Impedance Evaluation of the PF In-Vacuum Undulator: Theory, Simulations, and Measurements

Olga Tanaka

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Content

Introduction

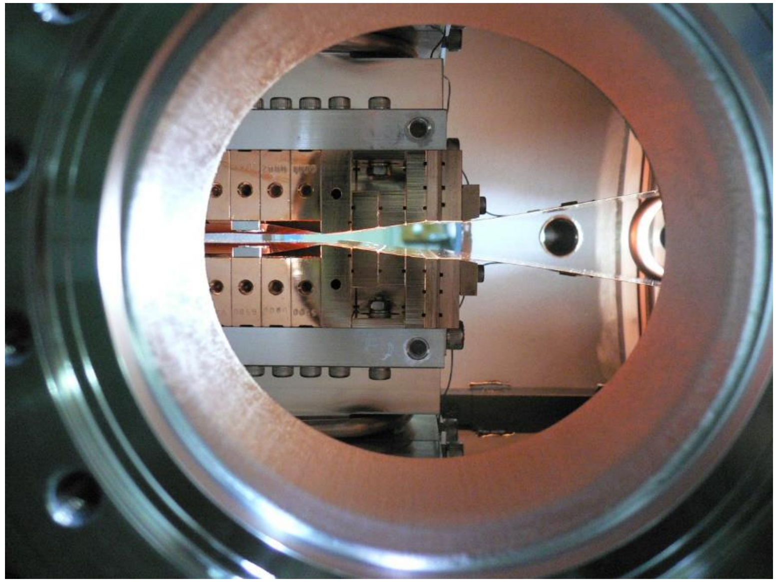
Motivation

- Four In-Vacuum Undulators (IVU) have been installed to PF recently
- They have RF shields using the standard design to reduce the impedance significantly
- These IVUs were installed to PF long after the construction of the ring itself was completed, and there was a need of the proper IVU's impedance evaluations
- The KEK future light source (KEK - LS) will include one IVU for each DQBA lattice cell (many IVUs are planned to be installed). Evaluation and improvement of their impedance is one more target of the present study
- This talk shows how we identify the major impedance contributors and evaluate their impedance using theoretical formulas, CST Studio simulations and measurements

Introduction

What is PF IVU?

- Three major impedance contributors of PF IVU:



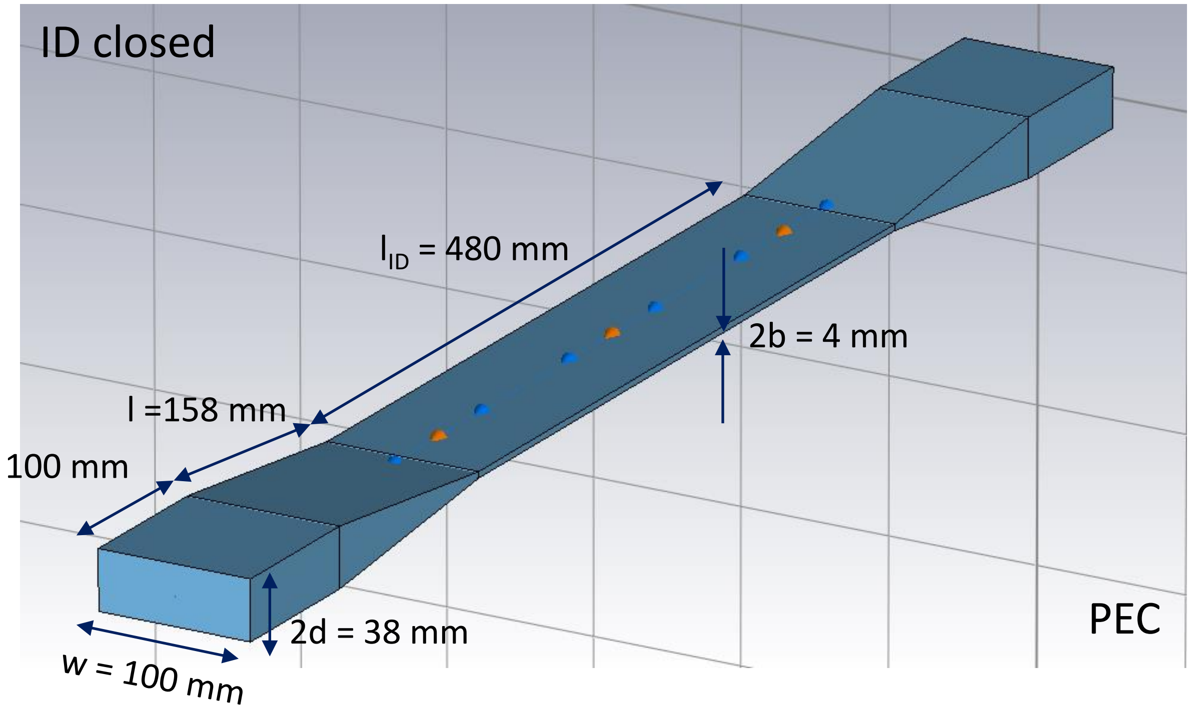
1. Taper between the flange and the undulator (200 μm thick) for the geometrical impedance

