HARMONIC UNDULATOR RADIATION WITH DUAL NON PERIODIC MAGNETIC COMPONENTS

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Outline

- Undulator field
- Electron trajectories
- Expression for Undulator radiation
- Analysis of radiation at Third Harmonic

- Undulator radiation at third harmonics coming out of harmonic undulator in the presence dual non periodic constant magnetic field has been analyzed.
- Electron trajectories along the 'x' and 'y' direction has been determined analytical and numerical methods.
- Generalized Bessel function is used to determine the intensity of radiation and Simpson's numerical method of integration is used to find the effect of constant magnetic fields.

Undulator field

$$B = \left[B_0\kappa_x, a_0B_0(sink_uz + \Delta sink_hz) + B_0\kappa_y, 0\right]$$

$k_u = \frac{2\pi}{\lambda_u}$ *and* $k_h = \frac{2\pi}{\lambda_h}$ *are undulator wave number with*

$\lambda_h = h\lambda_u(Undulator\ wavelength),$

B_0 is peak magnetic field

$\Delta = \frac{a_1}{a_0}$, a_0 *and* a_1 controls the amplitude of main undulator field and additional harmonic field

$\kappa_y, \kappa_x -$ magnitudes of constant non periodic magnetic field

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VELOCITY OF ELECTRON 'X' 'Y' AND 'Z'

$$\beta_x = -\frac{K}{\gamma}\left[\cos(\Omega_u)t + \Delta\frac{\cos(h\Omega_u)t}{h} - \kappa_y\Omega_u t\right]$$

$$\beta_y = -\frac{K}{\gamma}\kappa_x\Omega_u t$$

$$\beta_z = \beta^* - \frac{K^2}{2\gamma^2}\left[\left\{\frac{1}{2}\cos(2\Omega_u)t + \frac{1}{2}\left(\frac{\Delta}{h}\right)^2\cos(2h\Omega_p)t + \left(\frac{\Delta}{h}\right)\cos(1 \pm h)\Omega_u t - 2\kappa_y\Omega_u t\cos(\Omega_u t) - 2\kappa_y\Omega_u t\cos(h\Omega_u t)\right\} + (\kappa_x^2 + \kappa_y^2)\Omega_u^2 t^2\right]$$

$K = \dfrac{a_0 e B_0}{\Omega_u m_0 c}$ is the undulator parameter

$$\beta^* = 1 - \frac{1}{2\gamma^2}\left[1 + \frac{K^2 + K_1^2}{2}\right] \qquad \Omega_u = k_u c$$

TRAJECTORY ALONG Z

$$\frac{z}{c} = \beta^{*}t - \frac{K^{2}}{8\gamma^{2}\Omega_{u}}\sin(2\Omega_{u}t) - \frac{K_{1}^{2}}{8\gamma^{2}h\Omega_{u}}\sin(2h\Omega_{u}t) - \frac{KK_{1}}{2\gamma^{2}(1\pm h)\Omega_{u}}\sin(1\pm h)\,\Omega_{u}t + \frac{K^{2}\kappa_{y}t\sin(\Omega_{u}t)}{\gamma^{2}} + \frac{K^{2}\kappa_{y}\cos(\Omega_{u}t)}{\gamma^{2}\Omega_{u}} + \frac{KK_{1}\kappa_{y}t\sin(h\Omega_{u}t)}{h\gamma^{2}} + \frac{KK_{1}\kappa_{y}\cos(h\Omega_{u}t)}{h^{2}\gamma^{2}\Omega_{u}} - \frac{K^{2}\left(\kappa_{x}^{2}+\kappa_{y}^{2}\right)\ \Omega_{u}{}^{2}t^{3}}{6\gamma^{2}}$$

Lienard - Wiechart integral

$$\frac{d^{2}I}{d\omega d\Omega} = \frac{e^{2}\omega^{2}}{4\pi^{2}c}\left|\int_{-\infty}^{\infty}\{\hat{n}\times(\hat{n}\times\hat{\beta}\}exp\left[i\omega(t-\frac{z}{c})\right]dt\right|$$

