A 1d Time-Dependent Theoretical Model of X-Ray Free-Electron Laser Oscillator

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Outline

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1. Introduction
   - Background
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   - Initial start-up

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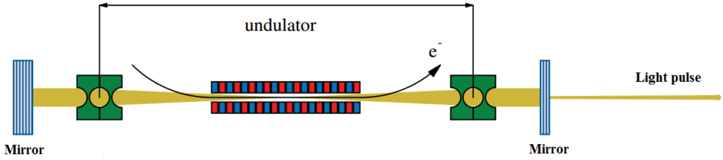

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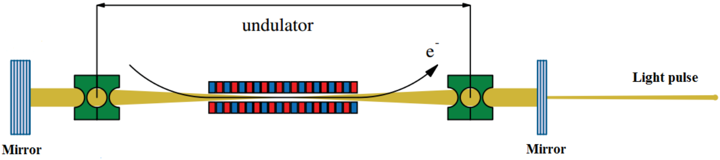

Development:

- 2008 Kwang-Je Kim made a proposal for XFELO.
- 2010 High-reflectivity high-resolution X-ray Bragg crystal diffraction.
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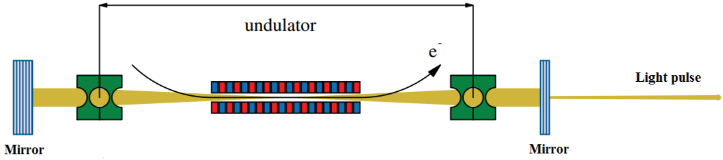

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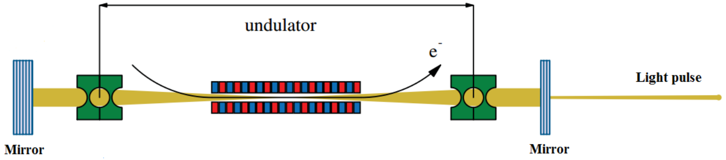

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Challenges:

- High repetition electron injector.
- Heat loading of the Bragg reflection crystal mirror.
- Crystal mirrors alignment.
- Time-consuming numerical simulation.

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The traditional numerical simulation methods (GENESIS+OPC) are time-consuming and hard for XFELO theoretical research and optimization.

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- Matlab codes for the cavity model.
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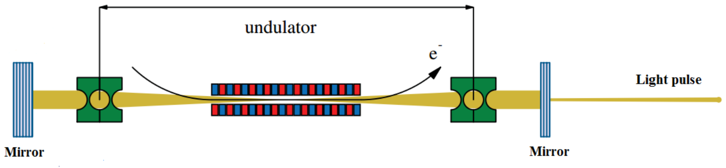
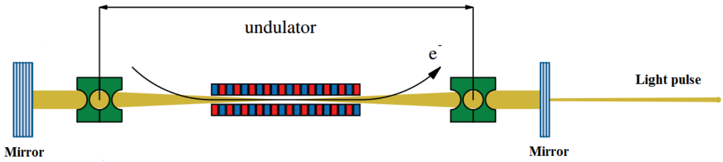
The new approach takes few minutes which makes the theoretical analysis of single-pass gain, power growth, time-dependent laser profile evolution and cavity desynchronism become more efficiently.



The radiation field inside the cavity for the (n+1)th pass at the entrance of undulator:

$$E_{n+1} = [E_n(t)g(t) + \delta E]R$$



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Three main steps for FELO new model:

- Calculating the single-pass gain theoretically.
- Initializing the start-up radiation field.
- Simulating the laser power and profile transformation.

Gain calculation

We take the advantages of electron distribution density function to get the single-pass gain. The motion of single electron in the phase space (θ, η) is described by the pendulum equation:

$$\frac{d\theta}{dz} = 2k_u\eta$$

$$\frac{d\eta}{dz} = -\frac{\epsilon}{2k_u{L_u}^2}\sin\theta$$

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Combining the equations above yields the following partial differential equation

$$\frac{\partial\rho}{\partial z'} + \eta'\frac{\partial\rho}{\partial\theta} + \sin\theta\frac{\partial\rho}{\partial\eta'} = 0$$

Gain calculation

[1] Boscolo I, et al,. IEEE Journal of Quantum Electronics, 1982, 18(11): 1957-1961.