2. Copper plate (60 μm copper and 25 μm nickel coating) on top of the undulator for the resistive-wall impedance

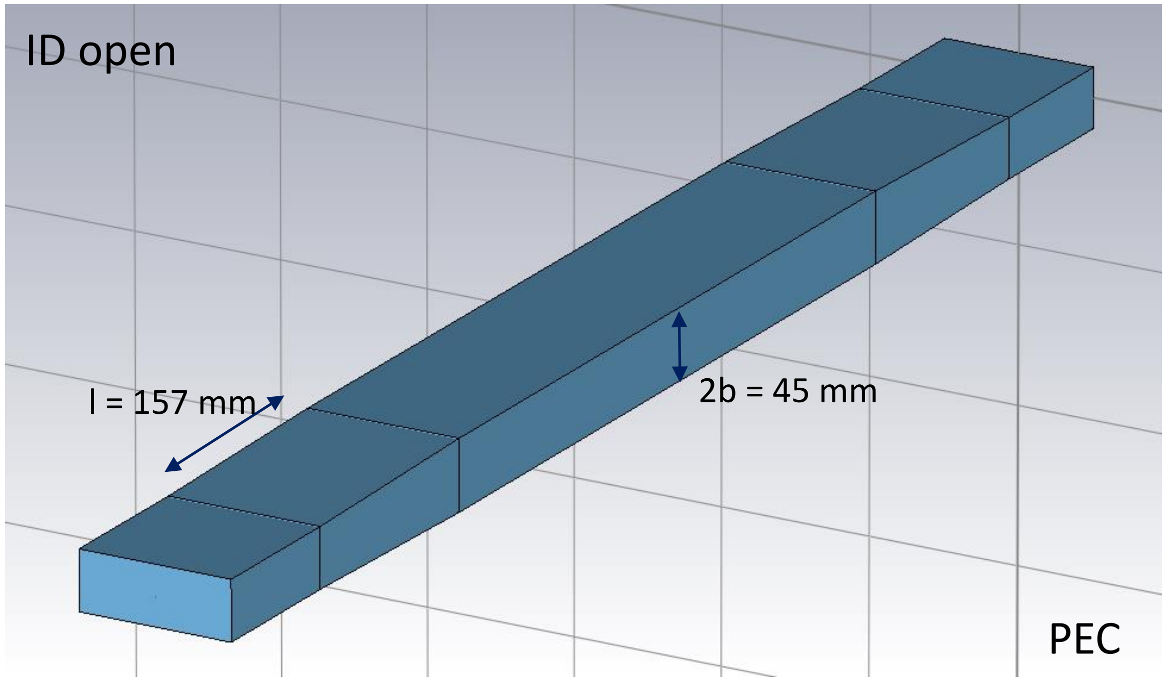
3. Step transition from the octagon to the rectangular chambers

Introduction
CST Studio Models of PF IVU

IVU Closed



IVU Open



1. Impedance Theory: Verification with Diamond IVU

1. Impedance Theory

Longitudinal Geometrical Impedance of Taper

- The taper structure is known to produce nearly pure inductive impedance even with a vessel included

$$Z_L = -i\omega L$$

$$W_{L0}(s) = Lc\frac{d}{ds}\delta(s/c)$$

- Theoretical formula for longitudinal impedance

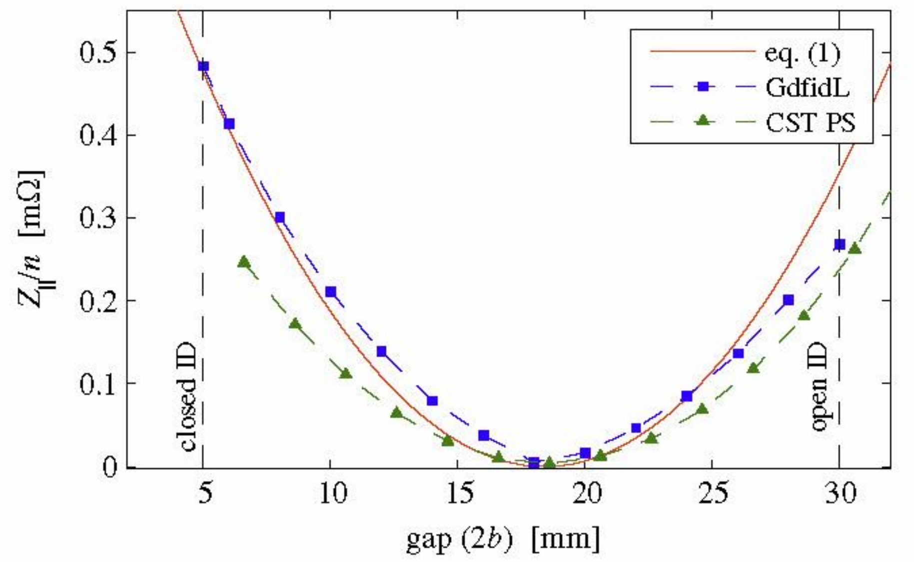
$b \ll w \ll l$

$$\frac{Z_l}{n} = -i\frac{Z_0\omega_0}{4\pi c}\int_{-\infty}^{\infty}(g')^2 F\left(\frac{g}{w}\right)dz,$$

$$F(x) = \sum_{m=0}^{\infty}\frac{1}{2m+1}\mathrm{sech}^2\left((2m+1)\frac{\pi x}{2}\right)\tanh\left((2m+1)\frac{\pi x}{2}\right).$$

G. Stupakov, Phys. Rev. ST Accel. Beams 10, 094401 (2007)

Diamond IVU



V. Smaluk, Phys. Rev. ST Accel. Beams 17, 074402 (2014)

1. Impedance Theory

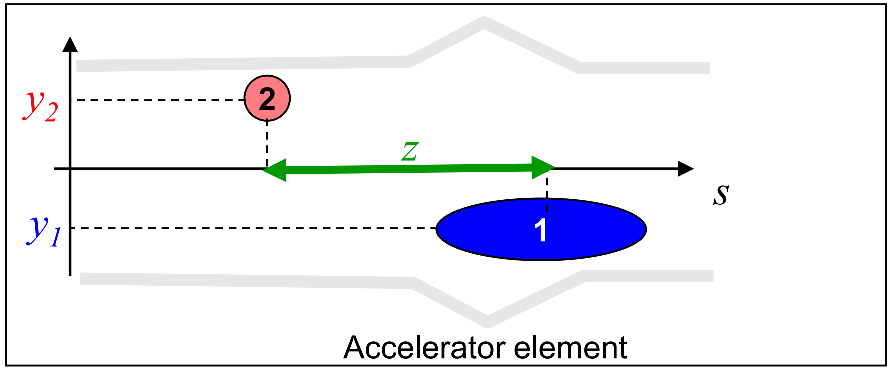
Transverse Geometrical Impedance of Taper (I)

