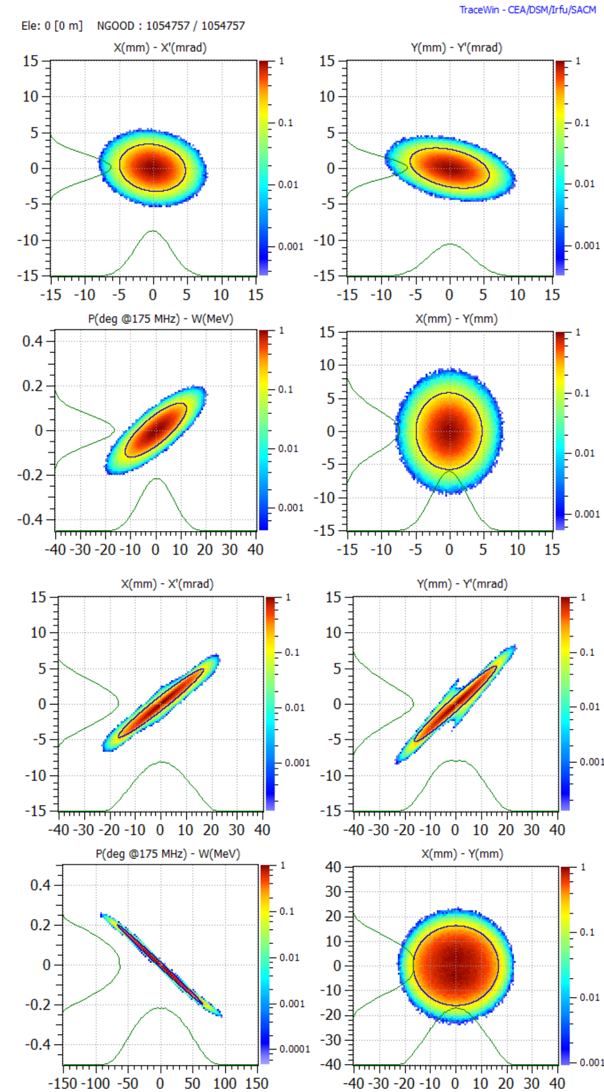


"Nominal" distribution

IFMIF
HEBT entrance
125 mA, 9 MeV
exactly same
α,β,γ,ε

3.5 m downstream
through 3 quadrupoles:
significantly different
beam outputs
α,β,γ,ε

Gaussian distribution



Beam transport: distribution dependent (2)

X-X'
ε = 0.3878 mm.mrad (Norm)
β = 1.4773 mm/mrad
α = 0.1201
Y-Y'
ε = 0.3736 mm.mrad (Norm)
β= 2.2011 mm/mrad
α= 0.3689

"Nominal" distribution

Exactly same
α,β,γ,ε

Gaussian distribution

← idem

Differences

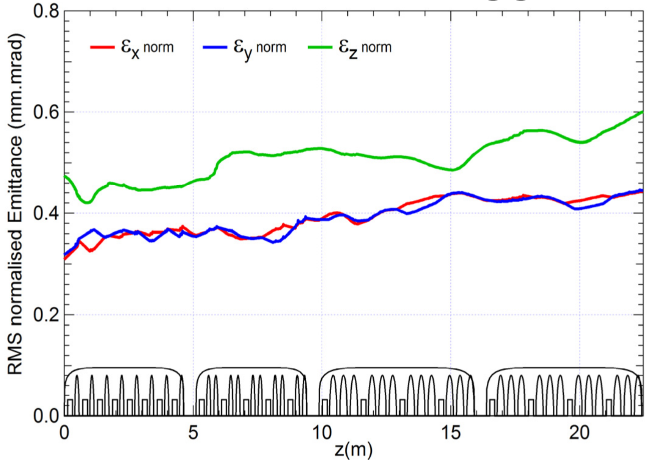
"Nominal" distribution	Differences	Gaussian distribution
X-X'		**X-X'**
ε = 0.4483 mm.mrad (Norm)	-1.3 %	ε = 0.4423 mm.mrad (Norm)
β= 15.5944 mm/mrad	+8.5 %	β= 16.9244 mm/mrad
α= -4.3853	+6.4 %	α= -4.6648
Y-Y'		**Y-Y'**
ε = 0.3770 mm.mrad (Norm)	+20.6 %	ε = 0.4547 mm.mrad (Norm)
β= 18.7219 mm/mrad	-15.3 %	β= 15.8499 mm/mrad
α= -5.8046	-15.3 %	α= -4.9160

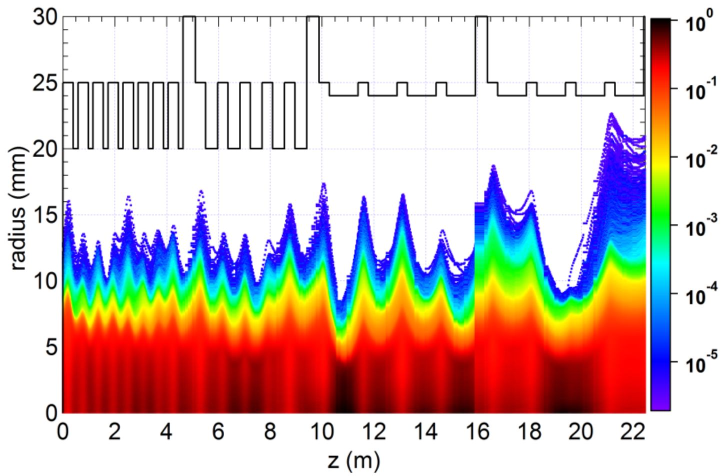
RMS Emittance and Twiss parameters are NOT ENOUGH for beam characterization

Beam Optimization (1)

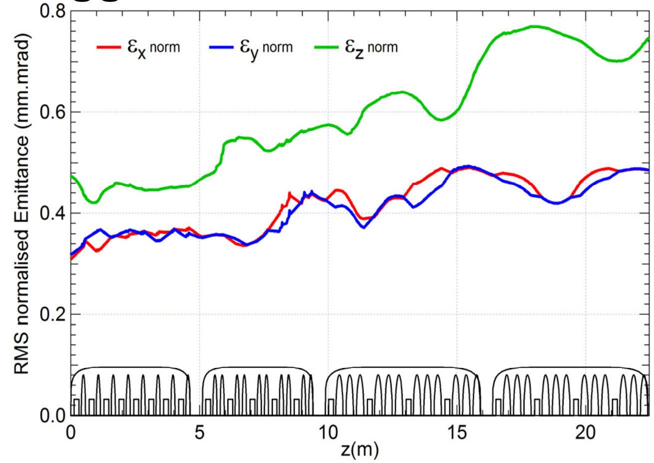
Example: IFMIF SRF Linac **(Laser Part. Beams 32, 10-118, 2014)**

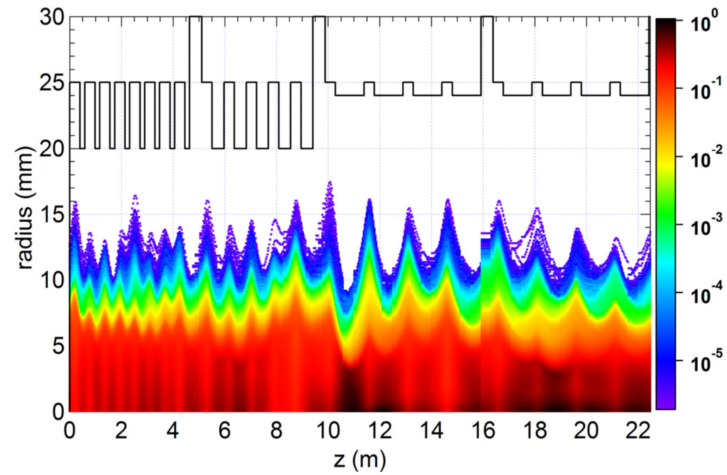
Smaller emittance: Bigger size





Bigger emittance: Smaller size

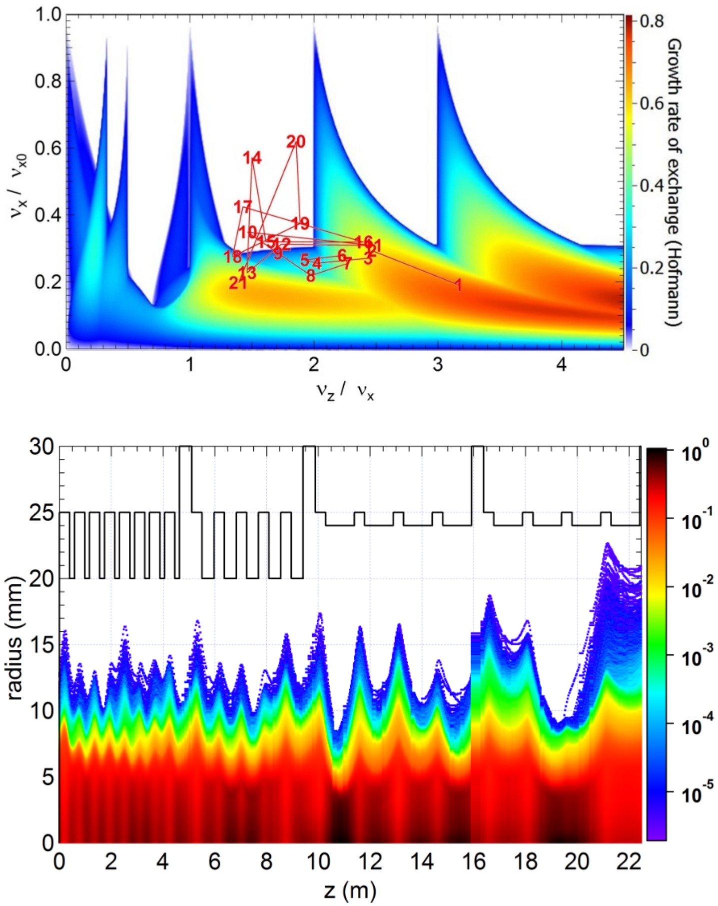




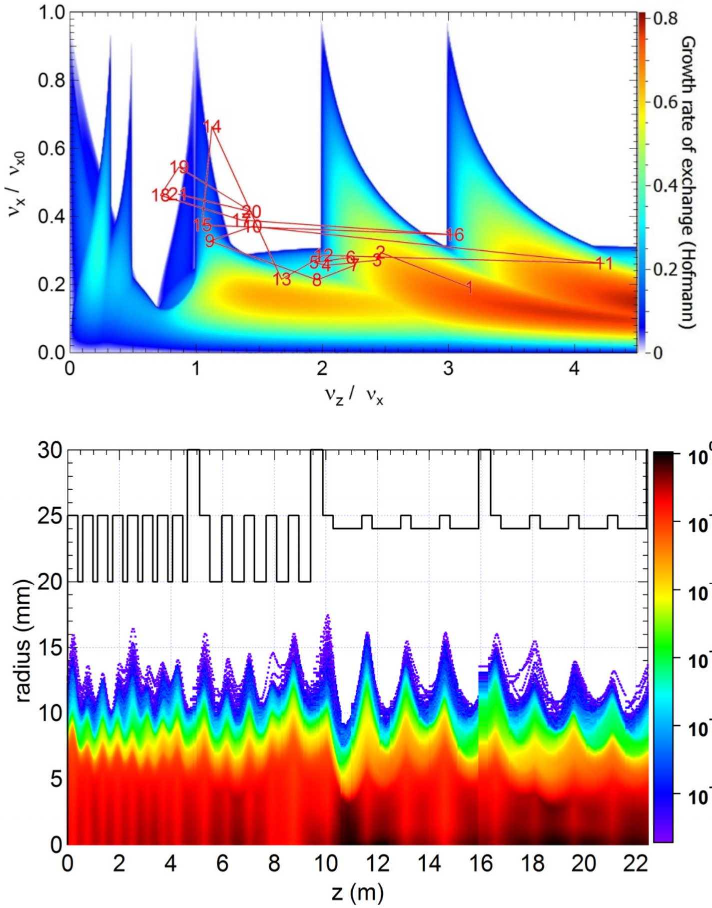
Beam Optimization (2)

Example: IFMIF SRF Linac (Laser Part. Beams 32, 10-118, 2014)