EXPRESSION FOR UNDULATOR RADIATION

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2T^2}{4\pi^2c}\left\{\left|I_{x1} + I_{x2} + I_{x3}\right|^2 + \left|I_y\right|^2\right\}$$

$$I_{x1} = \frac{K}{2\gamma}\left[\begin{matrix} J_{m+1}(0,\xi_1) + J_{m-1}(0,\xi_1)\}J_n(0,\xi_2)J_p(\xi_3) \\ J_q(\xi_4)J_r(\xi_5)J_s(\xi_5)J_u(\xi_6)J_v(\xi_6) \end{matrix}\right]S(\vartheta,\varphi)$$

$$\begin{aligned} I_{x2} \\ = \frac{K}{2\gamma}\Big[\frac{\Delta}{h}\{J_{n+1}(0,\xi_2) \\ + J_{n-1}(0,\xi_2)\}J_m(0,\xi_1)J_p(\xi_3)\, J_q(\xi_4)J_r(\xi_5)J_s(\xi_5)J_u(\xi_6)J_v(\xi_6) \\ + \Big]S(\vartheta,\varphi) \end{aligned}$$

$$I_{x3} = \frac{2i\pi K\kappa_y N}{\gamma}S'(\vartheta,\varphi) \qquad I_y = \frac{2i\pi K\kappa_x N}{\gamma}S'(\vartheta,\varphi)$$

$$S(\vartheta,\varphi) = \left|\int_0^1 \mathrm{e}^{(\vartheta'\tau+\varphi'\tau^3)}\mathrm{d}\tau\right| \qquad S(\vartheta,\varphi) = \left|\int_0^1 \mathrm{e}^{(\vartheta'\tau+\varphi'\tau^3)}\mathrm{d}\tau\right|$$

Argument of generalised Bessel Function

$$\xi_1 = -\frac{\omega K^2}{8\gamma^2\Omega} \qquad \xi_2 = -\frac{\omega K_1^2}{8\gamma^2 h\Omega_u} \qquad \xi_{3,4} = -\frac{\omega K K_1}{2\gamma^2(1\pm h)\Omega}$$

$$\xi_5 = -\frac{\omega K^2 \kappa}{\gamma^2\Omega_u} \qquad \xi_6 = -\frac{\omega K K_1 \kappa}{h^2\gamma^2\Omega_u}$$

Detuning parameters

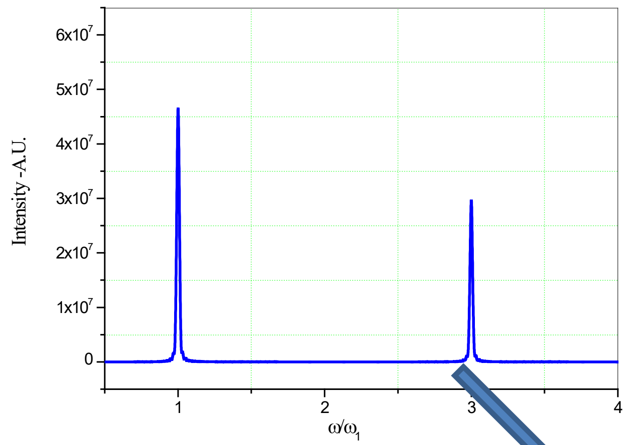
$$\vartheta = \frac{\omega}{\omega_1} - \{m + nh + p(1+h) + q(1-h) + r + s + uh + vh\}\Omega_u$$