Gain calculation

Assuming the initial condition of electron beam with a Gaussian distribution, the solution can be found by the method of characteristics[1]

$$\rho = \frac{1}{2\pi}\frac{1}{\sqrt{2\pi}\sigma_{\eta'}} \times \exp\left\{-\frac{1}{2\sigma_{\eta'}^2}\left[\frac{\eta'\mathrm{cn}(z';C) - \sin\theta\mathrm{sn}(z';C)\mathrm{dn}(z';C)}{1-\cos^2\frac{\theta}{2}\mathrm{sn}(z';C)} - \eta_0'\right]^2\right\}$$

where $C^2 = \frac{\eta'^2}{4} + \cos^2\frac{\theta}{2}$.

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Using the law of conservation of energy, the single-pass power gain is

$$G = \sqrt{m_e c^2 K[JJ]k_u}^{-1}\frac{I}{c\beta}\frac{1}{2\pi\sum^2}\frac{1}{\varepsilon_0 {E_0}^{3/2}}\langle\,\Delta\eta'\rangle$$

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Taking account of three dimensional effects, we use the equivalent formula

$$\frac{\sigma_E'}{E_0} = \sqrt{\left(\frac{\sigma_E}{E_0}\right)^2 + \left(\frac{\varepsilon\lambda_u}{4\lambda\beta}\right)^2}$$

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In this way, the complex reflectivity is simplified as

$$r(y) = \begin{cases} y - \sqrt{y^2 - 1} & \text{if } y > 1 \\ y - i\sqrt{1 - y^2} & \text{if } |y| \leqslant 1 \\ y + \sqrt{y^2 - 1} & \text{if } y < -1 \end{cases}$$

where $y = \frac{1}{|\chi_H|}\left[\frac{2(E-E_H)}{E_H} + \chi_0\right]$, E_H is the Bragg energy and χ_0 and χ_H are Fourier components of the dielectric susceptibility of the crystal.

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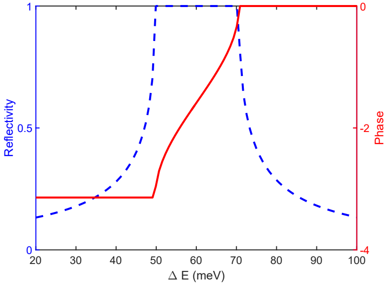

Figure: The complex reflectivity of Bragg crystal at various incident photon energy deviation from Bragg energy.

XFELO parameters

XFELO parameters

A typical XFELO is studied using parameters shown in Table bellow:

Parameter	Value	Unit
Beam energy E_0	7	GeV
Energy spread σ_E	1.4	MeV
Normalized emittance ε_n	0.2	μm-rad
Peak current I	10	A
Electron bunch length σ_t	1.0	ps
Undulator period λ_u	17.6	mm
Number of undulator N_u	3000	
Laser wavelength λ	0.1	nm
Cavity loss	5%	
Bragg mirror reflectivity R	94%	

Electron distribution and gain function

Electron distribution and gain function

The electron is trapped in the "bucket" and transform its energy to light like in the IR FELO case. However, the bucket which traps the electron becomes flatter, and the energy modulation is smaller due to the relative larger electron energy.

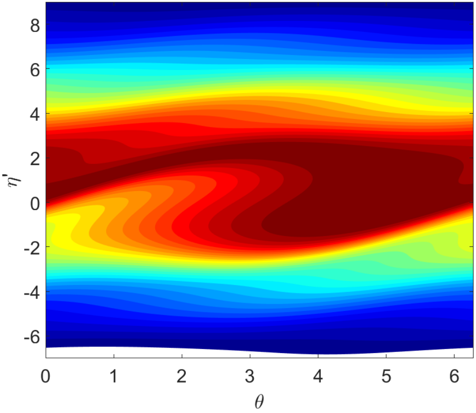

Figure: The electron density distribution function in phase space of one slice.