Smaller emittance: Bigger size



Bigger emittance: Smaller size



RMS Emittance and Twiss parameters are NOT ENOUGH for beam characterization

Beam Characterization

For high intensity beams

1) Characterize the beam by its actual number of particles

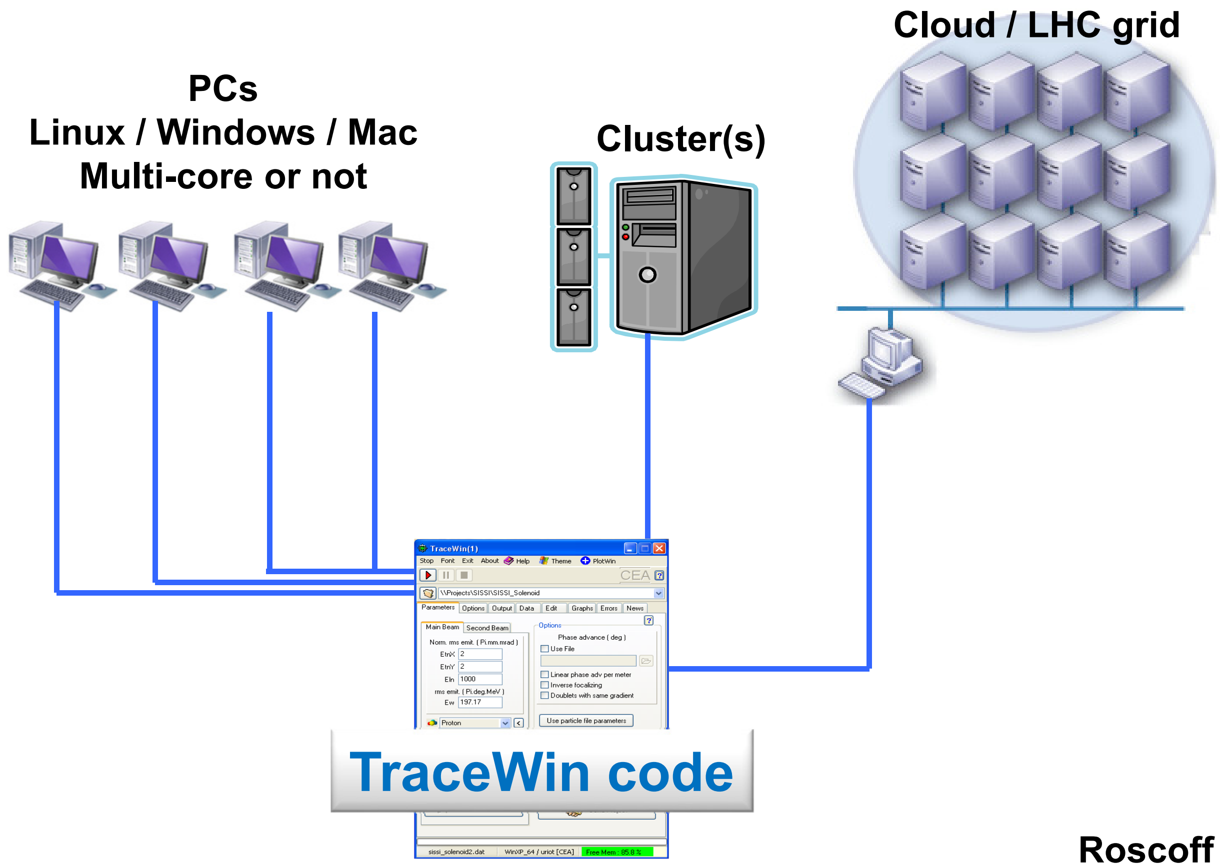
2) Characterize the beam by its projections onto a few axes

3) Characterize the beam by its core and halo separately

1. Beam characterization by the actual number of particles

Massive simulations (1)

Principle : share simulations on many computers



Roscoff 2015

Massive simulations (2)

(WEPOY032, IPAC'16)

Share particles on many computers

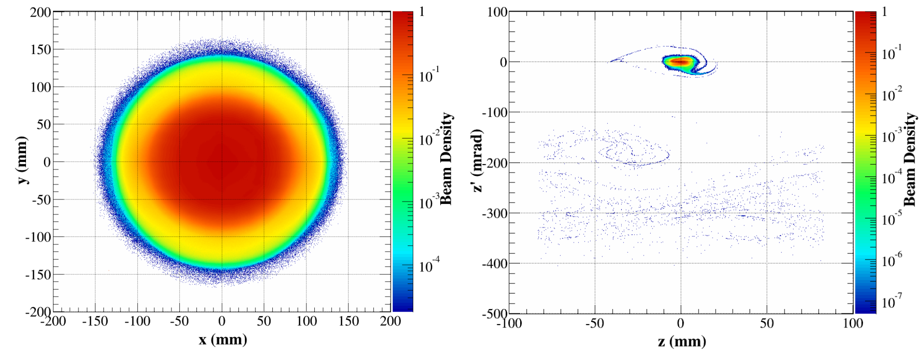
For IFMIF-LIPAc, D^+, 125 mA CW, 9 MeV

Actual number of particles: 5 10^9

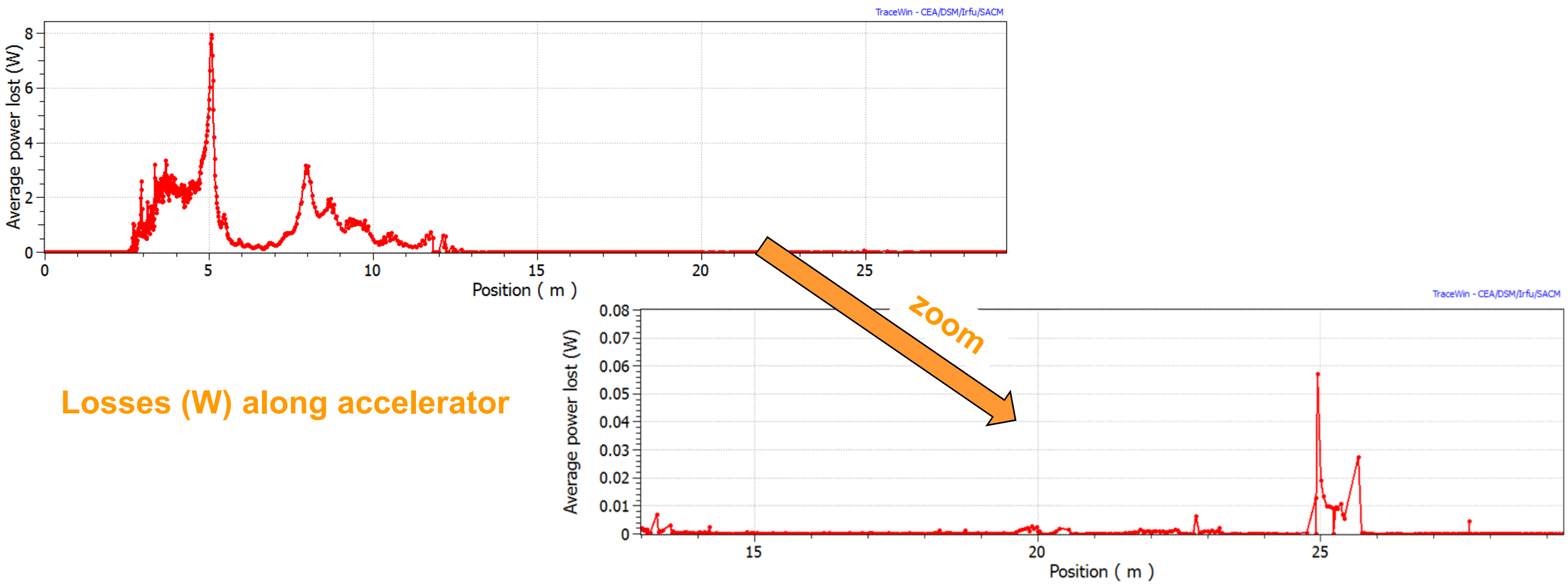
→ 170 processors for 25 days, storing 38 To

→ confirm losses < 10^{-7}

→ representative statistics of microlosses



Phase spaces on the beam dump (1.1 MW)

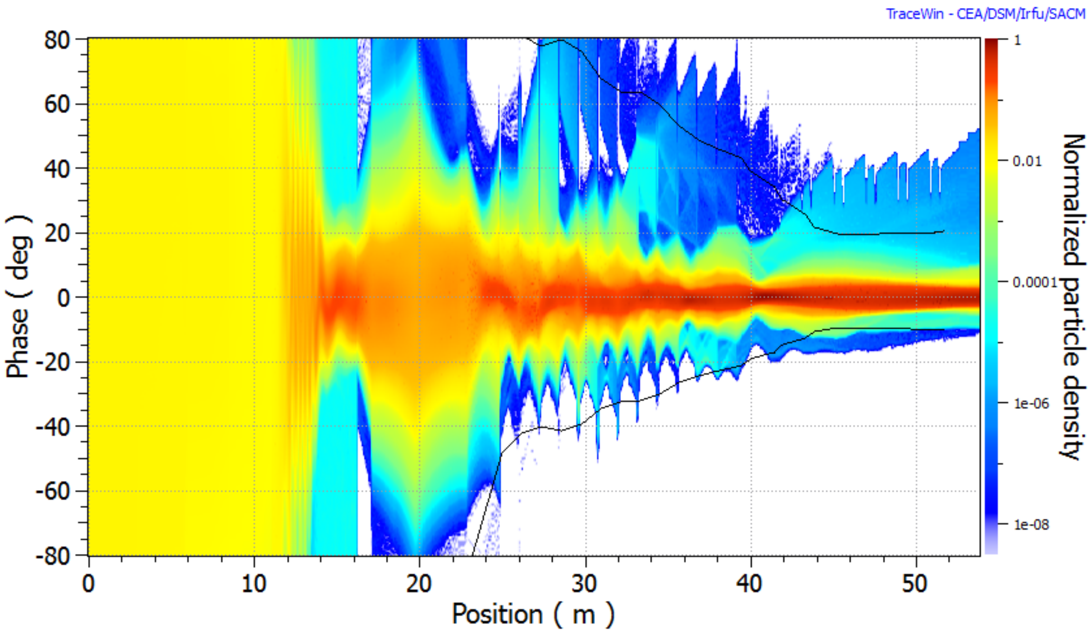


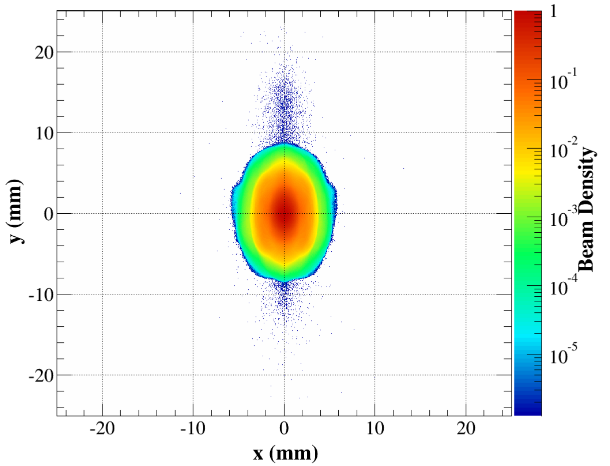
Losses (W) along accelerator

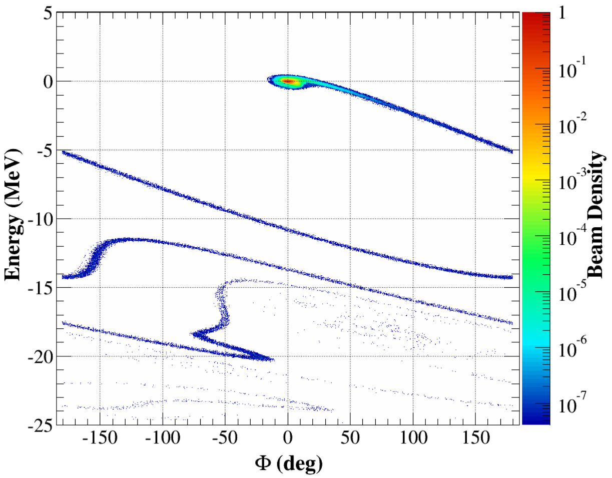
Massive simulations (3)

Share linacs on many computers, for error studies

Consequences of cavity errors (1°, 1%) on losses
for SPIRAL2, D^+, 5 mA CW, 40 MeV
number of macroprticles: 2.5 10^6, number of linacs: 1000
→ 150 processors for 7 days
→ Losses due to RFQ: 30%, due to cavity errors:70%







Cumulative results on 1000 linacs

Massive simulations (4)

- Allow very precise and detailed analysis of certain physical phenomena:

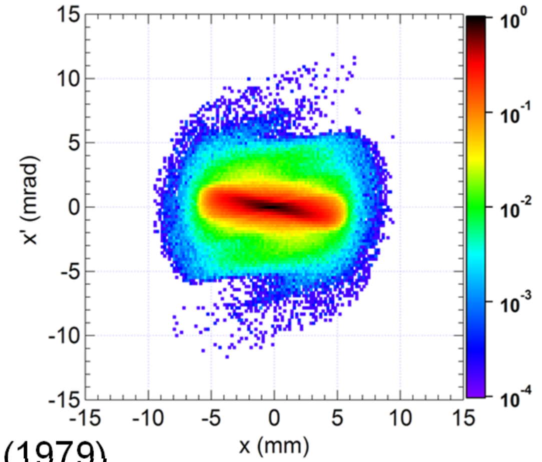
- Halo formation and evolution
- Microloss location (10^{-6} of the beam)

- More and more realistic beam transport

- Need very fine description of optics elements, realistic input particle distribution

2. Beam characterization by its projections onto a few axes

Beam Characterization by projections on a few axes (1)



Objective: Reconstructing the whole distribution from the only knowledge of its projections onto a few axes

Add supplementary hypothesis: where there is no data, the shape is the most regular possible
→ the dist. can be described by the least number of parameters
→ the dist. should have a Maximum ENTropy → MENT

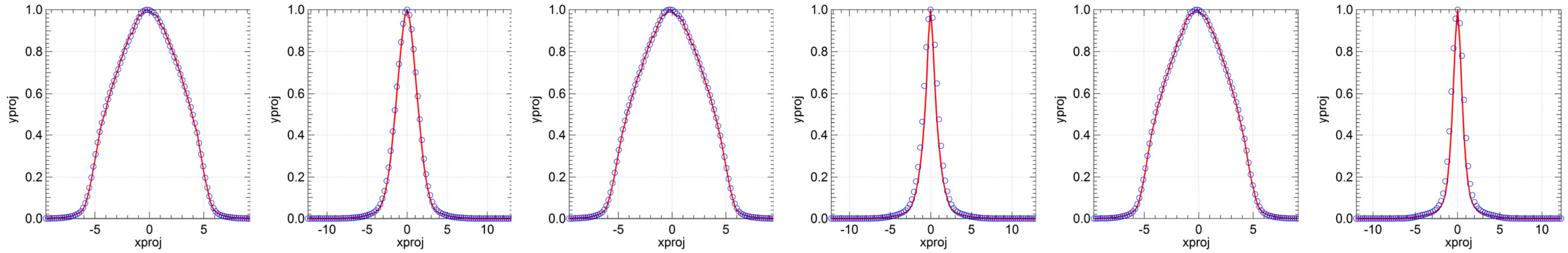
MENT method thoroughly developed in

G. Minerbo, Computer Graphics and Image Processing 10, 48-68 (1979)

then currently used in e.g.