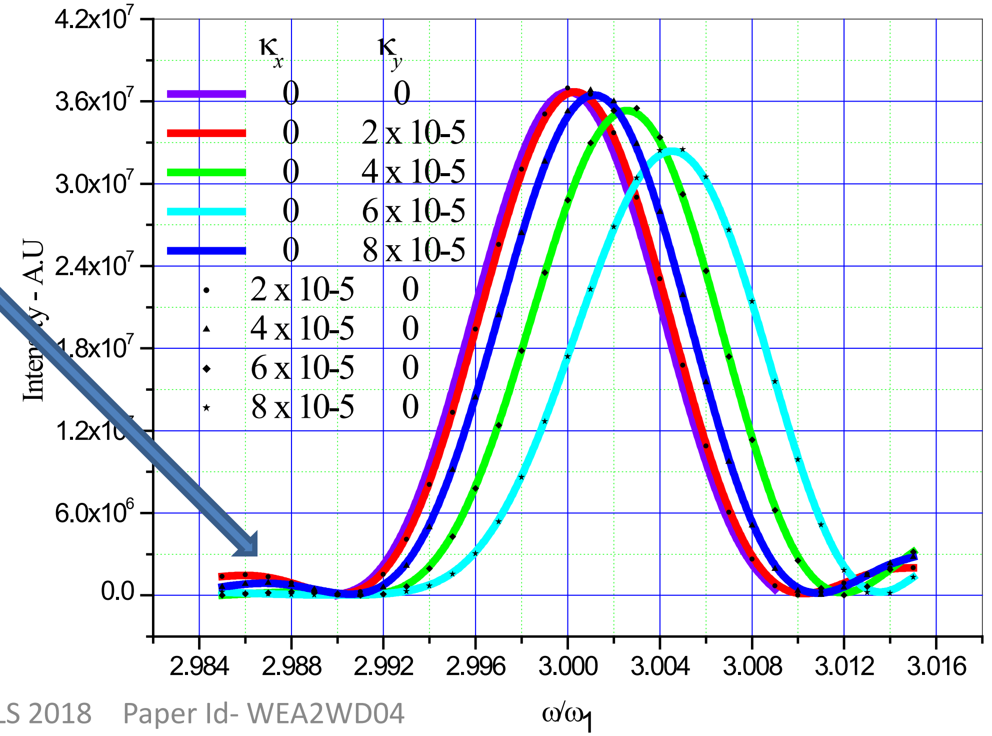
$$\varphi = \frac{\omega K^2\left(\kappa_x^2 + \kappa_y^2\right){\Omega_u}^2}{6\gamma^2}$$

Simulation parameters

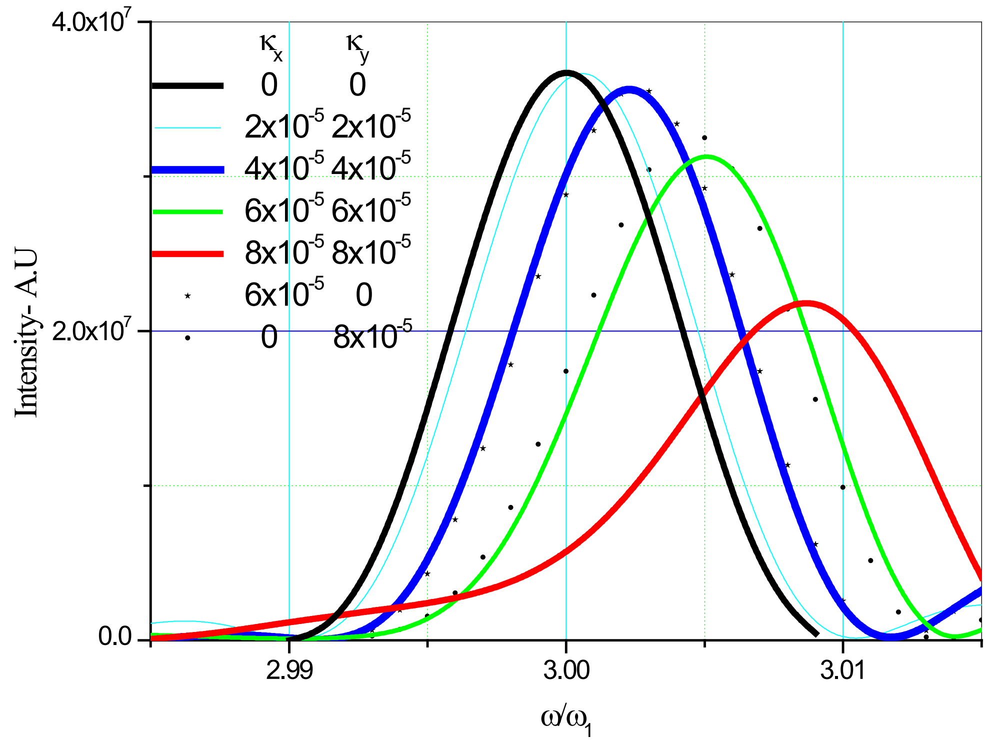
Parameter	Symbol
Undulator parameter	K=1
Electron bean relativistic parameter	20
Undulator wavelength	5 cm
Addition periodic harmonic field number	3
Harmonic field parameter	K_1=0- 0.11
Number of periods	100

