Longitudinal Dynamics of Superconducting Linacs

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Motivation

$$w' \equiv \frac{dw}{ds} = B(cos\varphi - cos\varphi_s)$$

$$\varphi' \equiv \frac{d\varphi}{ds} = -Aw$$

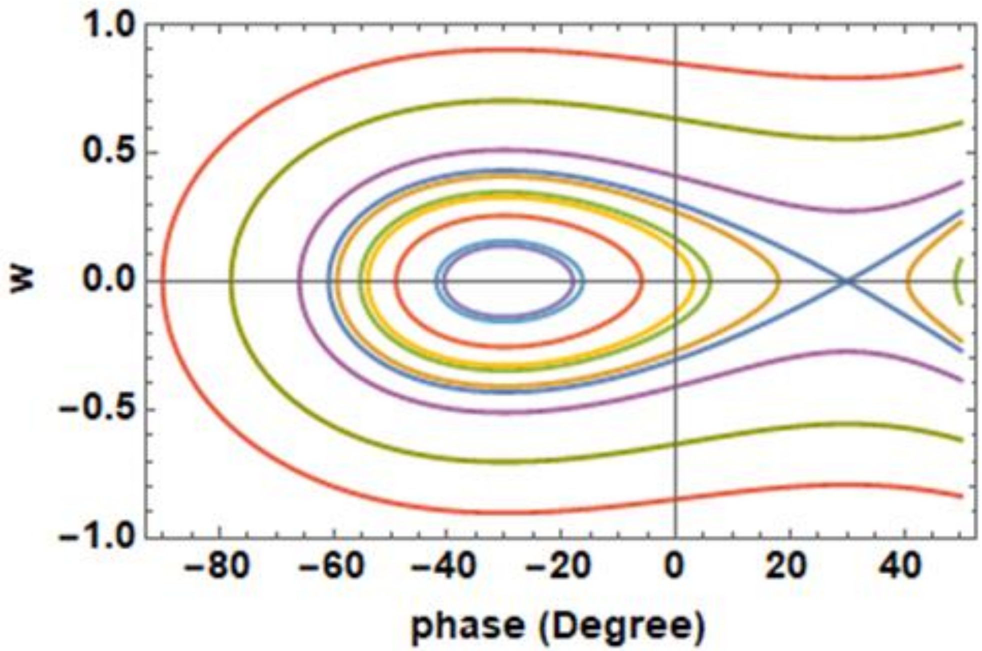
$$w = \frac{E}{m_0c^2} = \gamma,$$

$$A \equiv \frac{2\pi}{\beta_s^3\gamma_s^3},$$

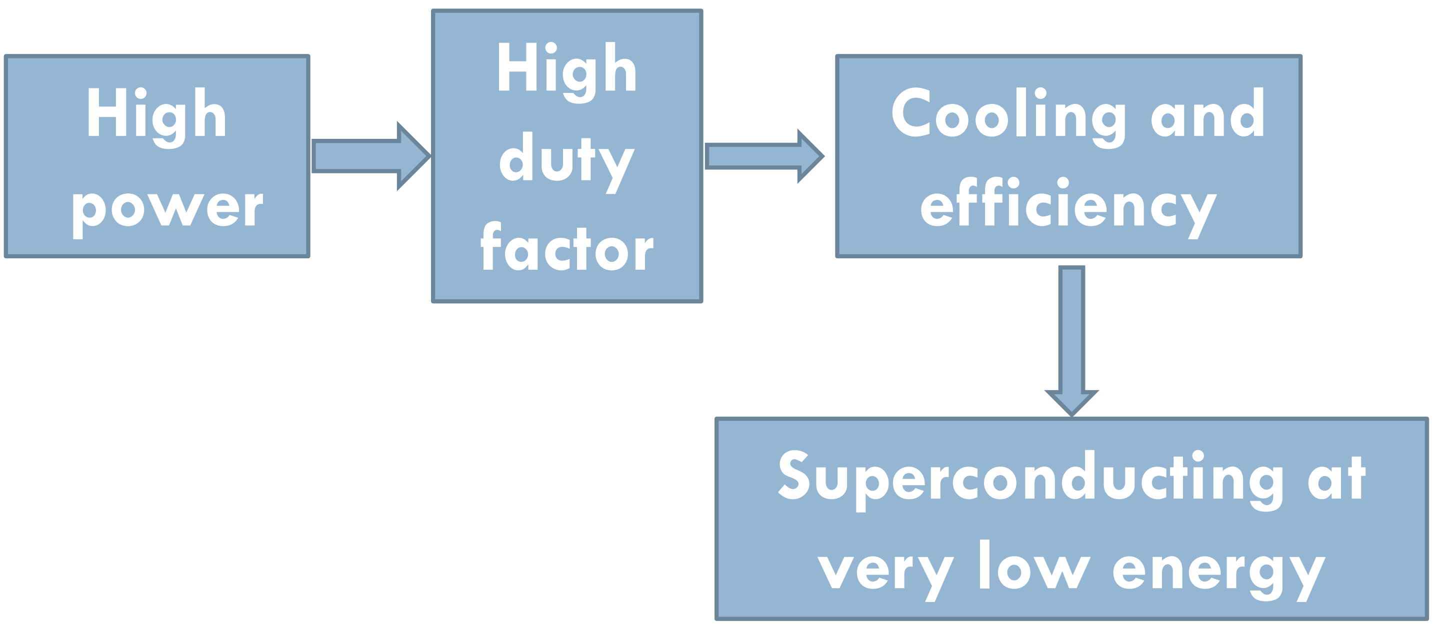
$$B \equiv \frac{qE_0T}{m_0c^2},$$

Motivation

- Constant focusing channel;
- Clear boundary of stable and unstable area;