C.T. Mottershead, IEEE Transactions on Nuclear Science NS-32 (1985)
D. Reggiani & M. Seidel, Proc. of IPAC'10, MOPE065, Kyoto, Japan (2010)
K.M. Hock et al., Nucl. Instru. Meth. Phys. Res. A 642, 36-44 (2011)
K.M. Hock et al., Nucl. Instru. Meth. Phys. Res. A 753, 38-55 (2014)

Quick convergence, in less than 5 iterations



Beam Characterization by projections on a few axes (2)

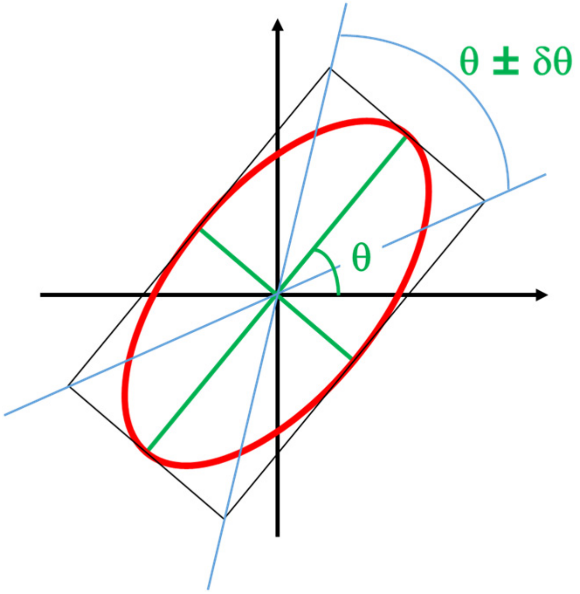
For correctly describing a given beam, the questions are:
- what is the minimum number of projections ?
- which projections to consider ?

We recommend:

1) Reconstruct the distribution with 4 axes regularly positioned within 360°, i.e. 0°, 45°, 90° and 135°.

2) Calculate the concentration ellipse of the obtained distribution then determine its axis angle θ and aperture $\delta\theta$

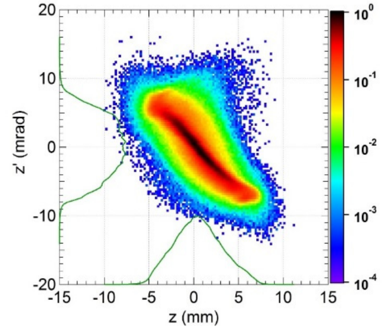
3) Reconstruct finally the distribution with projection axes regularly positioned within $\theta \pm \delta\theta$, and the same number of axes perpendicular to those ones.



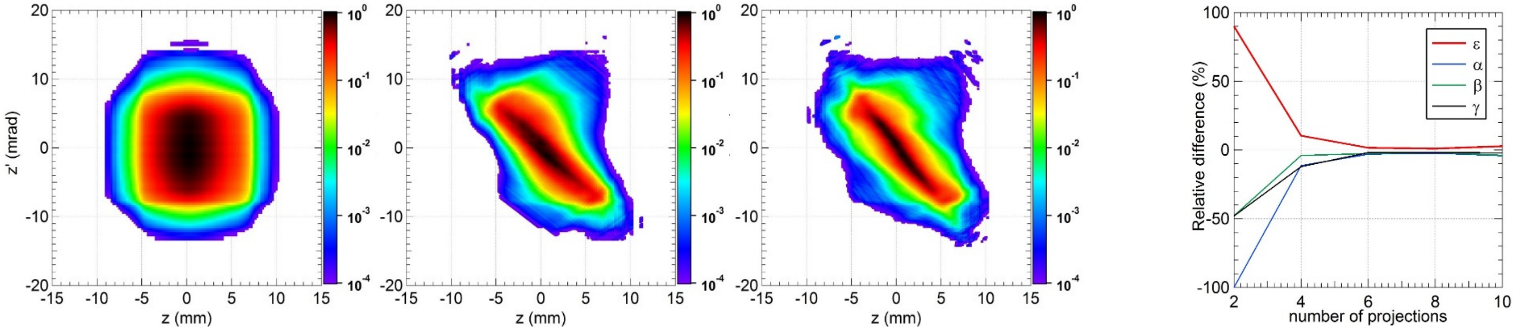
Beam Characterization by projections on a few axes (3)

IFMIF-LIPAc MEBT exit

Actual distribution

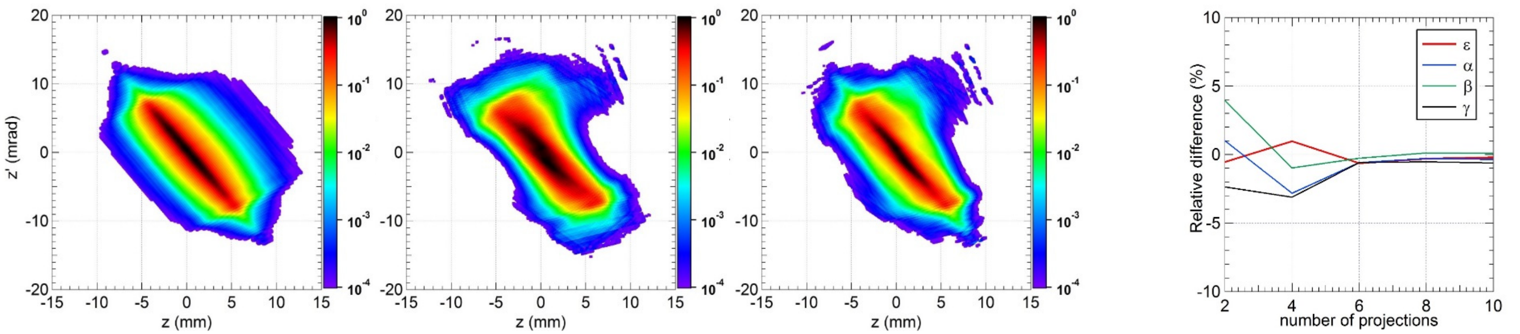


Projections within 360°



2 projections **4 projections** **6 projections**

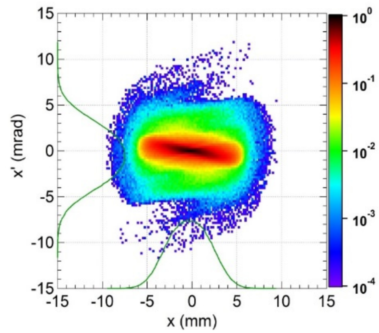
Projections within -57° ± 15° and ⊥



Beam Characterization by projections on a few axes (4)

IFMIF-LIPAc SC-Linac exit

Actual distribution



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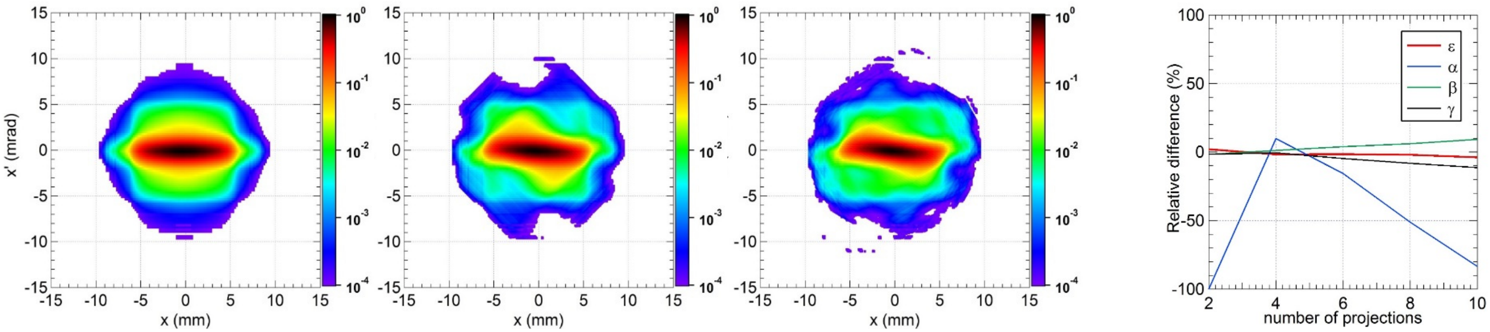
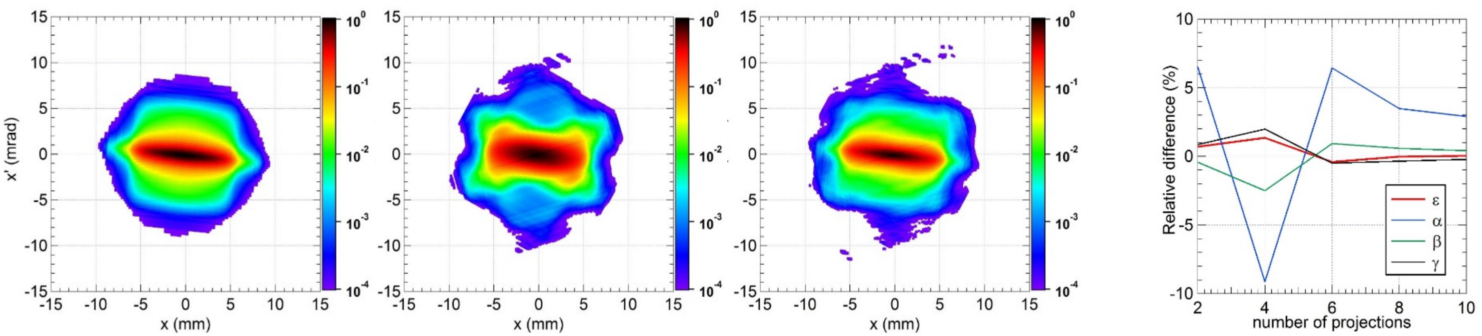


2 projections **4 projections** **6 projections**

Projections within -6,1° ± 22,8° and ⊥



Beam Characterization by projections on a few axes (5)

Reconstruct a distribution from its projections → MENT method:

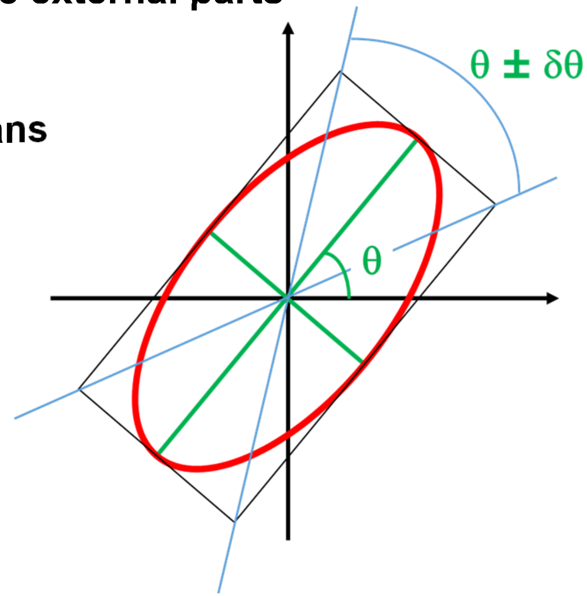
- **Projection axes correctly chosen in direction and angle range**
- **→ ~ 2 projections are enough for describing the internal, most dense parts**
- **→ ~ 6 projections are enough for describing in addition the external parts**

Each projection could be represented by generalized gaussians

$$f(x) = Ae^{-\left(\frac{|x-\mu|}{\alpha}\right)^{\beta}}$$

→ ~ 10 - 30 parameters for 2D

BUT: how to go to 4D, 6D ?



MOPOR032, IPAC'16

3. Beam characterization by its core and its halo separately

Beam Characterization by Core and Halo

Further reduce the number of parameters :
describe the beam by the global characteristics of its CORE and its HALO separately

→Fine details are lost BUT more insight in the physics of the beam can be gained

High intensity beam : competition of internal forces (space charge)
and external forces (focusing)

Result: growth or decay of Core or Halo

On the secret (!) relations between Emittance and Halo

Similar: Emittance and Halo can only be changed with non linear forces

ALLEN & WANGLER, PRSTAB,5,124202 (2002):
"As with emittance, the halo parameter is invariant under linear forces.
Thus, halo growth is necessarily the result of nonlinearities."

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Not similar: Their change can be not in the same sense or not at the same moment

1) ALLEN & WANGLER, PRSTAB,5,124202 (2002):
"The halo parameter contains additional information as to the beam state, since we find that it is possible to have emittance growth without halo growth (however, halo growth always implies emittance growth)."