Light power and profile

Light power and profile

The radiation in the cavity start from shot noise which has significant fluctuations, becomes smooth as passing number increases, and finally reaches saturation and remain steady state.

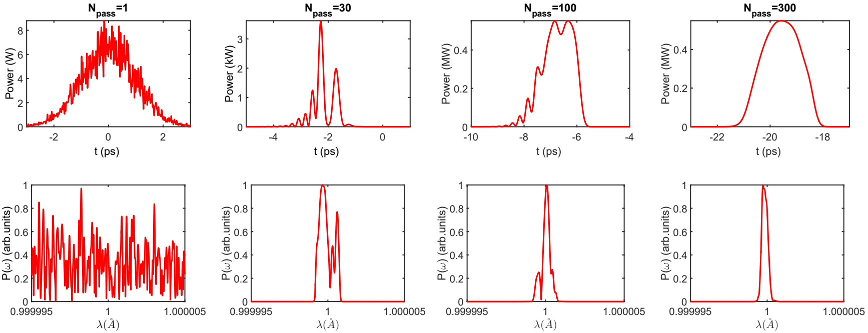

Figure: Snapshots of output radiation pulse for a typical X-ray FELO at 1.0 Å. The top and the bottom row show the longitudinal pulse temporal profile and corresponding spectrum respectively.

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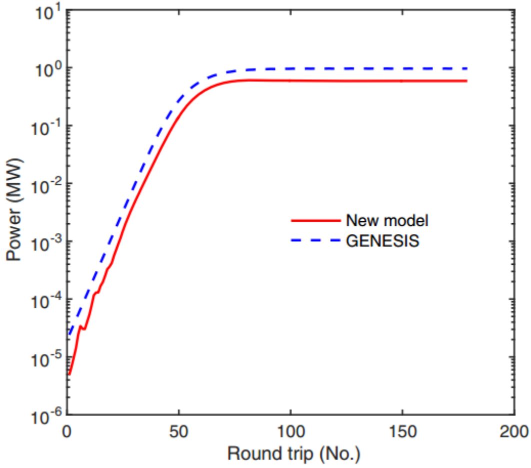

Figure: The enhancement of output laser peak power with various passes N_{pass}.

The complex reflectivity of the crystal mirrors causes an extra phase shift of optical field and leads to the pulse slides backward. In the theoretical model, the electron beam is constantly delayed a distance to overlap with the optical field.

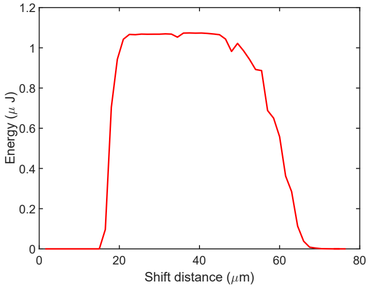

Figure: The output laser energy as a function of desynchronism.

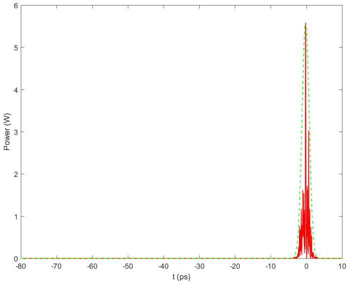

Figure: The electron beam and light power profile.

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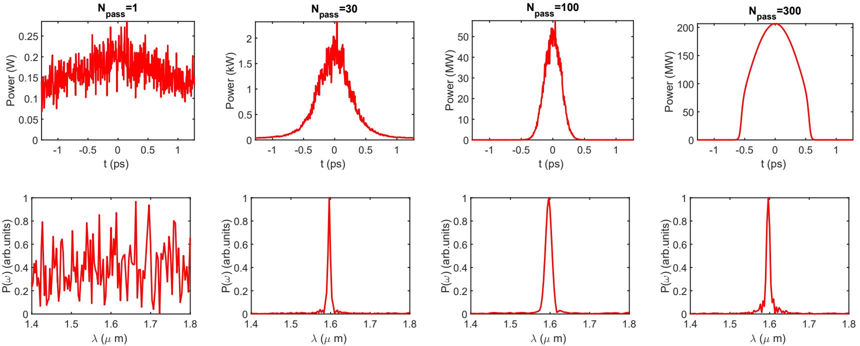

Figure: Snapshots of output radiation pulse for a typical infrared FELO at $1.6\mu m$. The top and the bottom row show the longitudinal pulse temporal profile and corresponding spectrum respectively.