- We need a careful treatment of the transverse impedance, since it includes both the dipolar and the quadrupolar components:

$$W_{y,tot}(y_1, y_2, z) = W_{y,dip}(z) y_1 + W_{y,quad}(z) y_2$$

Total wake Dipolar wake Quadrupolar wake

B. Salvant, Beam physics for FAIR



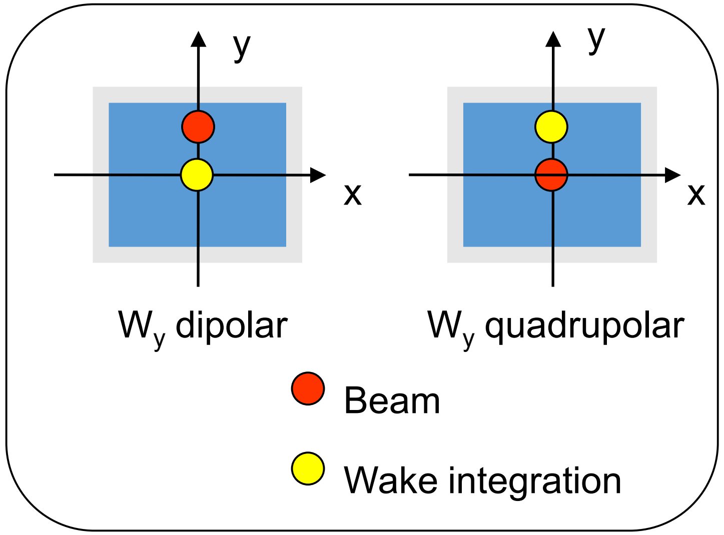
- They produce vertical kick factors

$$k_y = \frac{\operatorname{Im} Z_y c}{2\sqrt{\pi}\sigma_z}.$$

- Transversely, the calculation of kick factors is most important since it provides additional coherent vertical tune shift

1. Impedance Theory
Dipolar, Quadrupolar?

- In asymmetric structures, they are different concepts from dipole and quadrupole modes
- They can be calculated by displacing the beam and the wake integration path separately
- Machine measurements
    - o Tune shift
        - → dipolar + quadrupolar
    - o Instability growth rate
        - → dipolar



B. Salvant, Beam physics for FAIR

1. Impedance Theory

Transverse Geometrical Impedance of Taper (II)

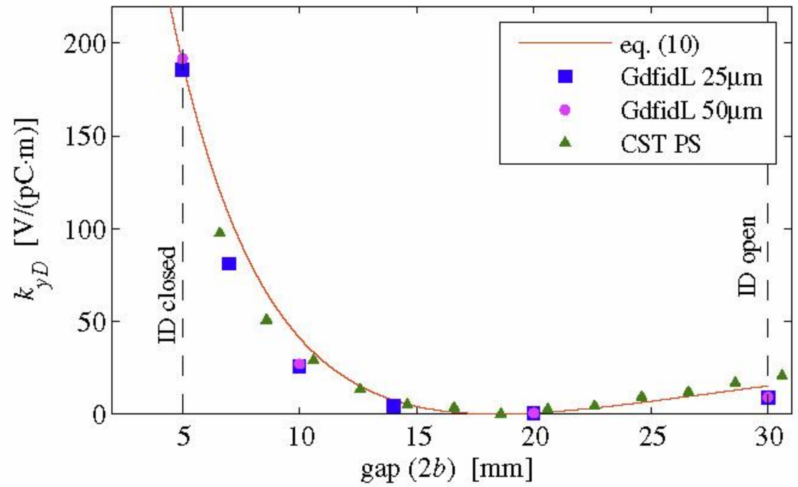
- Theoretical formula for dipolar impedance

$$Z_{yD}(k) = -i\frac{Z_0}{2\pi b}\int_{-\infty}^{\infty}\frac{\xi^2}{\sinh^2\xi}\sum_{n=0}^{\infty}\delta_n\frac{H(k_n,k)+H(k_n,-k)}{2ik_n b}d\xi$$

$$H(p,k) = \int_{-\infty}^{\infty}\int_{-\infty}^{z_1} S'(z_1)S'(z_2)e^{i(p+k)(z_1-z_2)}dz_1 dz_2, \qquad k_n b = \sqrt{(kb)^2-\xi^2-(\pi n)^2}$$

S. Krinsky, Phys. Rev. ST Accel. Beams 8, 124403 (2005) $w \to \infty$

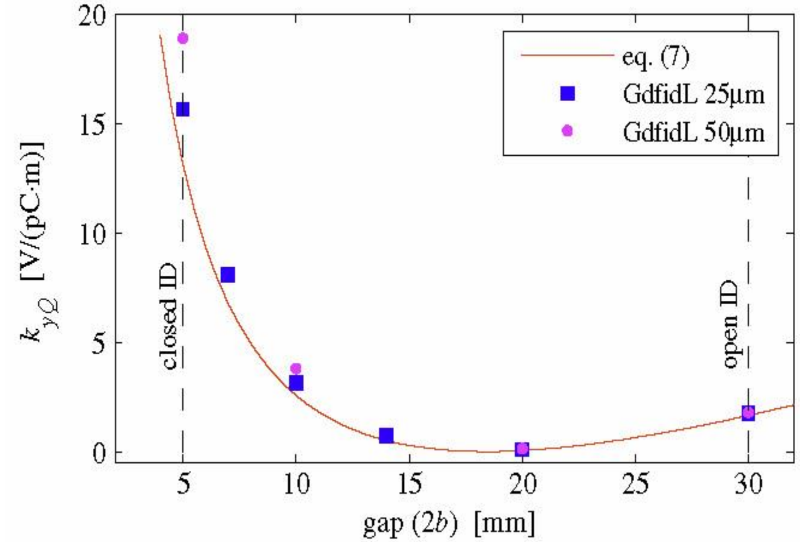
Diamond IVU



- Theoretical formula for quadrupolar impedance

$$Z_{yQ} = -i\frac{\pi Z_0}{4}\int_{-\infty}^{\infty}\frac{(g')^2}{g^2}G\left(\frac{g}{w}\right)dz,$$

$$G(x) = x^2\sum_{m=0}^{\infty}(2m+1)\times\mathrm{sech}^2\left((2m+1)\frac{\pi x}{2}\right)\tanh\left((2m+1)\frac{\pi x}{2}\right). \qquad b \ll w \ll l$$