2) Emittance can only increase, while Halo can increase or decrease

3) Well known "Beam redistribution" (Wangler, RF linear accelerators): Emittance increases very quickly, parallely to distribution reorganization to get more compact, decreasing the halo size

4) Use of external non linear forces, octupoles, duodecapoles, to decrease the halo (fold in the tails): Halo decreases and Emittance increases

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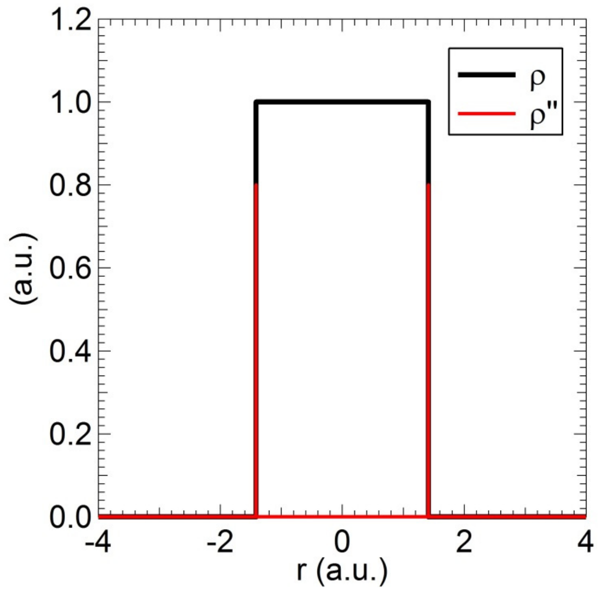
Emittance is NOT appropriate to characterize the beam size nor the halo

Precise determination of Core/Halo limit

For 1D: Core-Halo limit based on density profile

Appl. Phys. Lett. 104, 074109, 2014



<u>Extreme case:</u>

Core: uniform, sc force strictly linear

Halo: tenuous, sc force nonlinear

→ core-halo limit: very steep (infinite) variation of the slope

Precise determination of Core/Halo limit

For 1D: Core-Halo limit based on density profile

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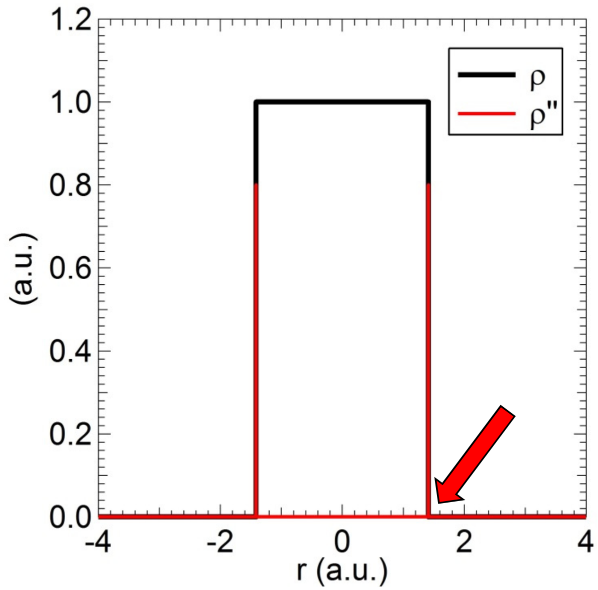

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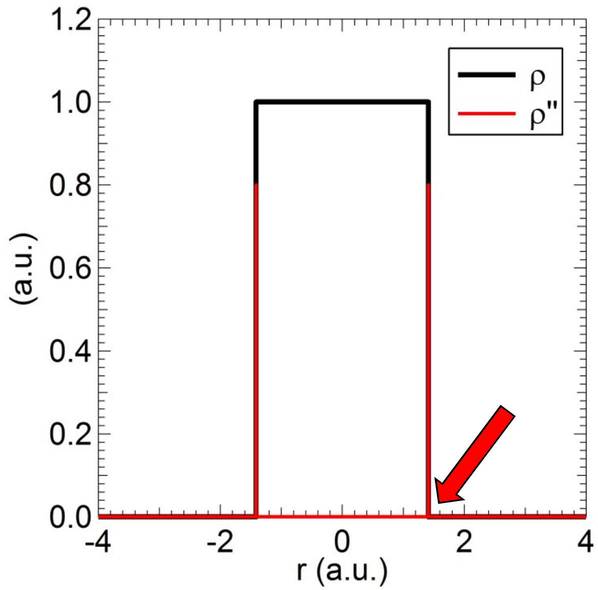

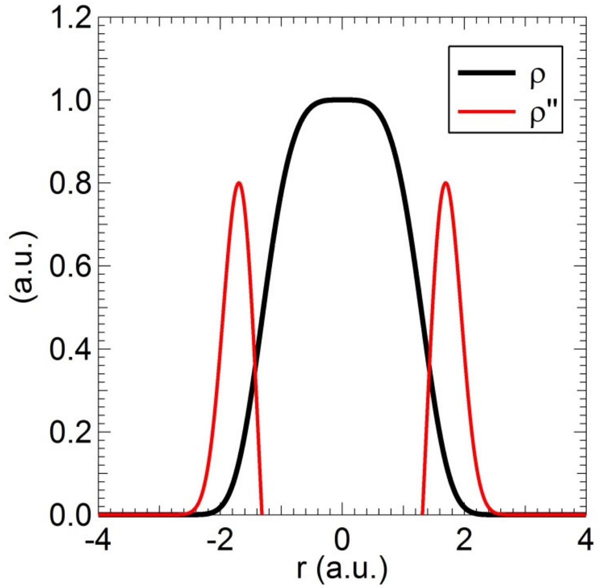

<u>Extreme case:</u>
Core: uniform, sc force strictly linear
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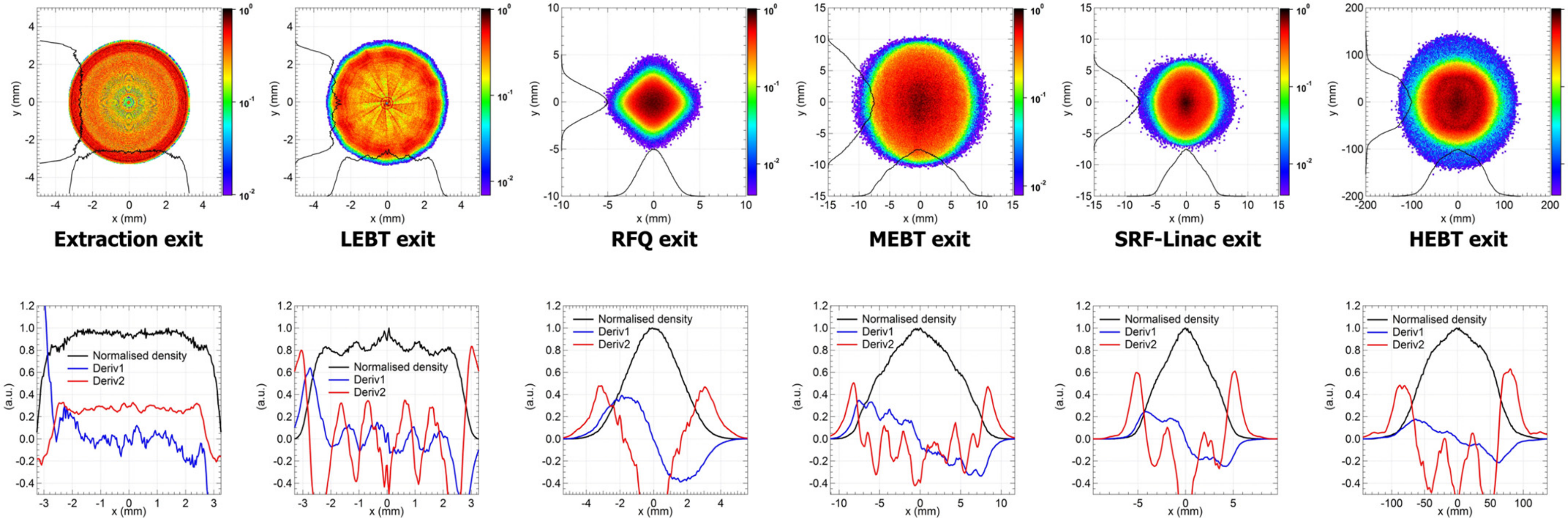
→ core-halo limit: very steep (infinite) variation of the slope

<u>General case:</u>
Continuously varying density
Core-Halo limit: steepest variation of the slope → <u>max of 2nd derivative</u>

Examples of Core/Halo limits

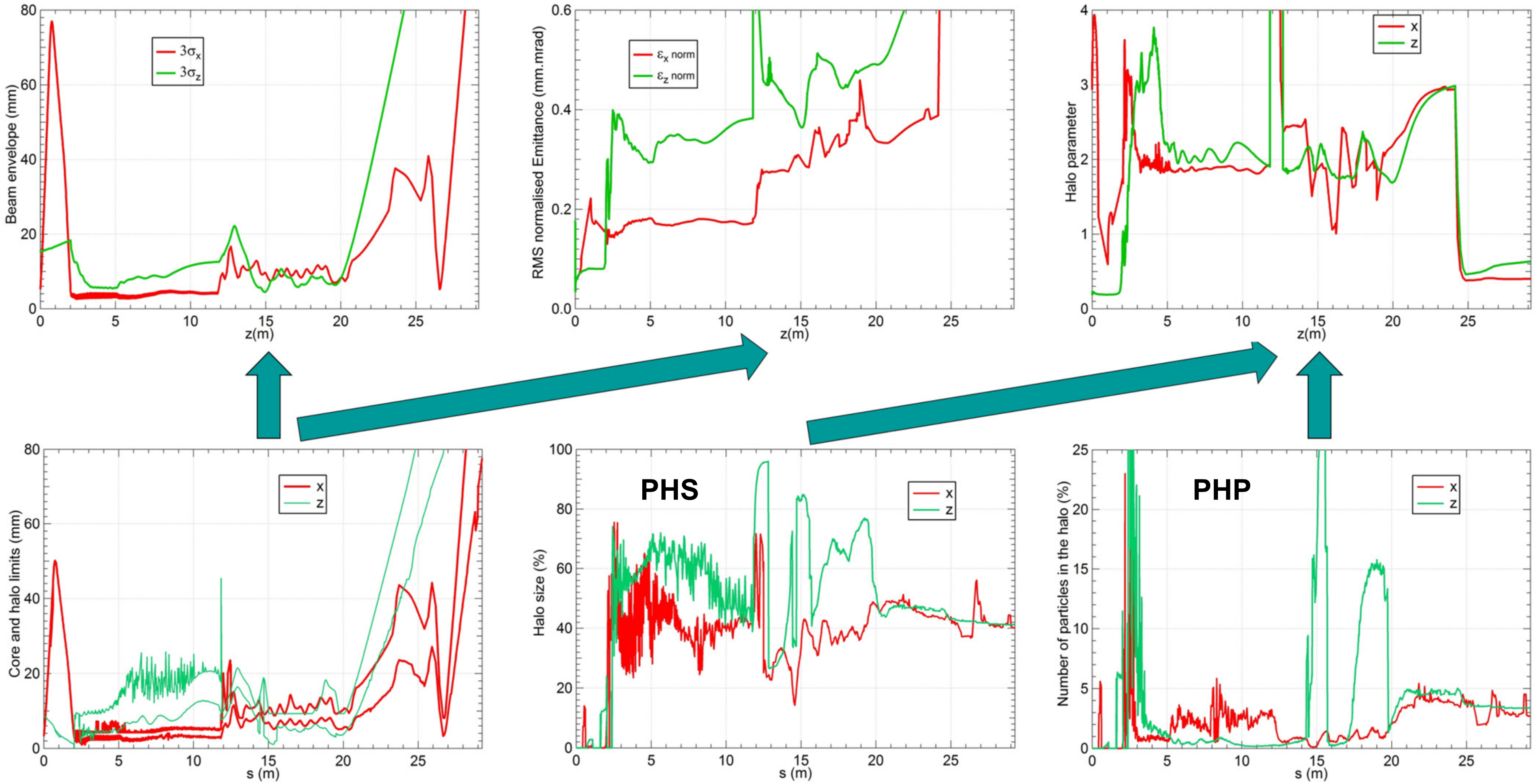
Example: Beam along the IFMIF prototype accelerator