Frequency Spectrum at third harmonic with parameter given in table 1 varying constant magnetic field parameter κ_y and κ_x

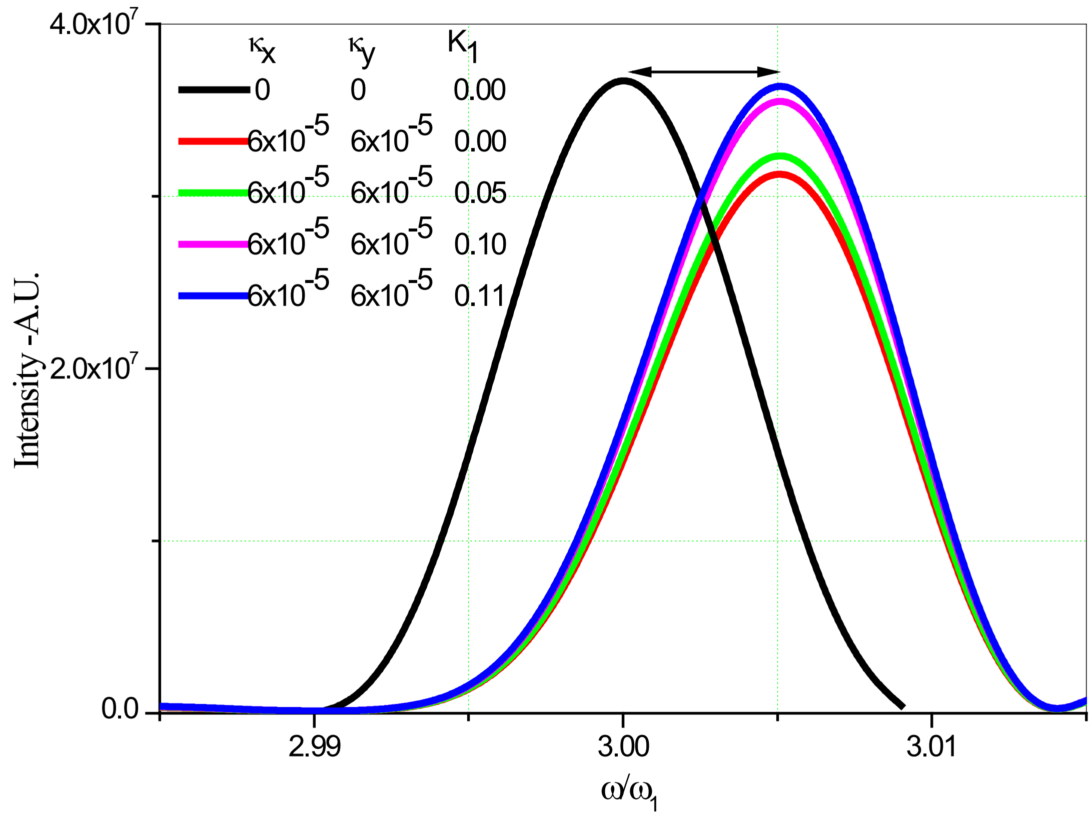