G. Stupakov, Phys. Rev. ST Accel. Beams 10, 094401 (2007)

1. Impedance Theory

Resistive-Wall Impedance of Undulator*

- Transverse impedance per unit length for a vertically displaced beam in a round chamber is

$$Z_y^{rnd}(\omega) = \frac{sign(\omega)+i}{\pi b^3}\sqrt{\frac{c\mu_r Z_0}{2\omega\sigma_c}}\frac{1+3(y/b)^2}{\left[1+3(y/b)^2\right]^3}$$

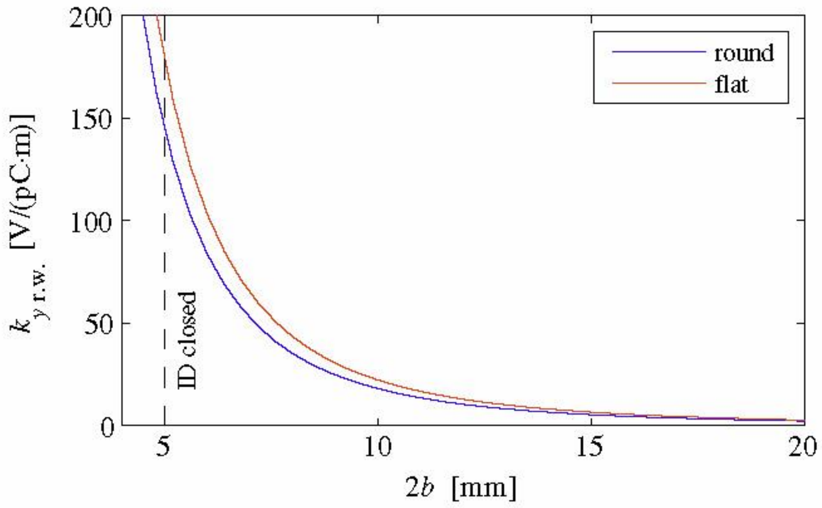
- Formula of resistive-wall impedance for a flat chamber formed by two infinitely wide plates

$$Z_y^{flat}(\omega) = \pi\frac{sign(\omega)+i}{8b^3}\sqrt{\frac{c\mu_r Z_0}{2\omega\sigma_c}}\frac{1+\frac{\pi y}{2b}\tan\left(\frac{\pi y}{2b}\right)}{\cos^2\left(\frac{\pi y}{2b}\right)}$$

$$Z_y^{flat}(\omega) = \frac{\pi^2}{8}Z_y^{rnd}(\omega)$$

A. Piwinski, Report No. DESY-94-068, Hamburg, 1994.

Diamond IVU



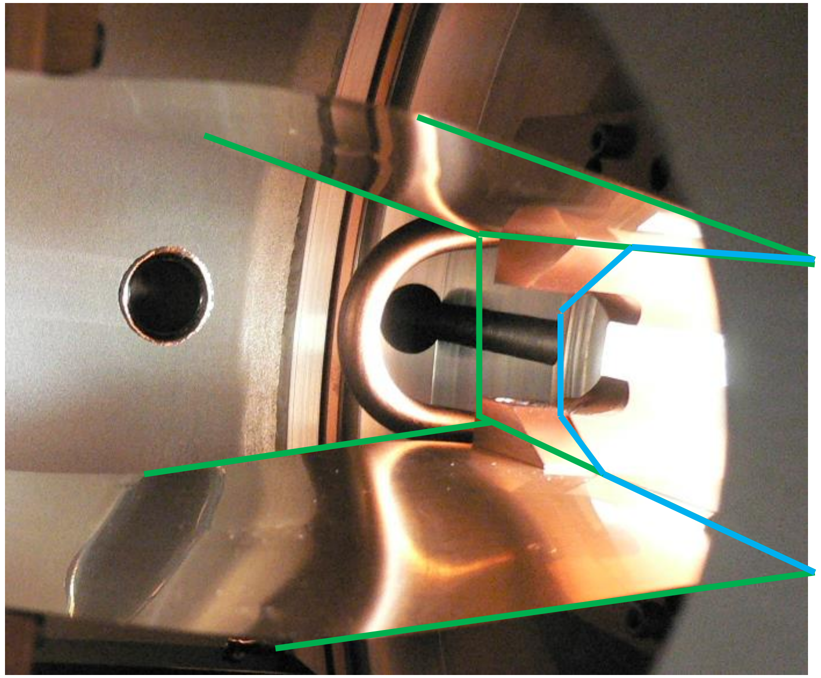
* RF shielded by Cu plate

1. Impedance Theory
Geometrical Impedance of Step Transition

- Low-frequency impedance of the step transition at the beginning of the taper can be roughly estimated using formula:

$$Z_y = i\frac{Z_0(d-b)}{\pi b^2}\frac{d^2-b^2}{d^2+b^2}$$

PF IVU



- Its power loss will be taken care by the cooling channel in the present design

1. Impedance Theory

Summary

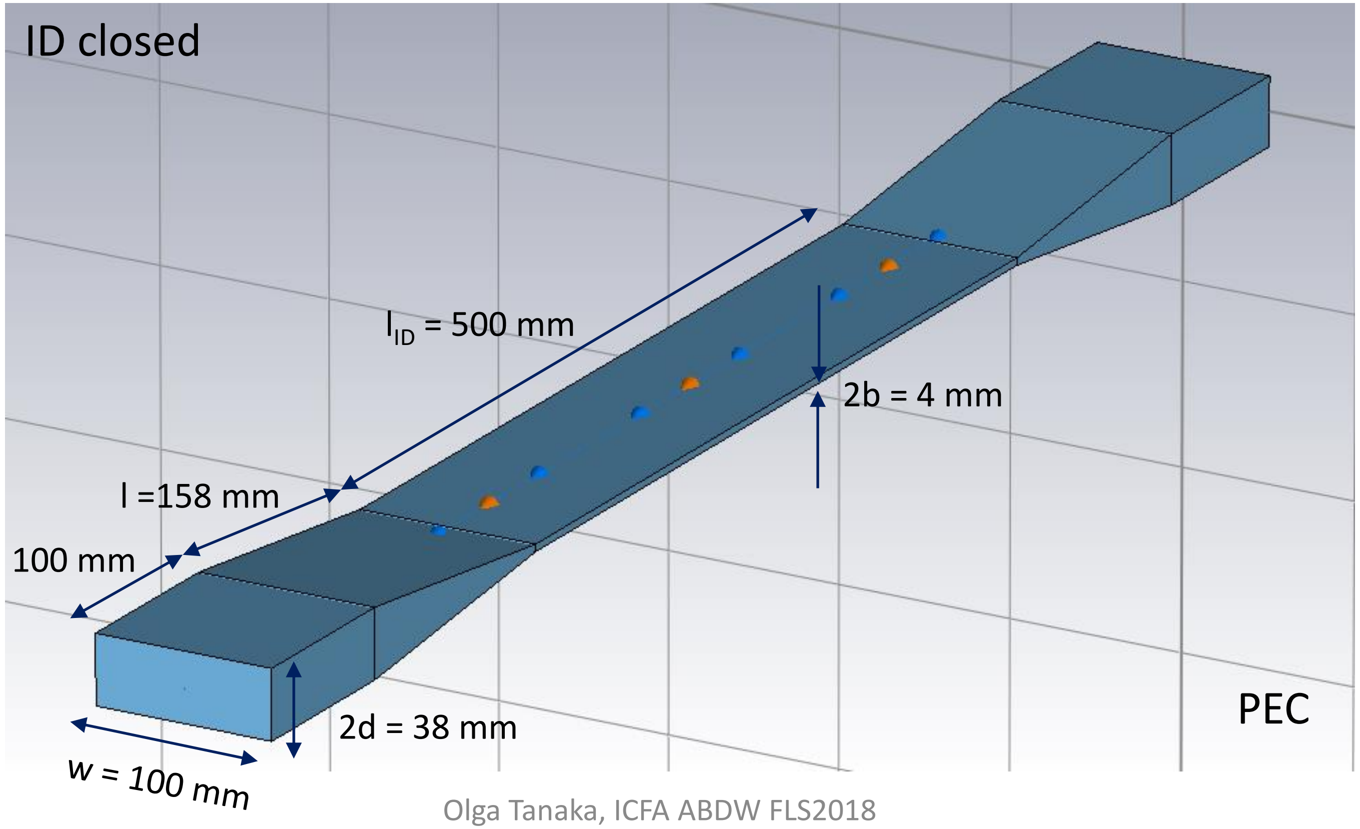
- There are the analytical formulas accurate enough for impedance calculations of all the 3 parts of IVU discussed above (geometrical impedance of the taper, resistive-wall impedance of the copper shield, and geometrical impedance of step transition between rectangular and octagonal beam chambers)
- PF IVUs follow the standard design, therefore we can apply the procedure outlined by Smaluk (Phys. Rev. ST Accel. Beams 17, 074402 (2014)) for the impedance evaluation plus some new formulas.
- The method is to calculate impedance of each part separately using CST and GdfidL and to compare it with the theoretical formula at each time

2. Impedance Evaluation for PF IVU by Simulations and Theory

2. Impedance Evaluation for PF IVU

Taper CST Model

- To calculate the pure geometrical impedance of the taper, we first assume the perfectly conductive material instead of using copper resistivity

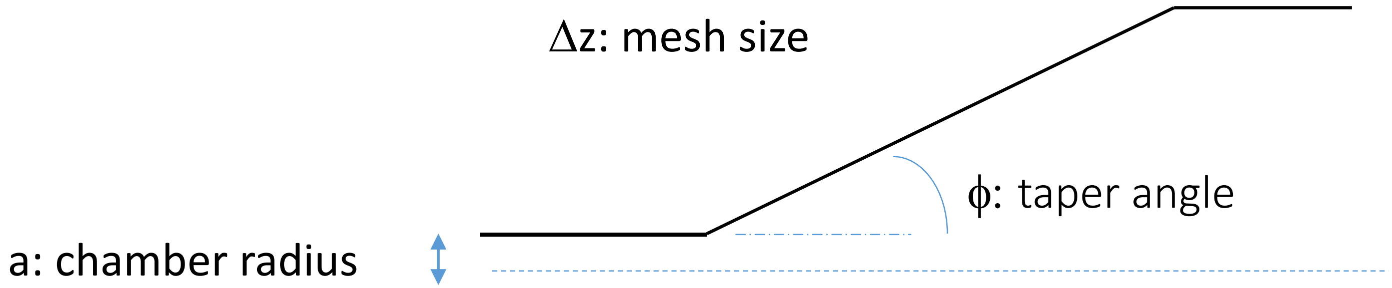


2. Impedance Evaluation for PF IVU

CST Studio Mesh Size

- It is known that a very fine mesh is needed for accurate calculations of the taper impedance
- The empirical formula