Beam Characterization by Core and Halo

Example: Beam along the IFMIF Prototype accelerator

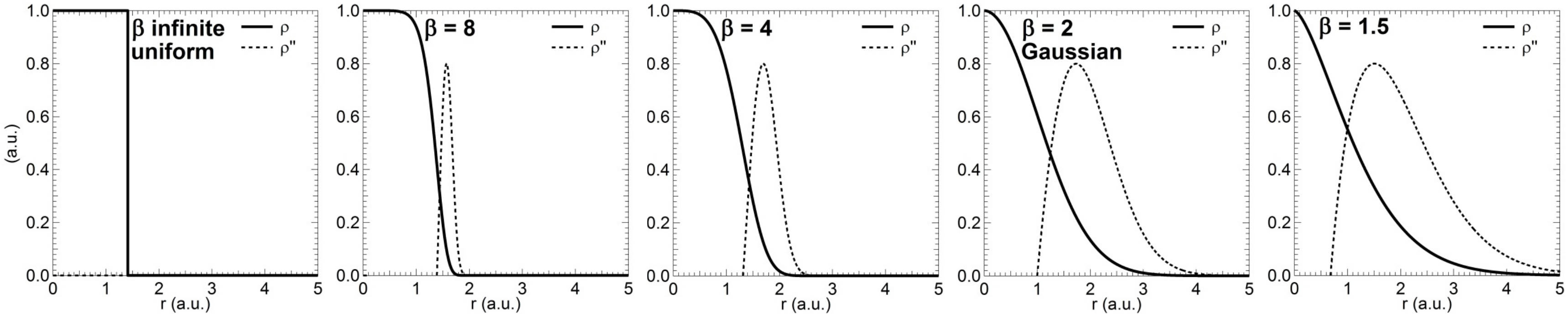
Classically

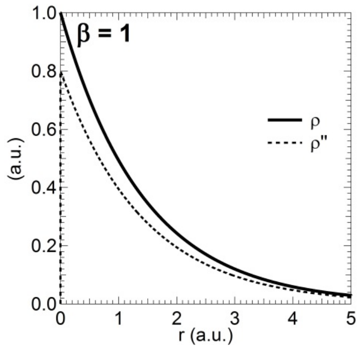


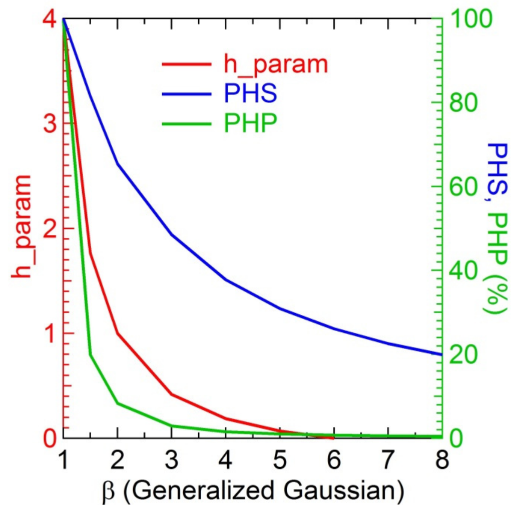
Advanced

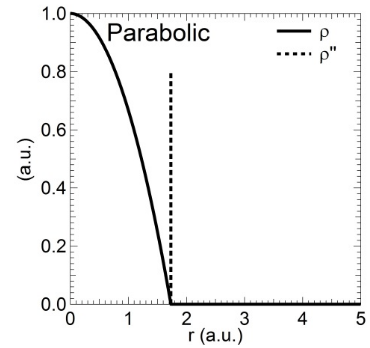
PHS, PHP vs h_parameter (1)

Generalised Gaussian profiles $\rho(r) = \rho_0 e^{-\left|\frac{r}{\alpha\sqrt{2}}\right|^{\beta}}$

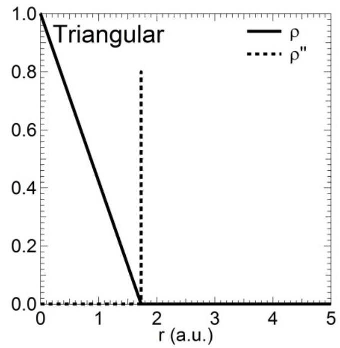








PHS = PHP = 0
h = 0.14

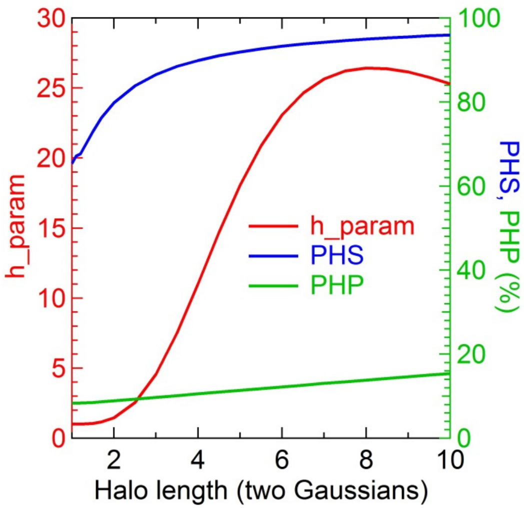


PHS = PHP = 0
h = 0.4

PHS, PHP vs h_parameter (2)

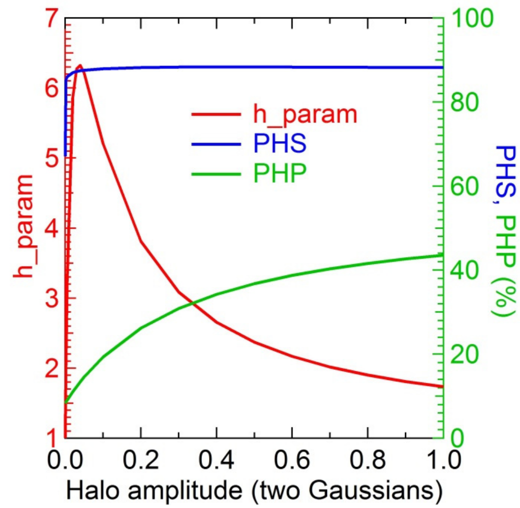
Sum of two Gaussian's $\rho(r) = \rho_1 e^{-\left(\frac{r}{\sigma_1\sqrt{2}}\right)^2} + \rho_2 e^{-\left(\frac{r}{\sigma_2\sqrt{2}}\right)^2}$

Halo with longer and longer tail



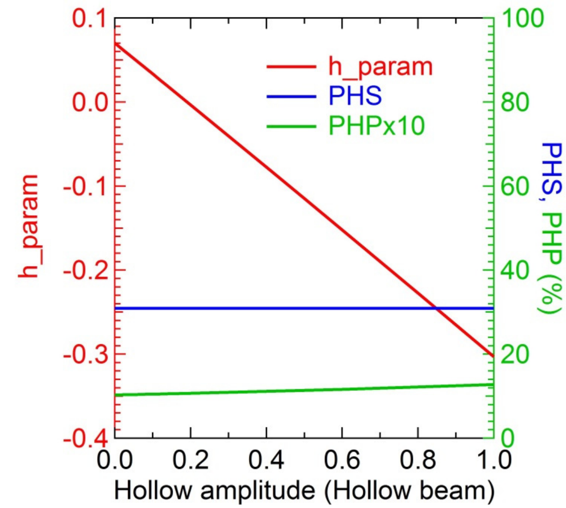
σ_2/σ_1

Halo with higher and higher amplitude



ρ_2/ρ_1

Hollow with higher and higher amplitude



$-\rho_2/\rho_1$

The KEY QUESTION

- Core-Halo limit corresponds well to visual inspection of density profile ρ
- PHP and PHS vary as expected for every type of density profile ρ

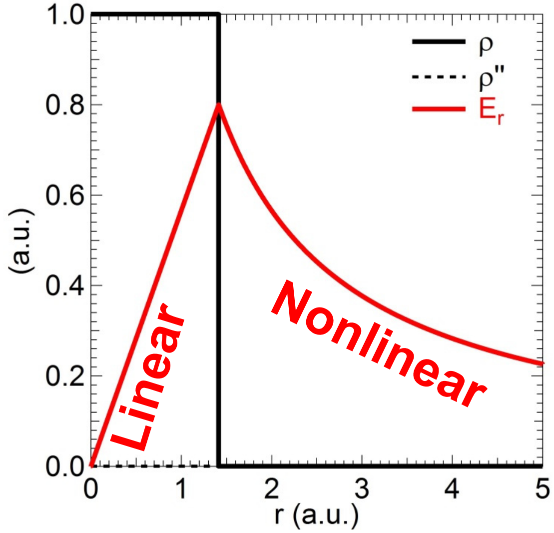
Important question:
Core-Halo ≡ Indicator of the internal dynamics of the beam ?

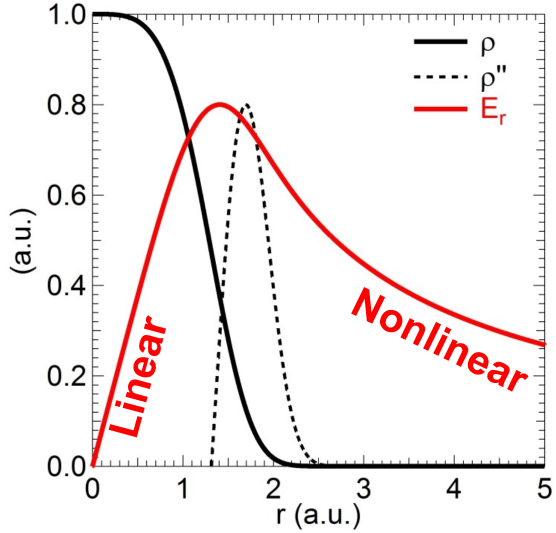
Governed by the Space Charge self field E_{SC}

- **Core: E_{SC} is "mainly" linear**
- **Halo: E_{SC} is "mainly" nonlinear**

DENSITY & S.C. FIELD

Gauss' law: $E_r(r) = \frac{1}{\varepsilon_0 r} \int_0^r \rho(x) x dx$



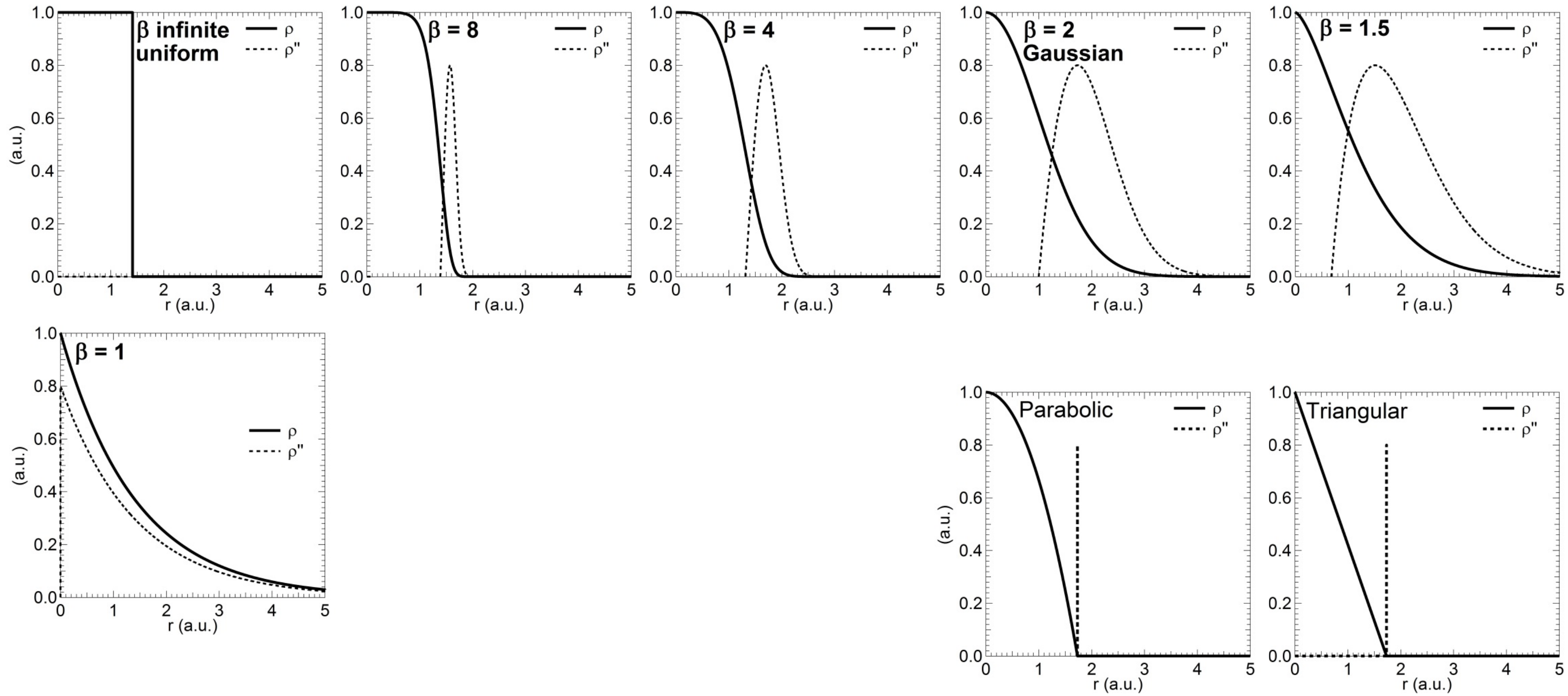


Core-Halo limit:
Discontinuity of ρ, E_r'
Biggest change in slope

Core-Halo limit:
Maximum of ρ'', E_r'''

For different density profiles

Generalised Gaussian profiles $\rho(r) = \rho_0 e^{-\left|\frac{r}{\alpha\sqrt{2}}\right|^{\beta}}$