- No limitation on the amplitude of the acc. gradient;
- Bucket area is determined by syn. phase and acc. gradient;



Motivation

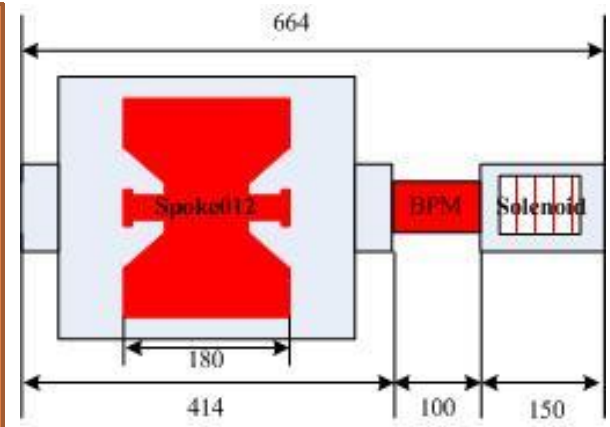

High power
High duty factor
Cooling and efficiency
Superconducting at very low energy

Motivation

- High voltage->strong defocusing;
- Long period length;
- Low cavity filling factor

$$\eta = L_c/L$$

- Large phase acceptance requirement、high acceleration efficiency;
  - Large phase advance;