$$100 \le \frac{a\phi}{\Delta z} \cdot \frac{\sigma_z}{\Delta z}.$$



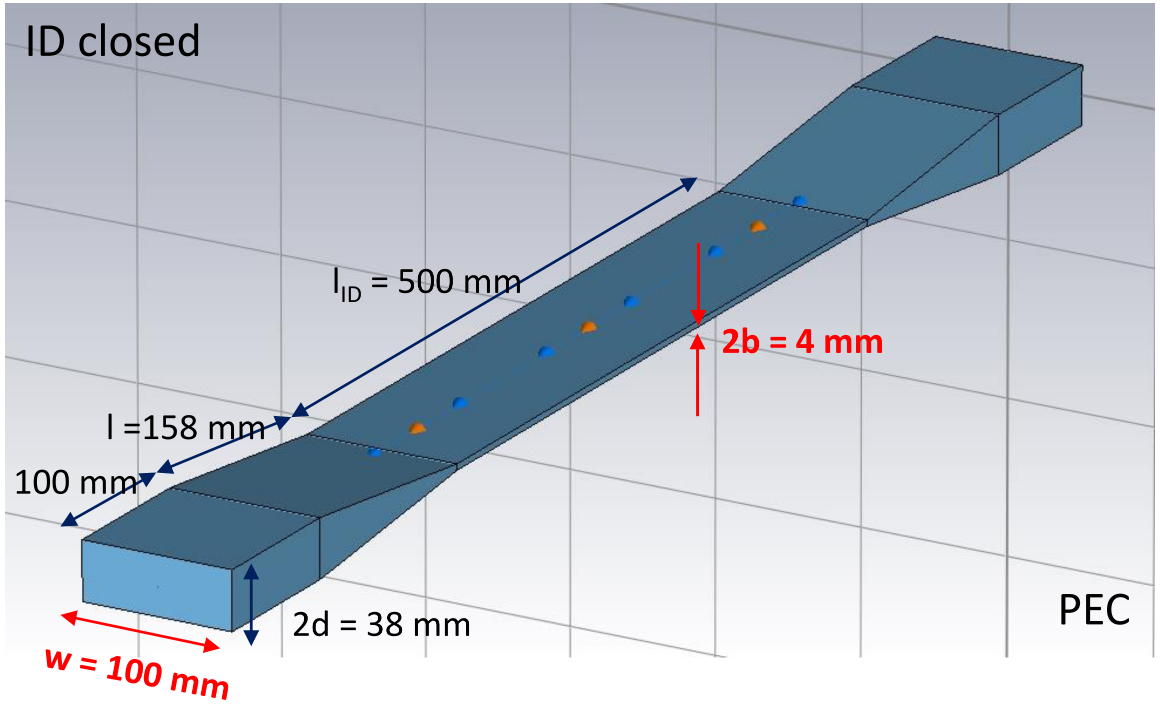
- We need $\Delta z < 150\ \mu m$

Frasciello's slide at SIF2014
on wakes of LHC collimators

2. Impedance Evaluation for PF IVU

Parameter Scan

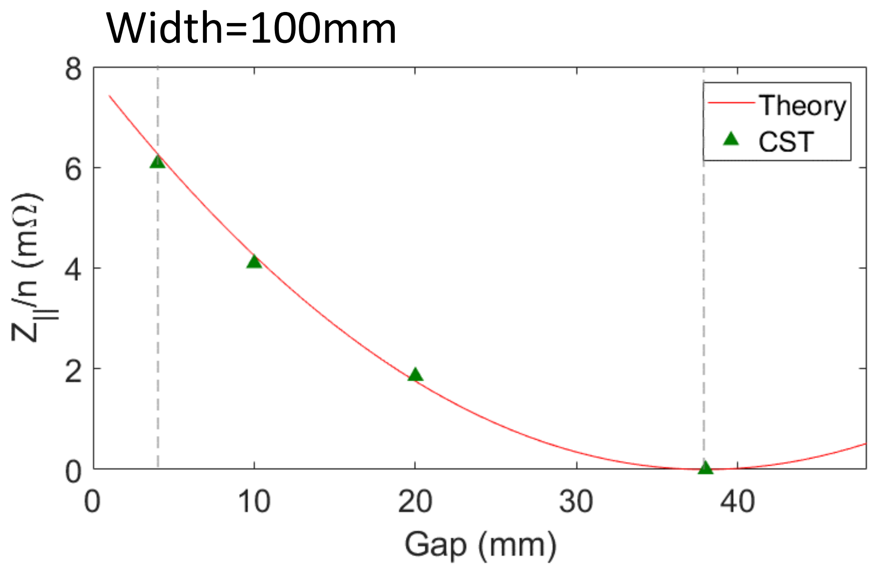
- IVU impedance affected greatly by the size of its gap. When ID is closed the difference even in 0.5 mm yields a drastic increase of impedance
- For a better and more economical design in future, we also studied the dependence of kick factors on the taper width. Conclusion before the results are shown: the present 100 mm is reasonable and close to optimal width