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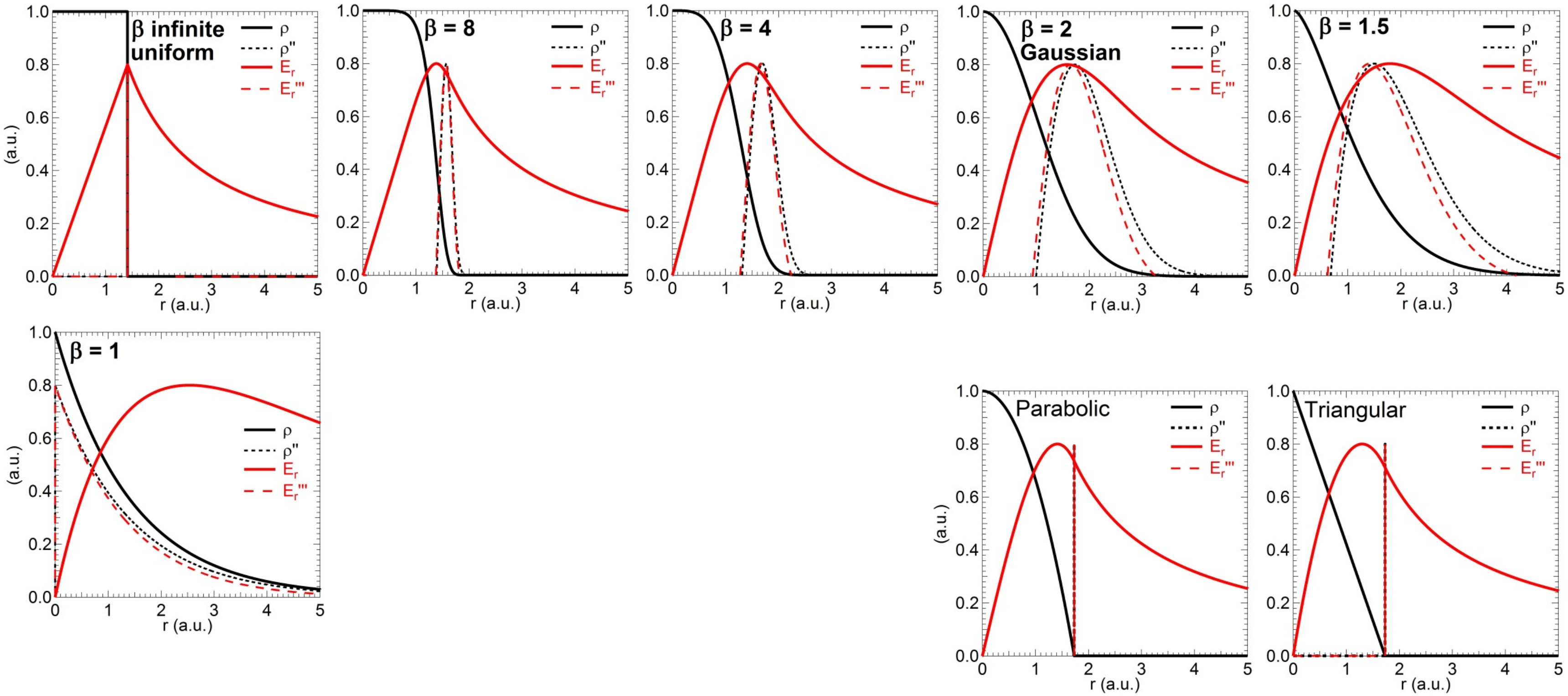

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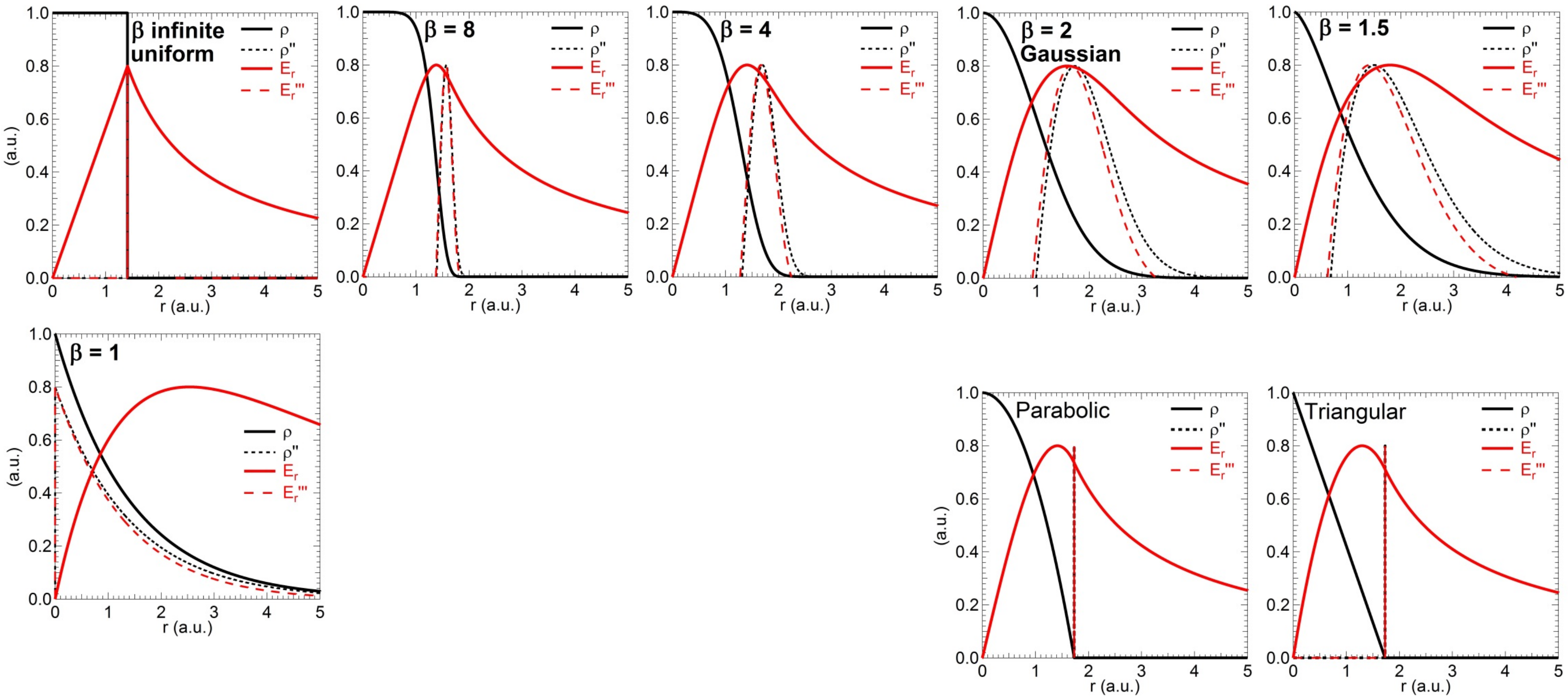


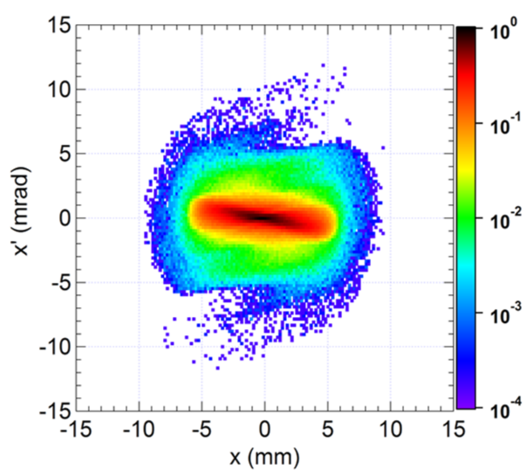
Phys. Plasmas 22, 083115, 2015: **This core-halo limit ≡ good indicator of beam internal dynamics**

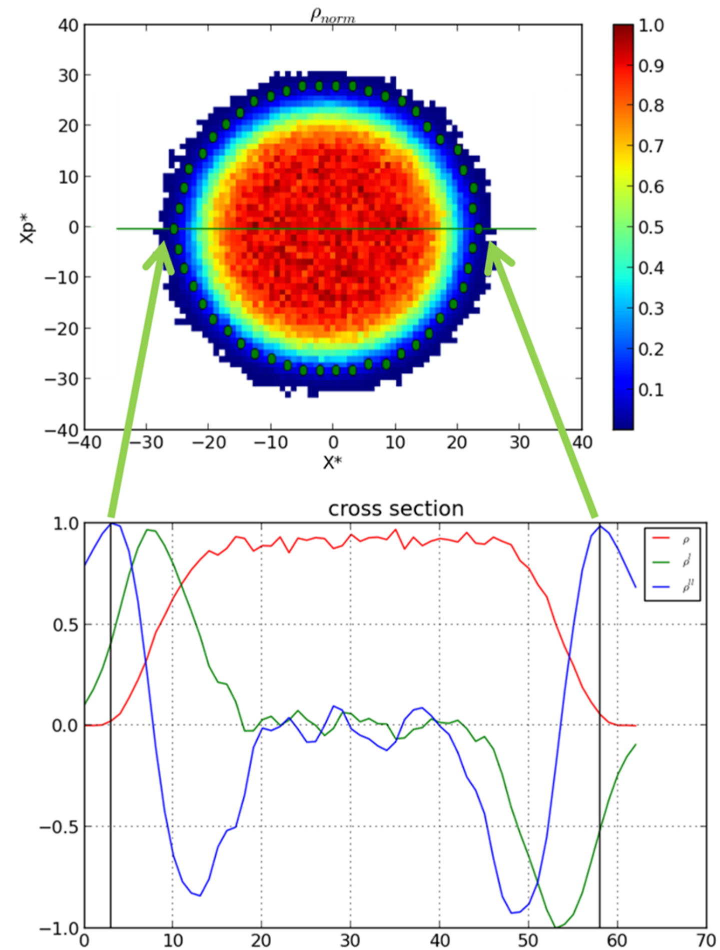
Precise determination of the Core/Halo limit in 2D

MOPWA010, IPAC'15: Extension to 2D

"Wheel algorithm": Max of second derivative along many sections

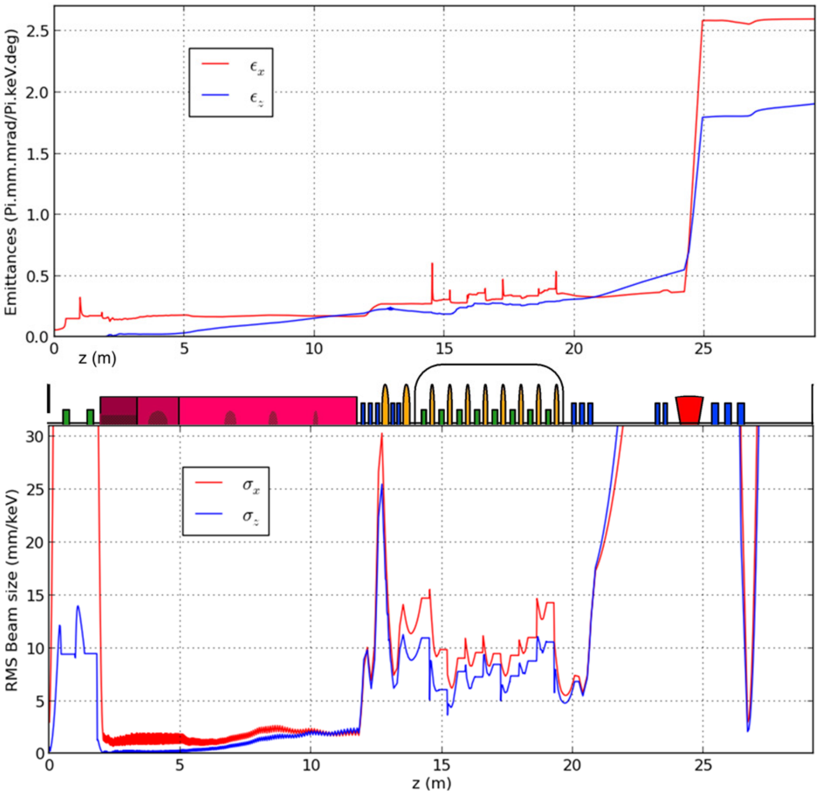
- **Core-Halo limit contour**
- **PHS, PHP**
- **Emittance ε and Twiss parameters α, β, γ of the core and the halo separately**



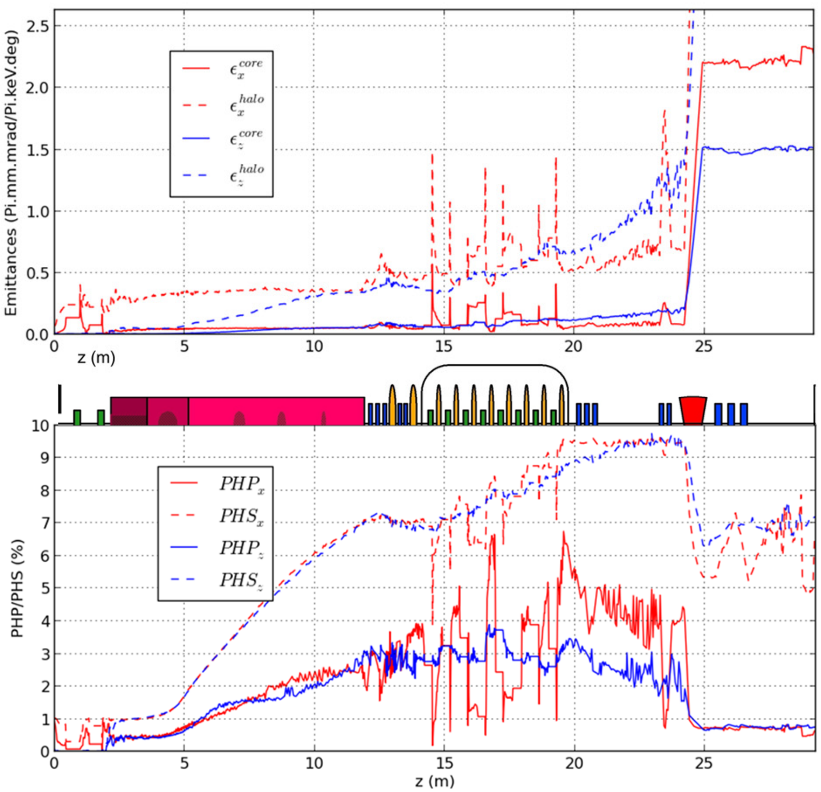


Beam Characterization by Core and Halo in 2D

Classically



Advanced



Important question:
Core-Halo ≡ reflect the halo formation dynamics?