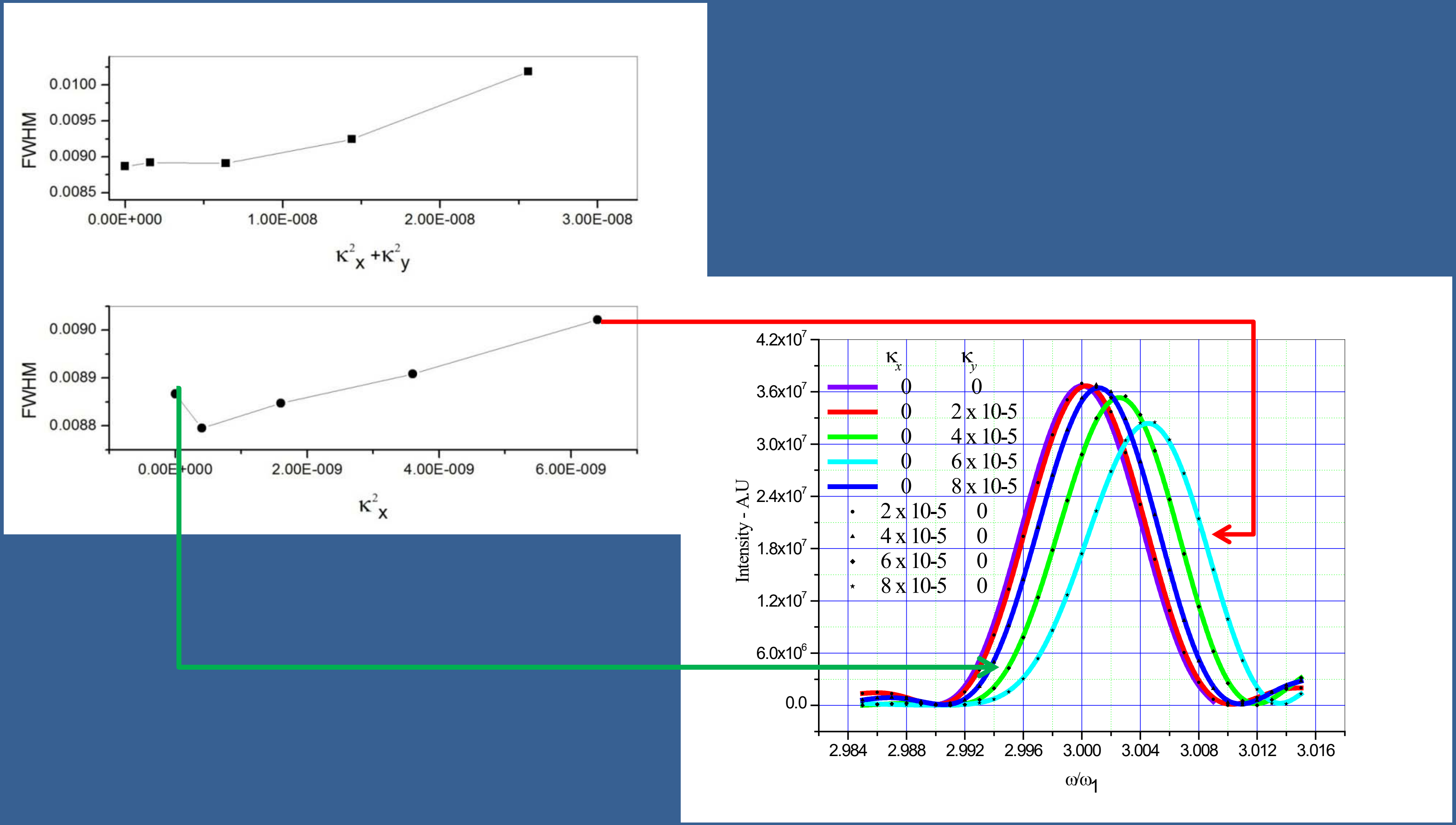
Frequency Spectrum at third harmonics with parameters same given in table 1, varying $\kappa_y = \kappa_x$