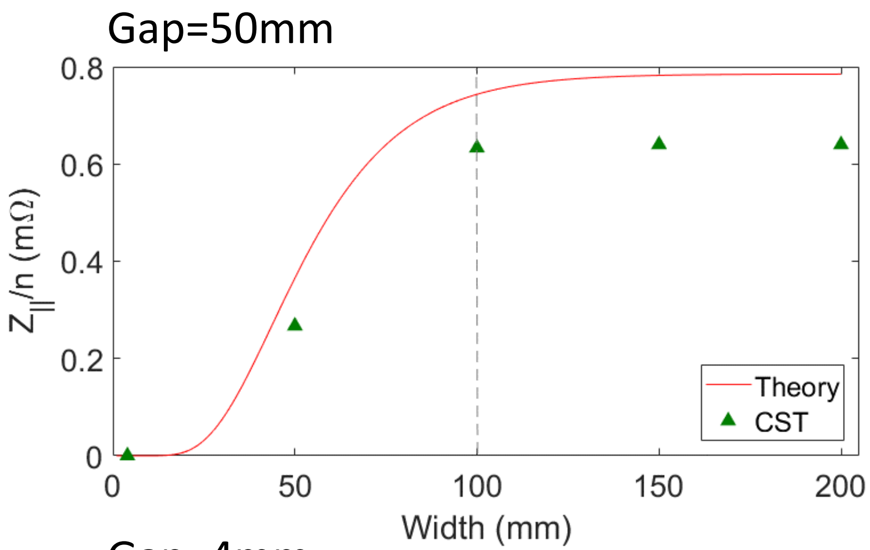


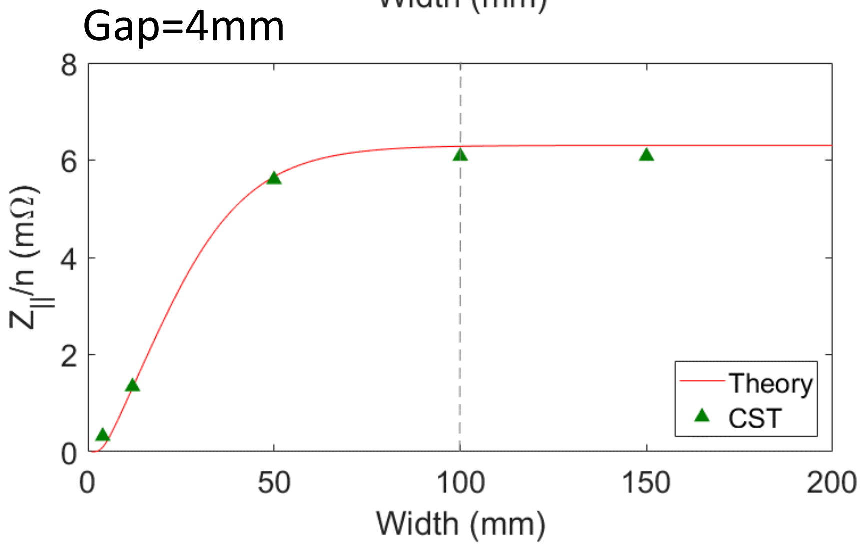
- For the future IVU designs a length of the taper (or its angle) is one of the key parameters of impedance evaluation. Its consideration was excluded from the present study because IVUs were already designed and installed

2. Impedance Evaluation for PF IVU

Longitudinal Geometrical Impedance of Taper







G. Stupakov

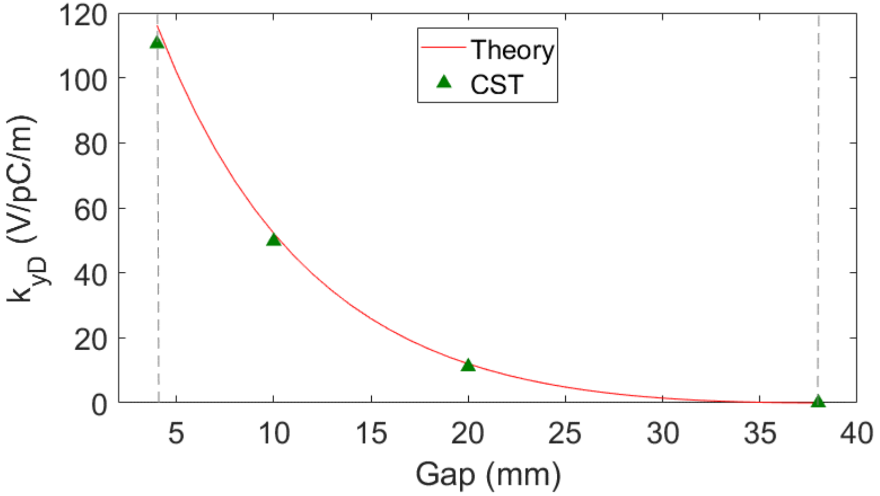
$$\frac{Z_l}{n} = -i\frac{Z_0\omega_0}{4\pi c}\int_{-\infty}^{\infty}(g')^2 F\left(\frac{g}{w}\right)dz, \quad b \ll w \ll l$$

$$F(x) = \sum_{m=0}^{\infty}\frac{1}{2m+1}\operatorname{sech}^2\left((2m+1)\frac{\pi x}{2}\right)\tanh\left((2m+1)\frac{\pi x}{2}\right).$$