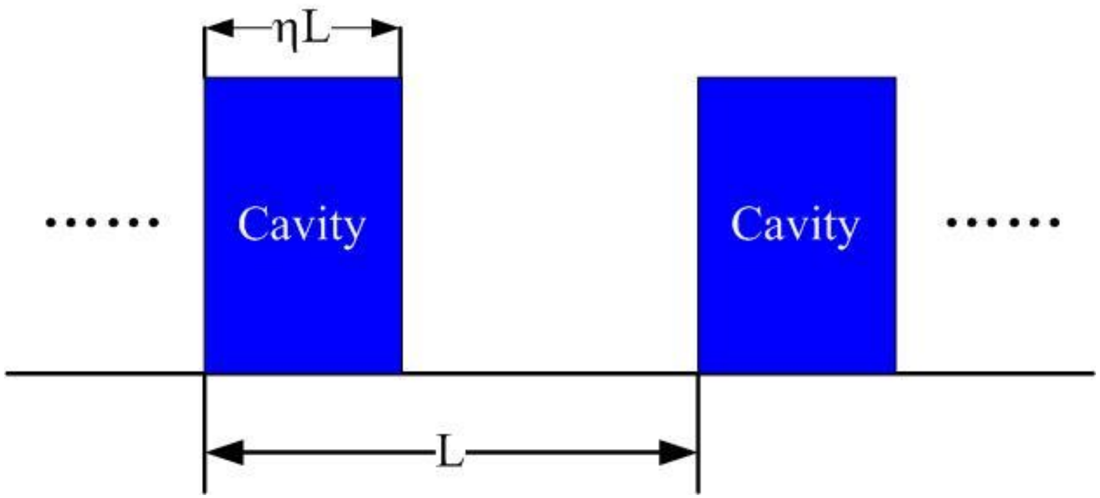


$L_c \approx \beta_g \lambda \approx 110\ mm$

$\eta \approx 110/664 \approx 1/6$

Smooth approximation is still valid and for low current is there limitation on phase advance per period?

Model



$$w' \equiv \frac{dw}{ds} = B(cos\varphi - cos\varphi_s)$$

$$\varphi' \equiv \frac{d\varphi}{ds} = -Aw$$

$$\mathrm{B} = \begin{cases} \frac{qE_0T}{mc^2}, 0 < s < \eta L \\ 0, \quad \eta L < s < L \end{cases}$$

Linear dynamics

$$cos\varphi = \cos(\varphi_s + x) \approx cos\varphi_s - sin\varphi_s x$$