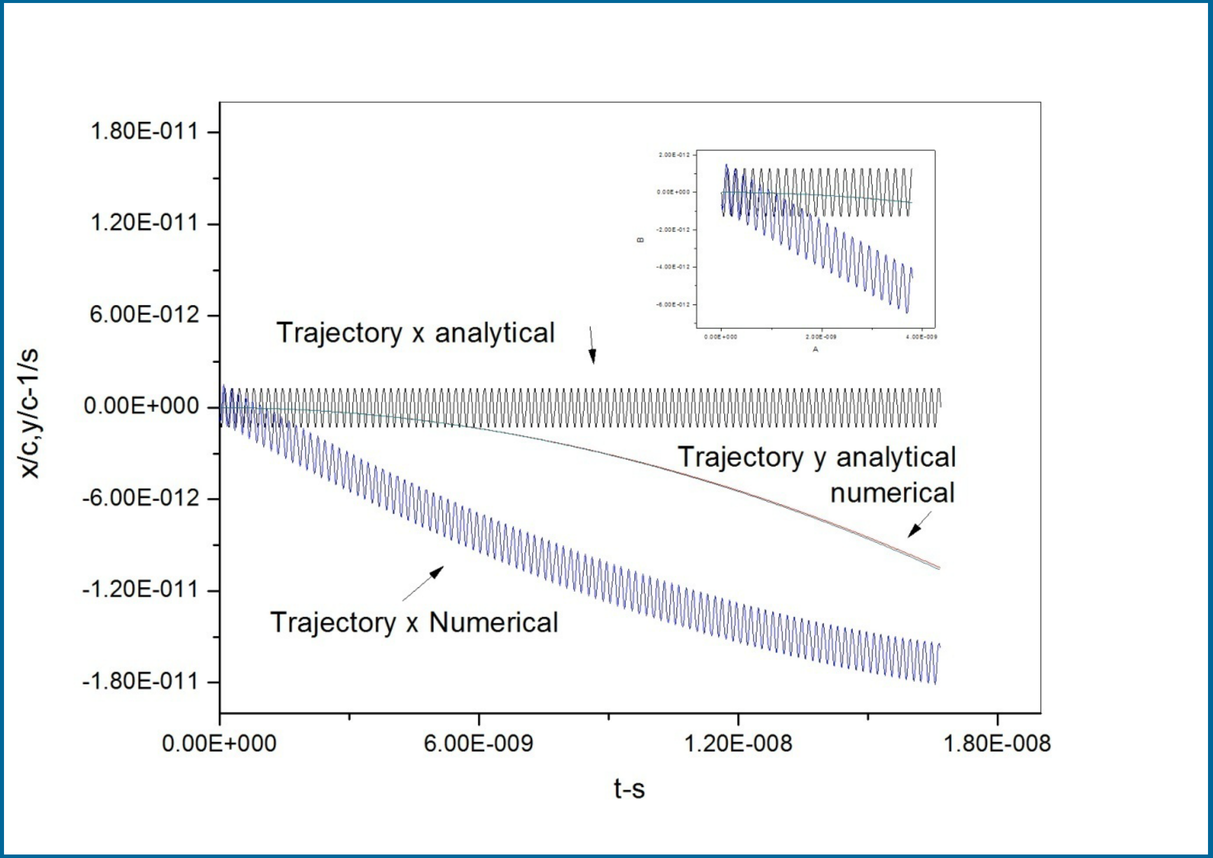
Frequency Spectrum at third harmonics with varying harmonic Field amplitude as K_1=0 and 0.11 and κ_y *and* $\kappa_x = 0.00006$



Variation of FWHM at third harmonics with varying κ_y and κ_x



Trajectory of electron along x and y directions at $\kappa_y = \kappa_x = 0.00004$ by analytical and numerical method



Conclusions

- The expression for on axis spontaneous radiation by UR with harmonic and dual constant magnetic field component has been derived.
- There is as an intensity reduction and line shape broadening due to presence of constant magnetic field, along the main field due to error in design and perpendicular to main field due to horizontal component earth's Magnetic field.
- Enhancement in intensity at third harmonics can be done by additional harmonic field where as shift in resonance remains unaltered.
- Analytical and Numerical approach has been used to find the trajectory of electron in Multiple magnetic field.