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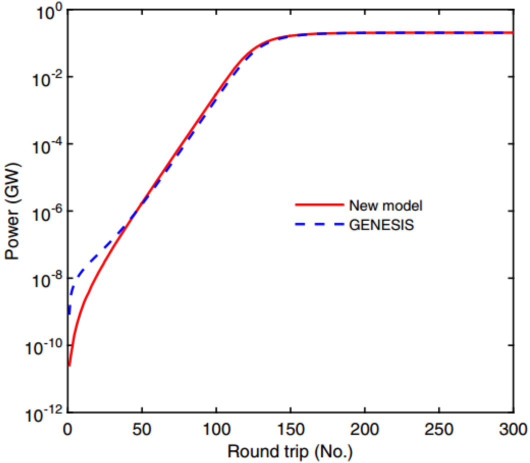

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Solution[2]:

- Solving the electron density partial differential equation to get single-pass gain.

Gain function

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- Producing the initial electric field by sampling according to its probability distribution function.
- Simulating the evolution of the light power inside the cavity using the single-pass gain function.

Light power profile evolution equation

$$E_{n+1}(t) = \left[E_n(t) g(t) + \delta E(t) \right] R_{total}$$

[2] Li K, et al., Physical Review Accelerators and Beams, 2017, 20(3): 030702.

Utilizing FEL oscillator with multi-stage undulators enables gain cascading in a single-pass, making it possible to achieve shorter single pulse lengths, higher peak power, and even higher pulse energy than normal FEL oscillator.[3]

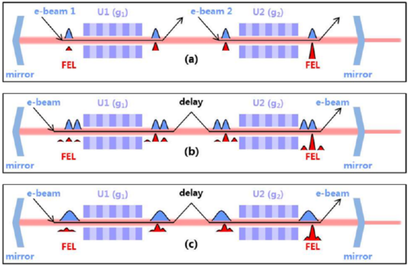

Figure: The schematic view of cascaded FELO.

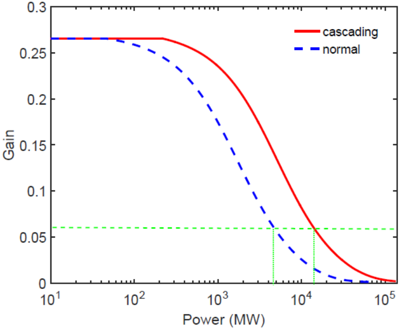

Figure: The gain of cascaded FELO calculated by new model.

[3] Li K, et al., Physical Review Accelerators and Beams, 2017, 20(11): 110703.

Applications: XFELO design at SCLF

The quasi-CW, FEL quality electron beams at Shanghai Coherent Light Facility (SCLF) is suitable to consider an X-ray free electron laser oscillator (XFELO) operation.

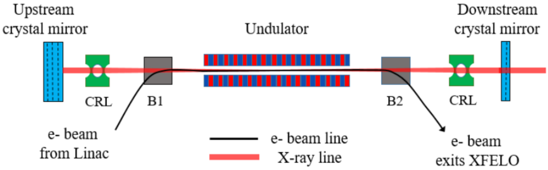

Figure: Schematic view of XFELO for SCLF.

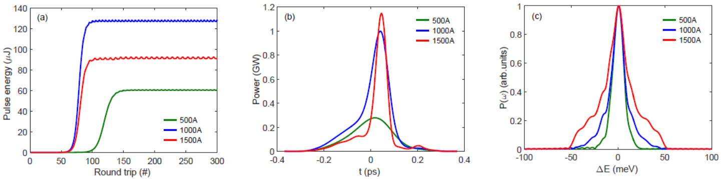

Figure: Performances of XFELO for SCLF.

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Proposal for an X-ray free electron laser oscillator with intermediate energy electron beam.
Physical review letters, 2012, 108(3): 034802.

Thank you!

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