$$\boldsymbol{x'' + k^2 x = 0}$$

$$\boldsymbol{k^2 = \begin{cases} -ABsin\varphi_s, & 0 < s < \eta L \\ 0, & \eta L < s < L \end{cases}}$$

- ✓ The system is equivalent to a periodic solenoid channel;
- ✓ The linear dynamic properties can be deduced from transform matrix;

Linear dynamics

- The transform matrix:

$$T = T_d T_c$$

- The phase advance per period is:

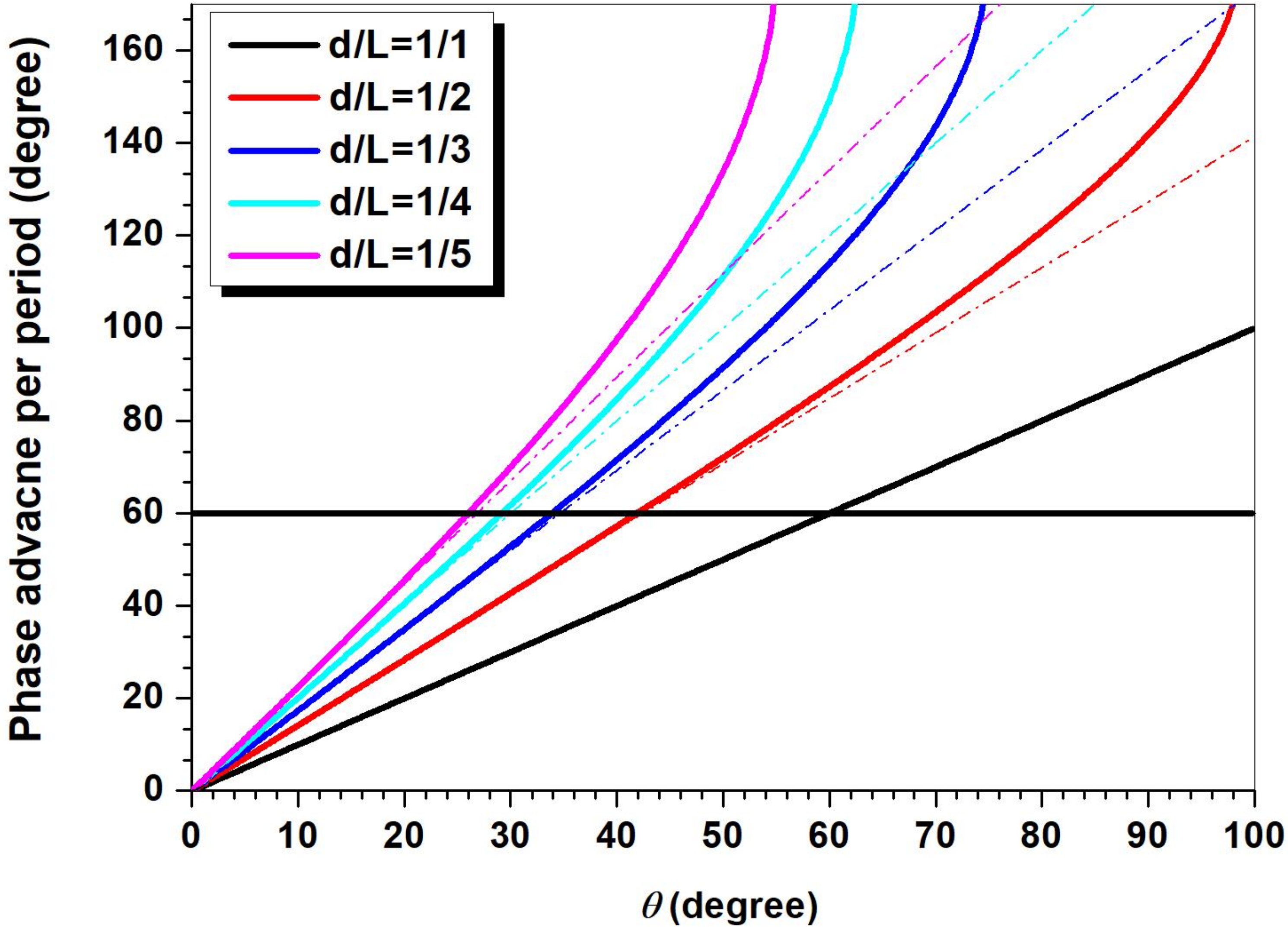
$$\cos\sigma = \cos\theta - \frac{1}{2}\frac{1-\eta}{\eta}\theta\sin\theta$$