Classical case of halo formation:

Transport of a mismatched beam through a continuously focusing channel

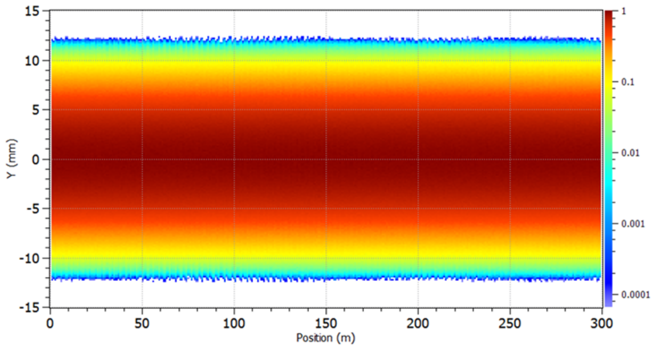
Protons 5 MeV, 100 mA
10^6 macroparticles, uniform distribution in 6D ellipse

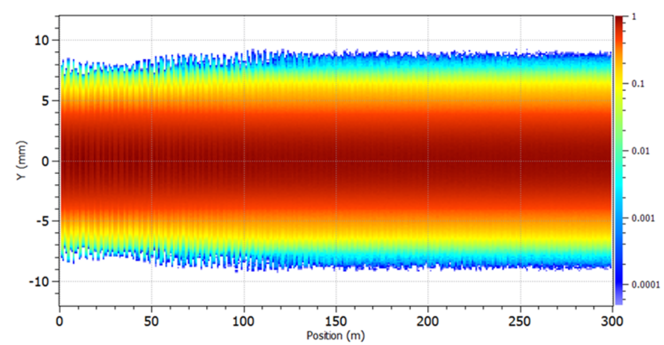
Matched transport:

	x	*y*	*z*
μ_0 (d.m^{-1}):	80	65	30
μ/μ_0:	0.9	0.9	0.9
ε_{init} (µm):	1	2	10
ε_{fin} (µm):	1	2	9.9

Mismatched transport:

	x	*y*	*z*
μ_0 (d.m^{-1}):	80	65	30
μ/μ_0:	0.86	0.8	0.86
ε_{init} (µm):	1	0.7	10
ε_{fin} (µm):	1	0.8	9.9





The KEY QUESTION

Important question:
Core-Halo ≡ reflect the halo formation dynamics?

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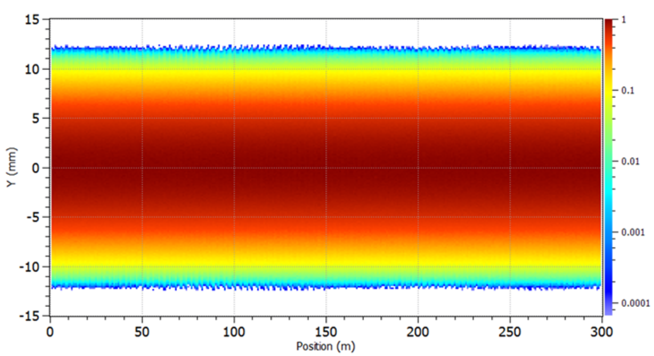
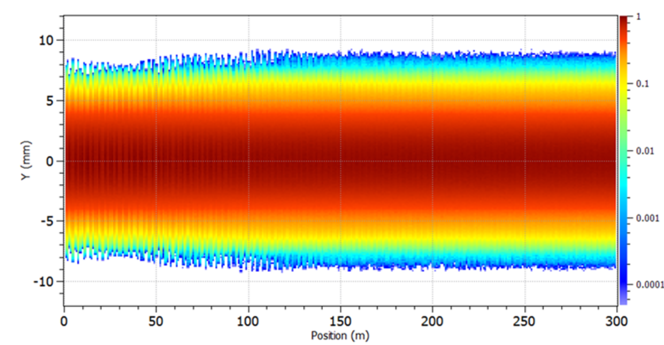
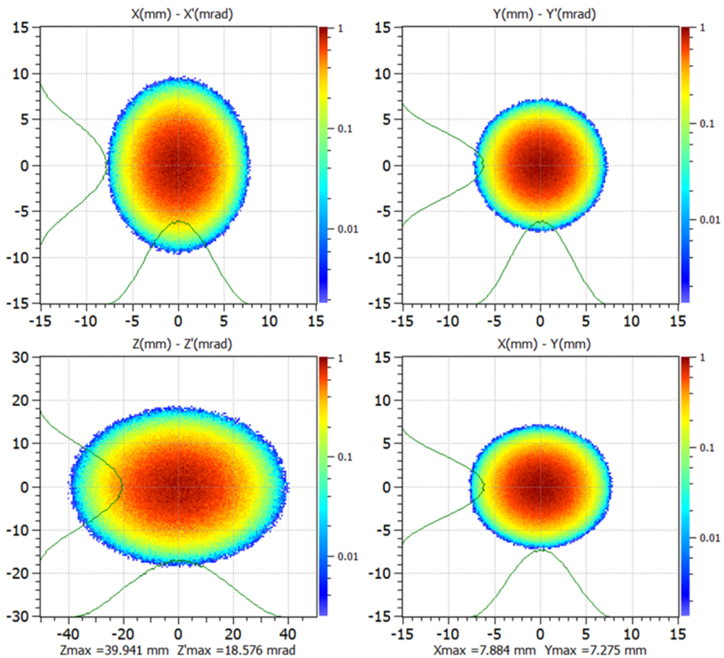
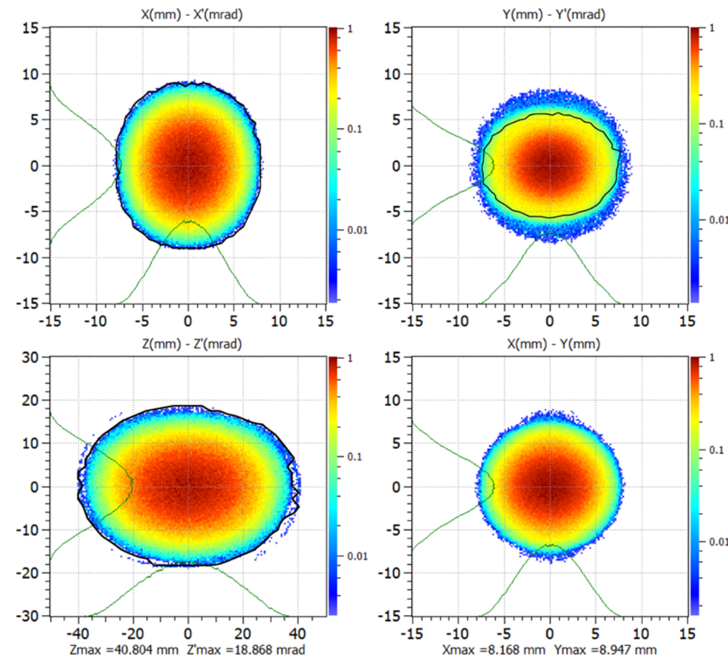




input and output when matched

output when mismatched





Core Halo limit in each phase space

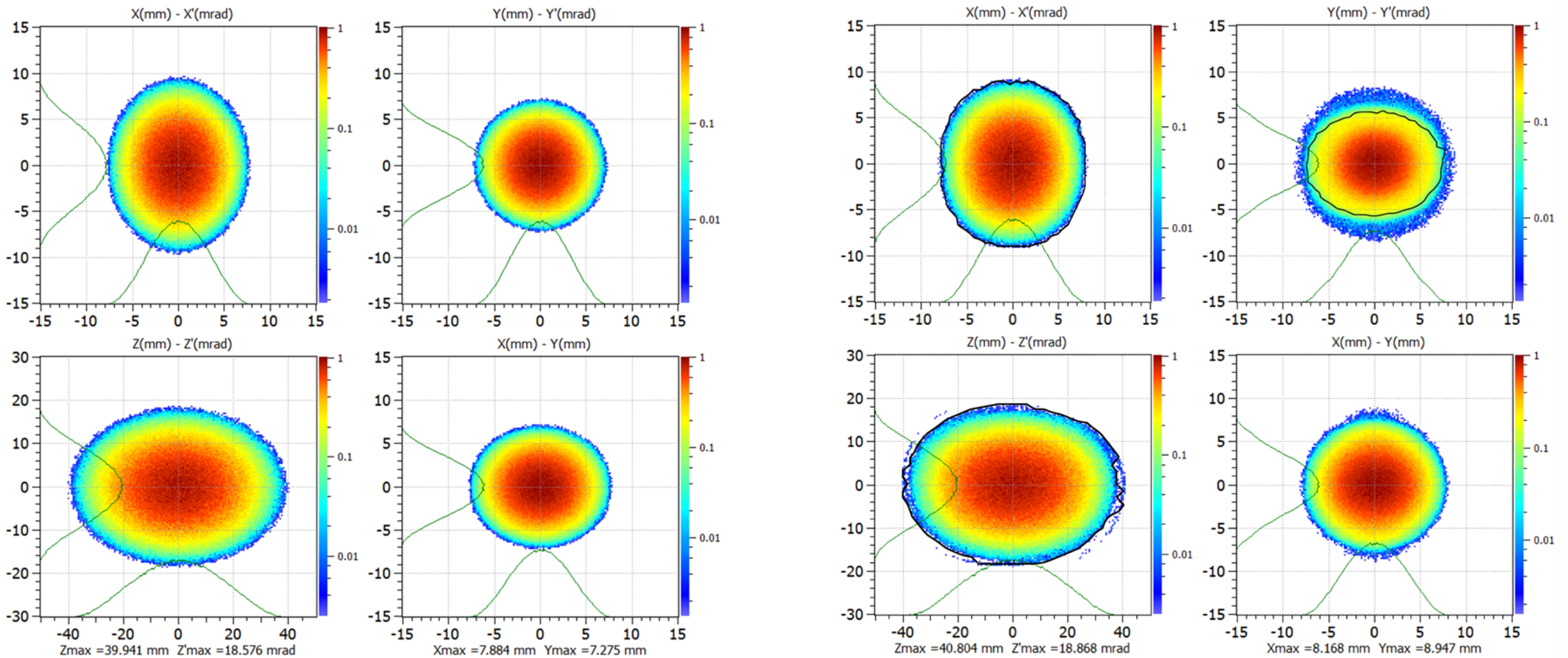
Matched transport:

	x	*y*	*z*
PHP_{init} (%):	0	0	0
PHP_{fin} (%):	0	0	0
Npart halo	13	29	44

Mismatched transport:

	x	*y*	*z*
PHP_{init} (%):	0	0	0
PHP_{fin} (%):	0	0.3	0.01
Npart halo	17	4328	1489

input and output when matched / output when mismatched



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Mismatched transport:

	x	*y*	*z*
PHP_{init} (%):	0	0	0
PHP_{fin} (%):	0	0.3	0.01
Npart halo	17	4328	1489

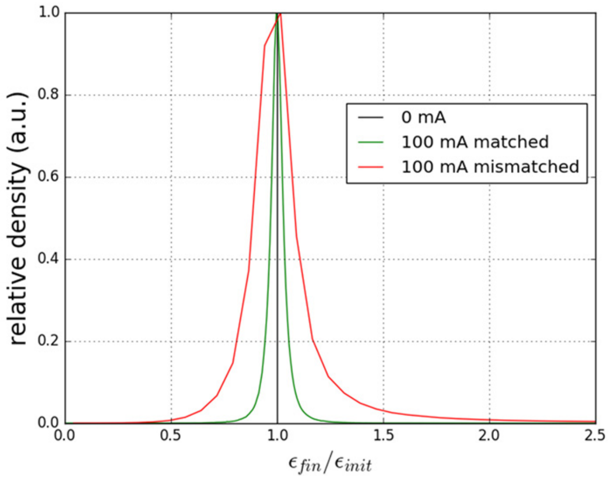
→ Qualitatively consistent with the well known halo formation mechanism

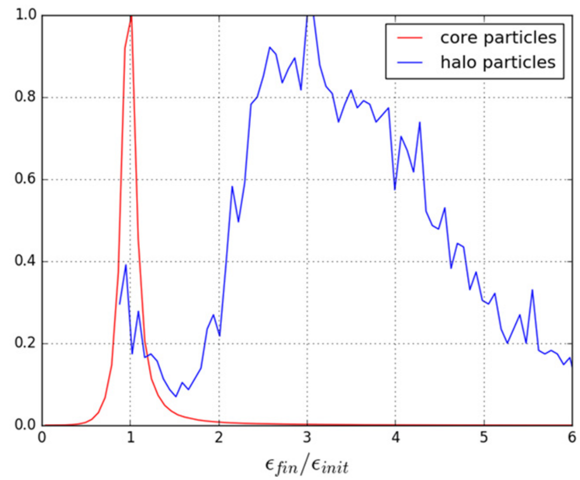
Histogram of ε_{fin} / ε_{init}:
growth ratio of the individual particle's emittances (or action)

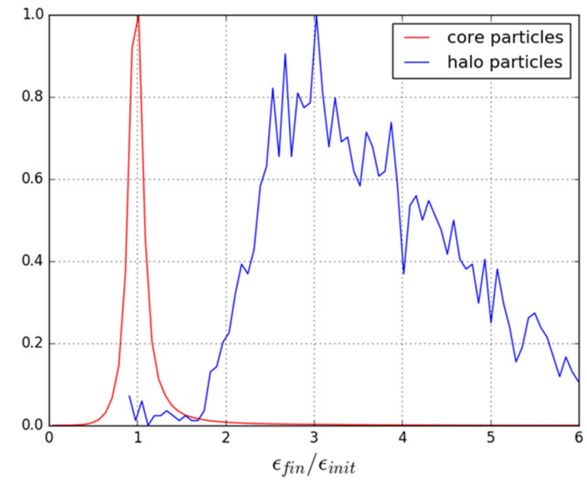
- 0 mA (no space charge): the ratio is 1 for every particle.
- 100 mA, matched beam: the particles exchange transverse energy, widening the histogram.
- 100 mA, mismatched beam: space-charge excites some particles, undergoing non-linear transport.