$$\theta = \sqrt{k}L_c$$

- If both θ<<1 and σ <<1, we can get:

$$\sigma = \theta/\sqrt{\eta}$$

Linear dynamics

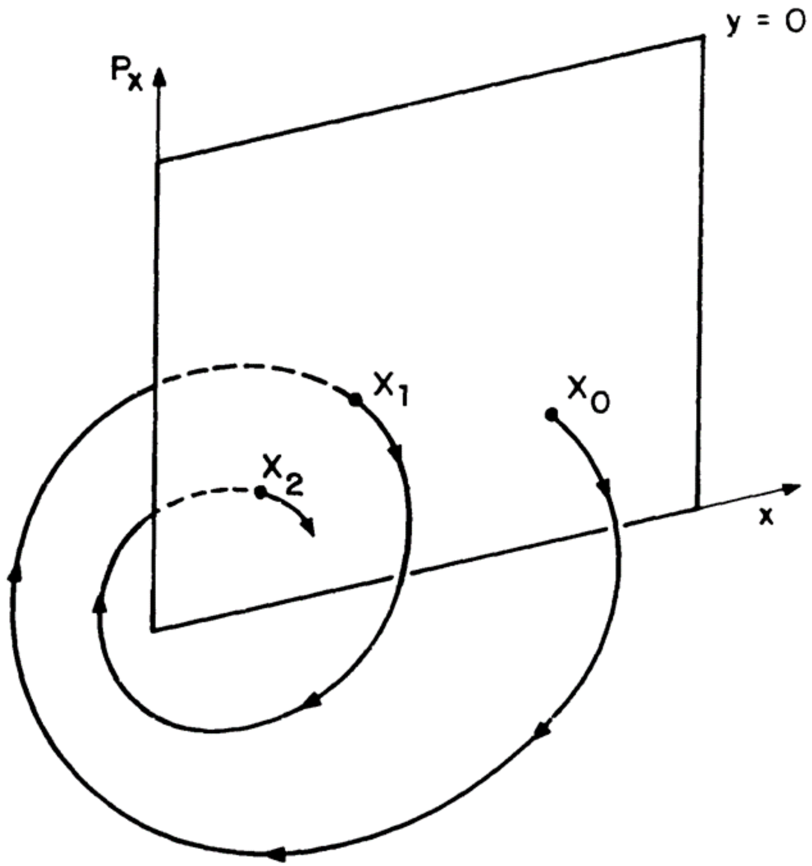


Nonlinear dynamics

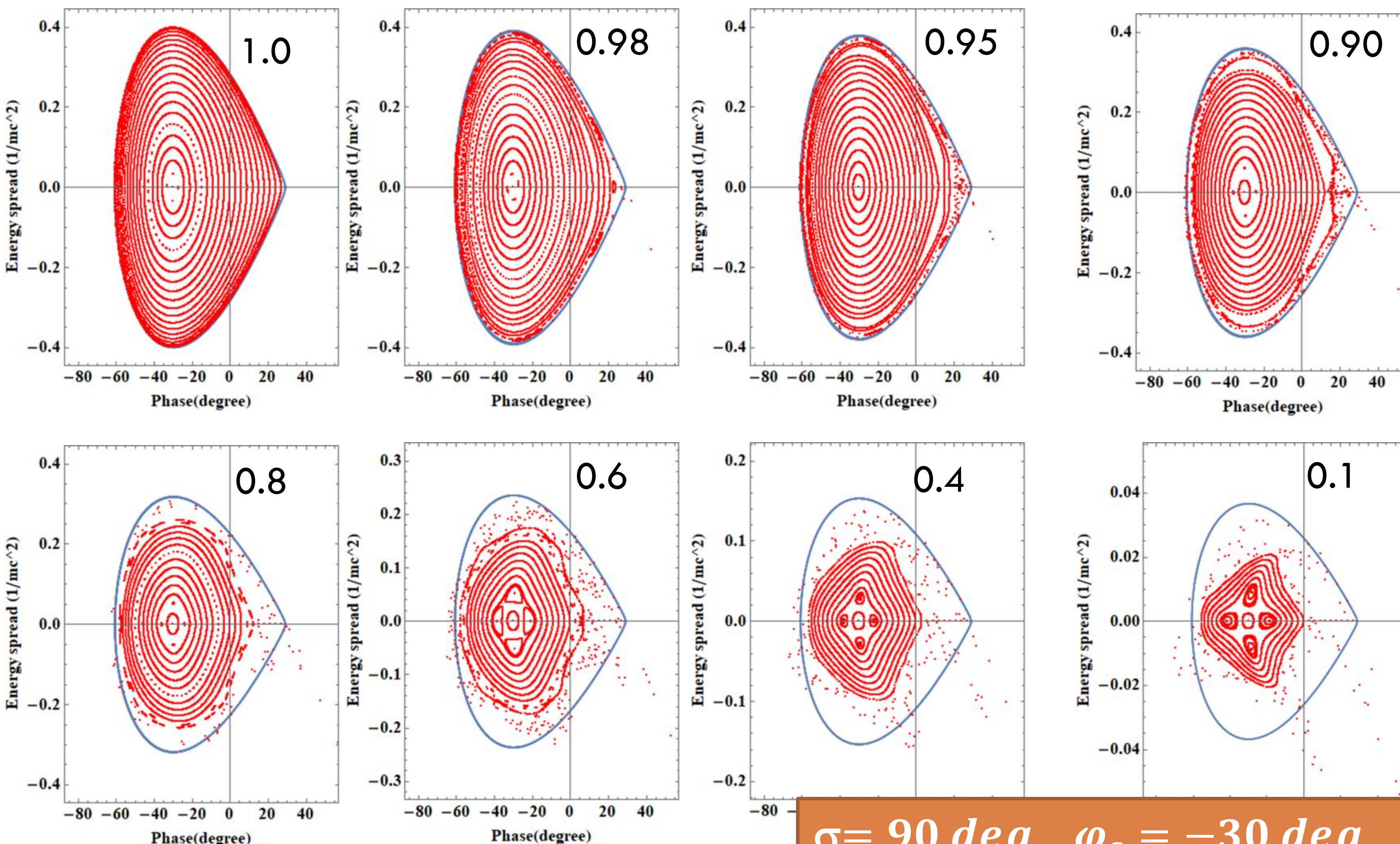
- Poincare section

$$(\varphi_{n+1}, w_{n+1}) = T(\varphi_n, \varphi_n)$$

- ✓ Integrate with 4th order Runger-kutta;
- ✓ Particle loss when $|\varphi|$ >10;
- ✓ 1000 iterations per particle;
- ✓ 500 particles uniformly located along phase axis between S.F.P and U.F.P;

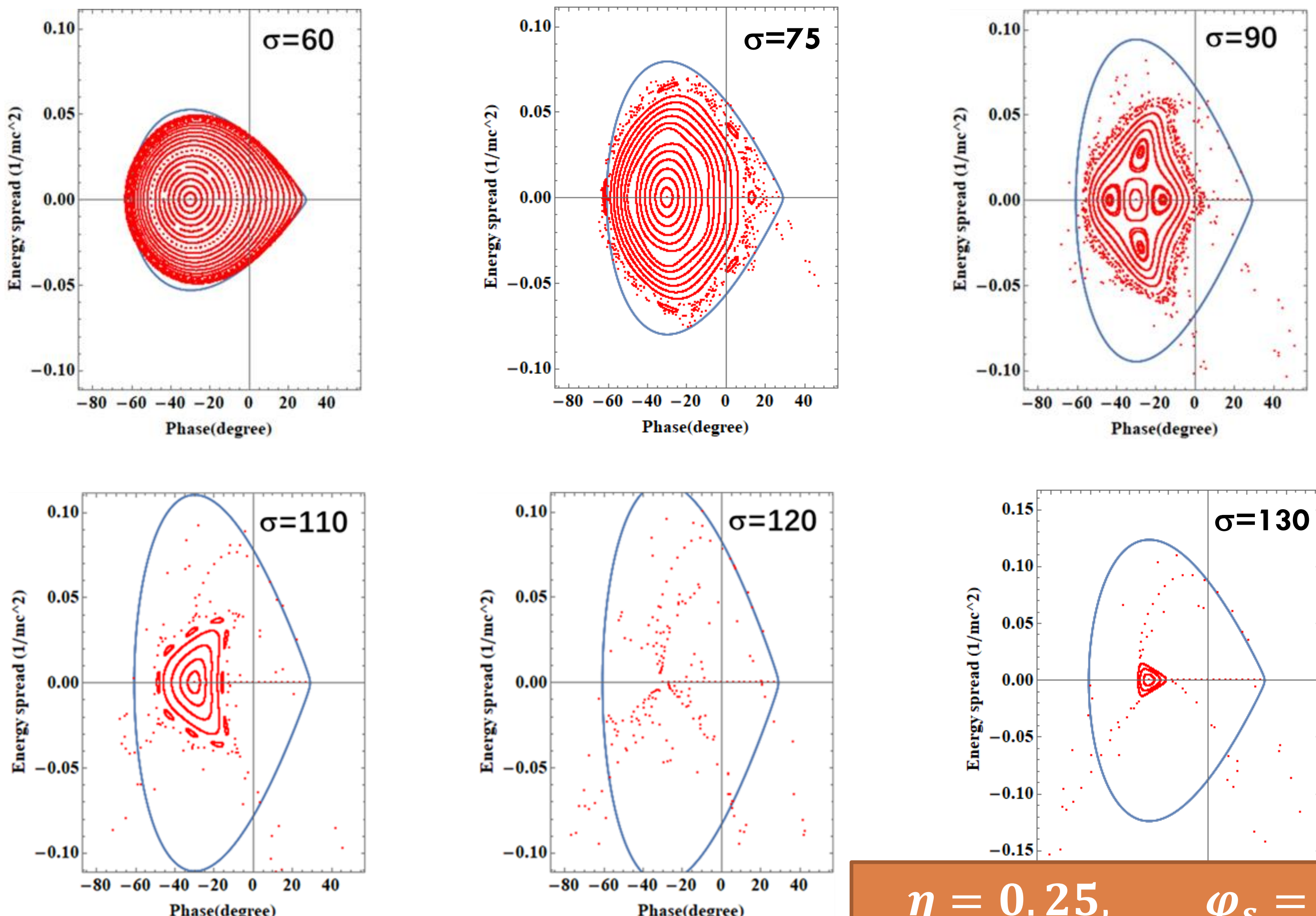


Dynamics properties as function of filling factor



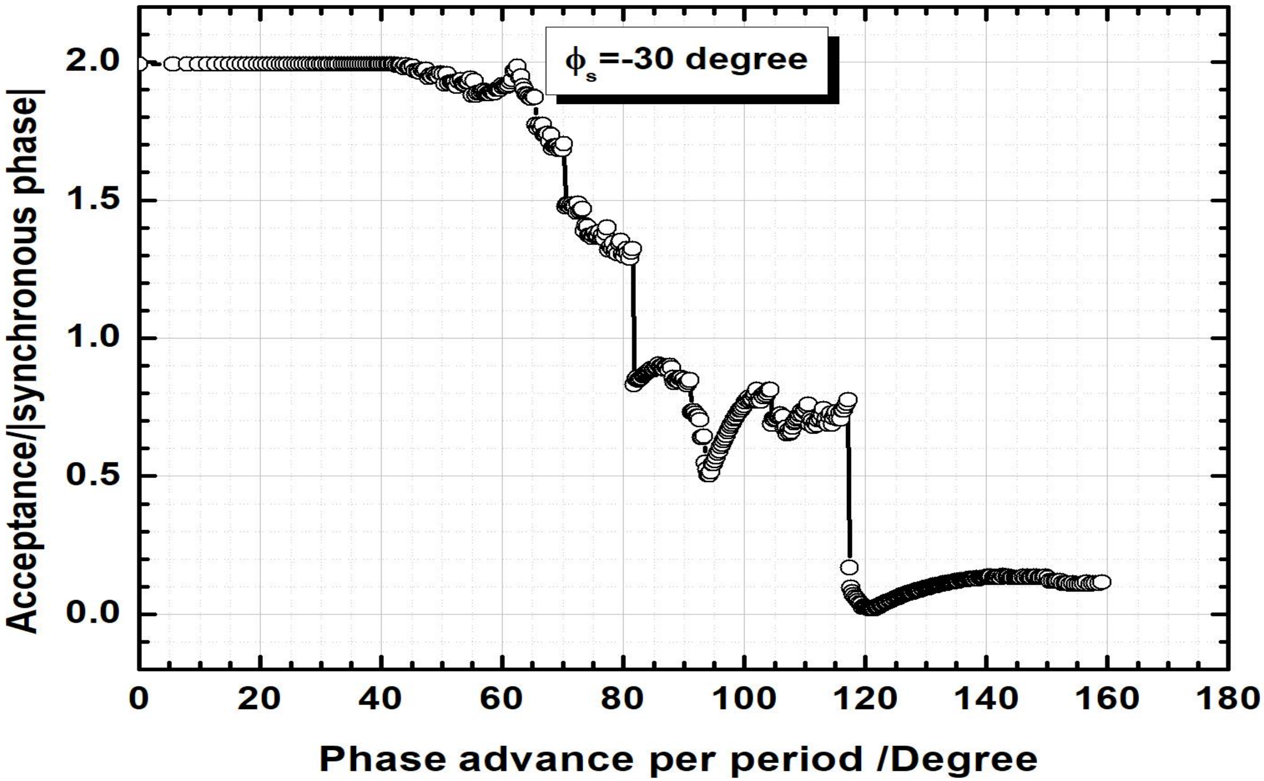
$\sigma = 90\ deg.,\ \varphi_s = -30\ deg.$

Dynamics properties as function of phase advance per period



$\eta = 0.25, \qquad \varphi_s = -30\ deg.$

Phase acceptance



Higher order fixed point of the map T^n;
M. Henon, Quarterly of applied mathematics, vol. XXVII, 1969

Conclusions

- When phase advance is greater than 60 degree, the smooth approximation is no longer valid;
- The longitudinal acceptance is decreased as the phase advance per period increase and it becomes zero when phase advance per period is 120 degree;
- The compact lattice structure is preferred;

Thanks for your attention!

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