In the mismatched case:
-Core particles have ratio ~1
-Halo particles gained energy
Due to numerical errors a small error is made on the contour, including some core particles.

By dilating
the core-halo limit of 4%:
-Core particles have ratio ~1
-Halo particles are exactly the outer particles having gained transverse energy through the instability.







CONCLUSION: (THPMR014, IPAC'16)
The PROPOSED CORE-HALO LIMIT IS CONSISTENT with the well-established HALO FORMATION DYNAMICS

Beam Characterization by Core and Halo

Core-Halo limit as maximum of ρ'':

- **Corresponds well to visual inspection of density profile**
- **Varies as expected for every type of density profile**
- **Good indicator of the internal dynamics of the beam**
- **Totally consistent with halo formation dynamics**

Summary

Characterization of high intensity beam:

1) By the actual number of particles

precise and detailed description of halo formation, microlosses

but needs massive simulations and more realistic input distribution

2) By projections onto a few axes

truththful description of 2D distribution with reasonable number of parameters

but how to go to 4D, 6D ?

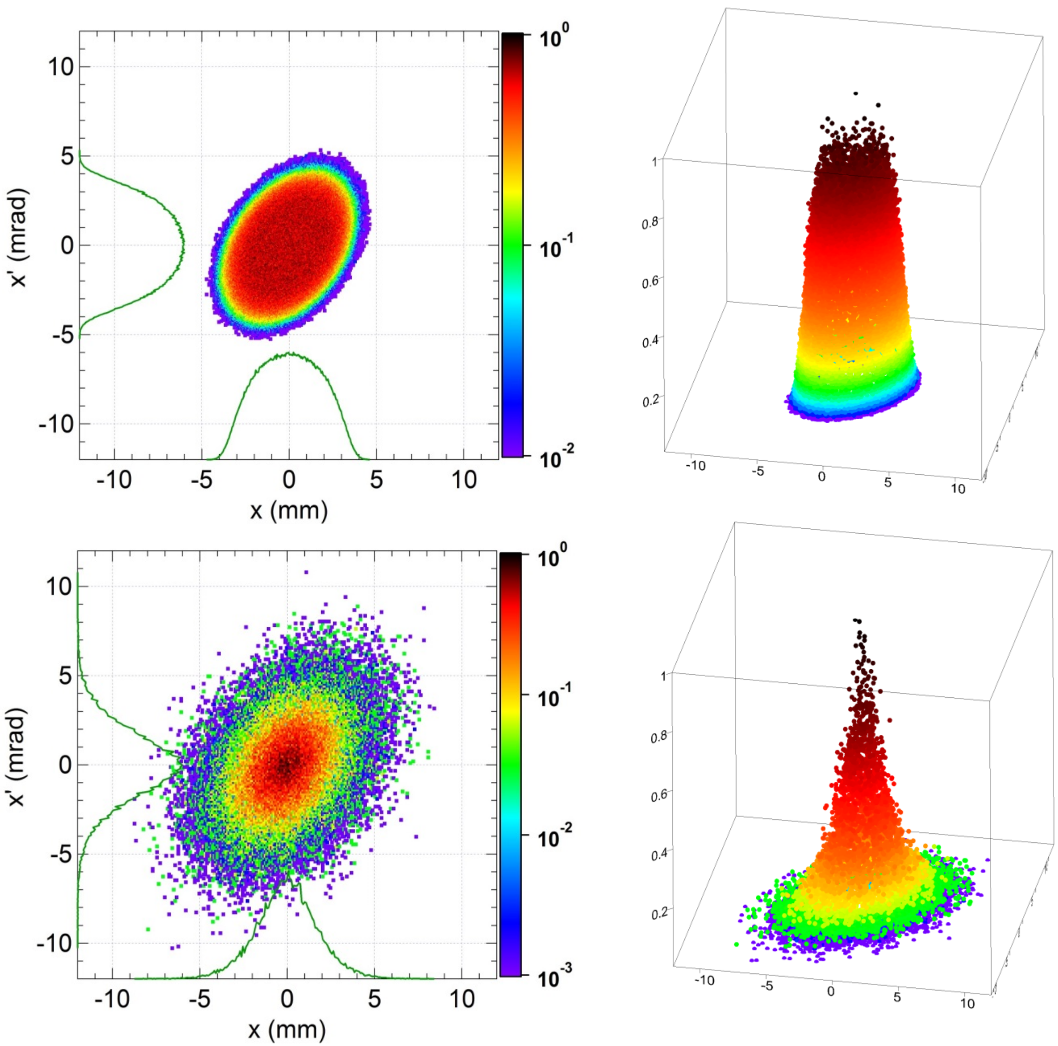
3) By global properties of the Core and the Halo separately

more insight in physical properties of the beam

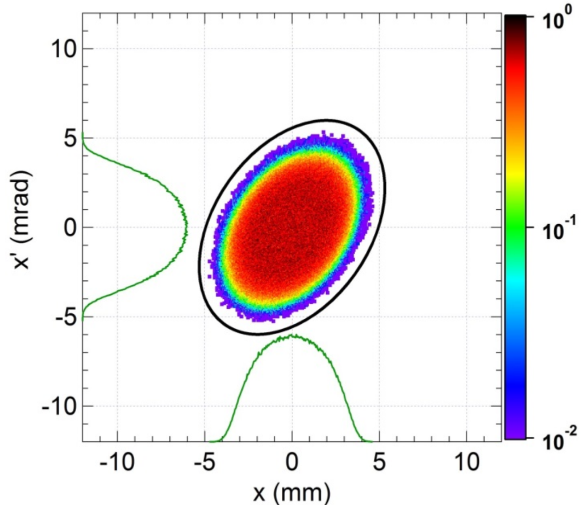
but compared to above methods, fine details are lost

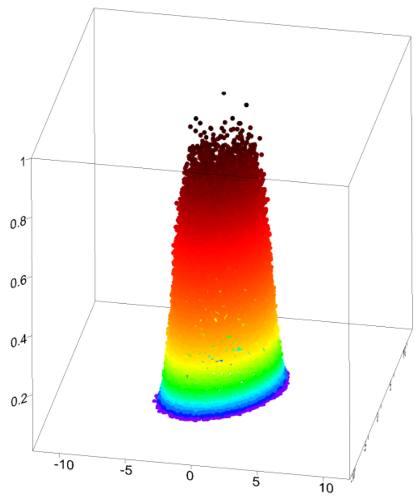
Extra

Two different beams...

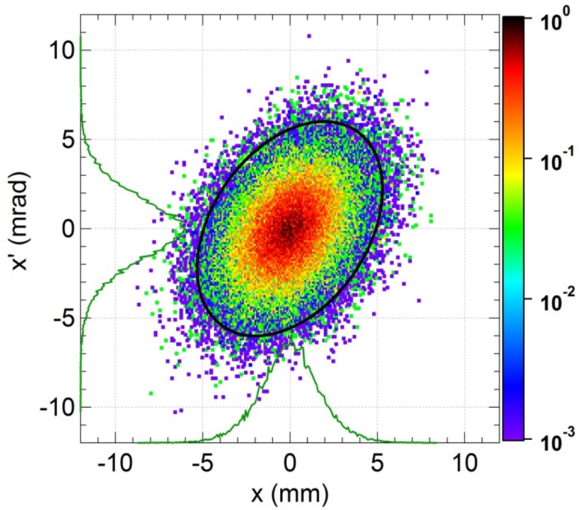


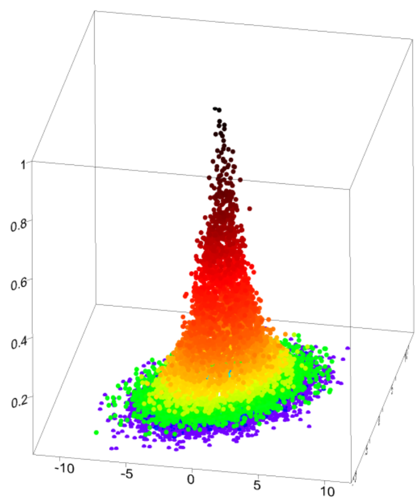
Same Emittance but different Halos



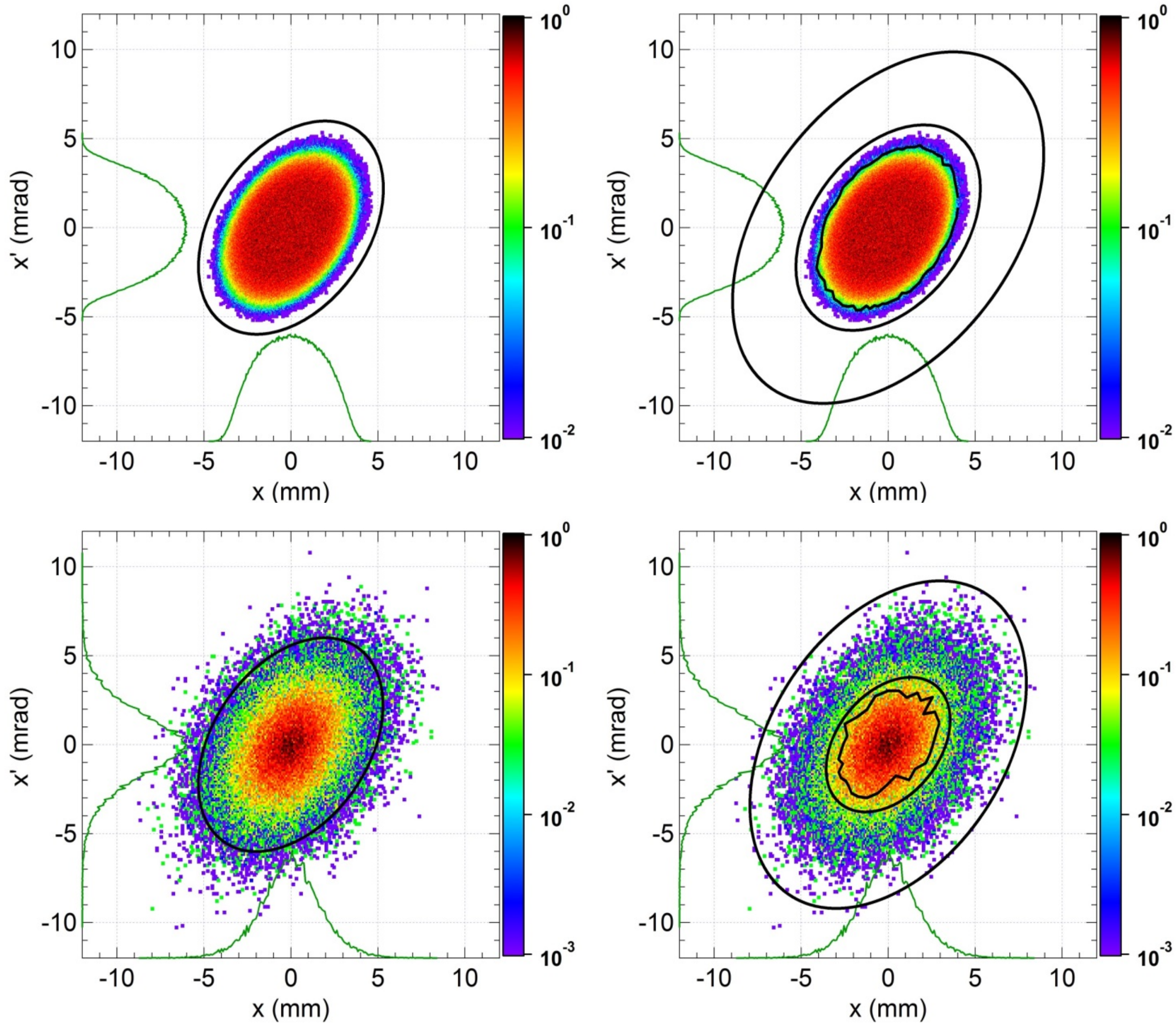


ε = 3.30, α = -0.40, β = 0.95





ε = 3.30, α = -0.40, β = 0.96



	ε	α	β
Global	**3.30**	**-0.40**	**0.96**
Core	**3.15**	**-0.41**	**1.00**
Halo	**8.95**	**-0.46**	**1.00**

PHS=28.9% PHP=3.2%

	ε	α	β
Global	**3.31**	**-0.40**	**0.95**
Core	**1.40**	**-0.39**	**1.01**
Halo	**7.56**	**-0.40**	**0.93**

PHS=90.1% PHP=30.9%