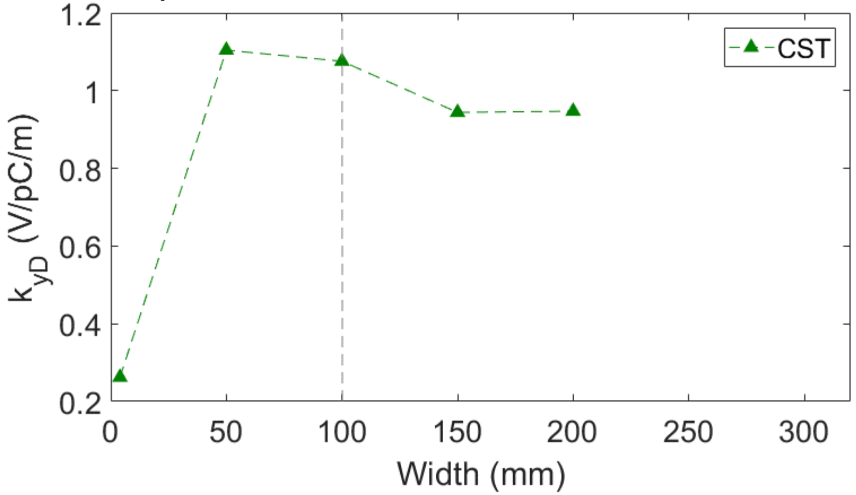
2. Impedance Evaluation for PF IVU

Dipolar Geometrical Impedance of Taper

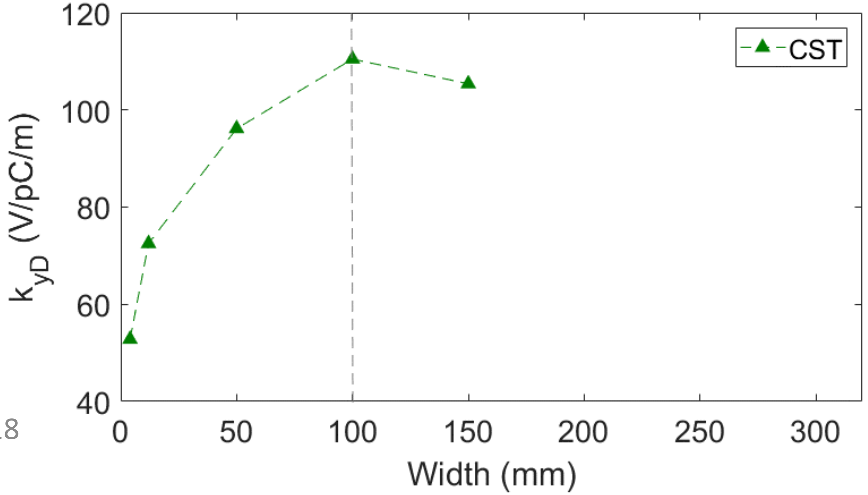
Width=100mm



Gap=50mm



Gap=4mm



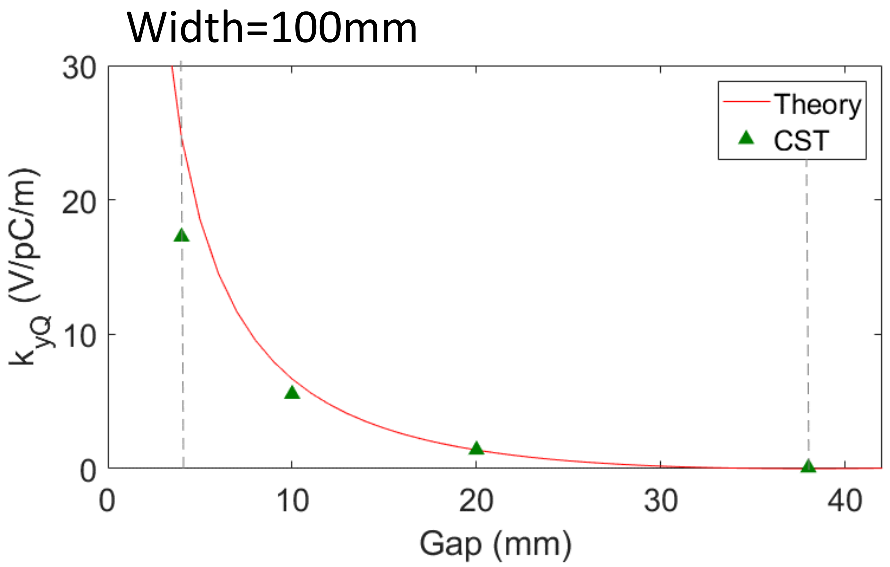
S. Krinsky
$$Z_{yD}(k) = -i\frac{Z_0}{2\pi b}\int_{-\infty}^{\infty}\frac{\xi^2}{\sinh^2\xi}\sum_{n=0}^{\infty}\delta_n\frac{H(k_n,k)+H(k_n,-k)}{2ik_n b}d\xi$$

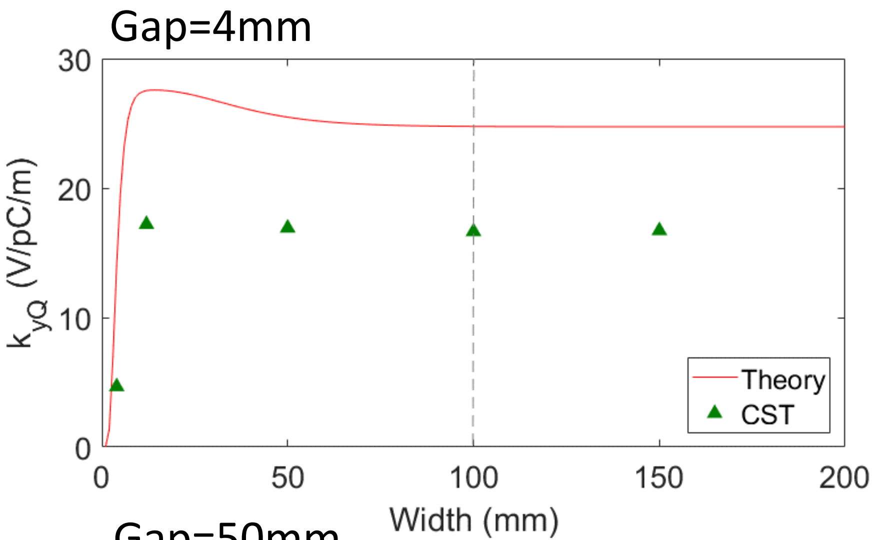
$$H(p,k) = \int_{-\infty}^{\infty}\int_{-\infty}^{z_1} S'(z_1)S'(z_2)e^{i(p+k)(z_1-z_2)}dz_1 dz_2, \quad k_n b = \sqrt{(kb)^2-\xi^2-(\pi n)^2}$$

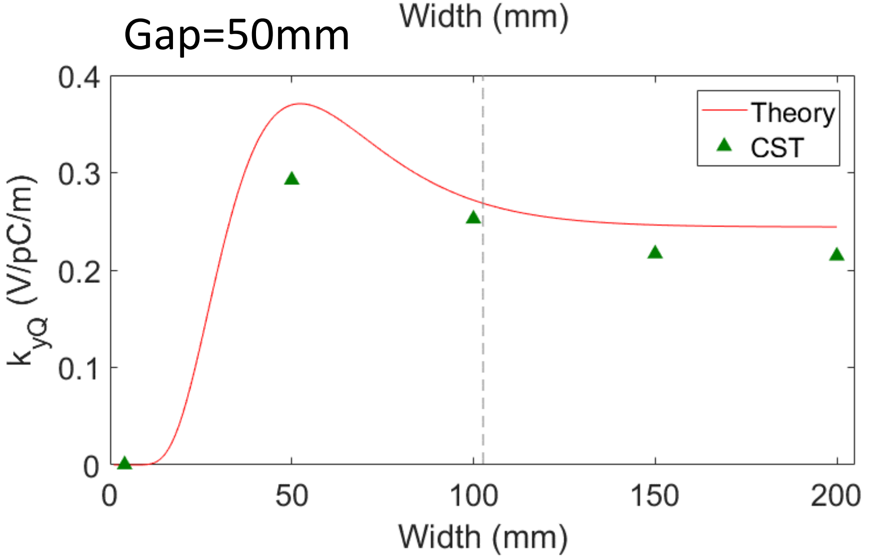
$$w \to \infty$$

2. Impedance Evaluation for PF IVU

Quadrupolar Geometrical Impedance of Taper







G. Stupakov

$$Z_{yQ} = -i\frac{\pi Z_0}{4}\int_{-\infty}^{\infty}\frac{(g')^2}{g^2}G\left(\frac{g}{w}\right)dz,$$

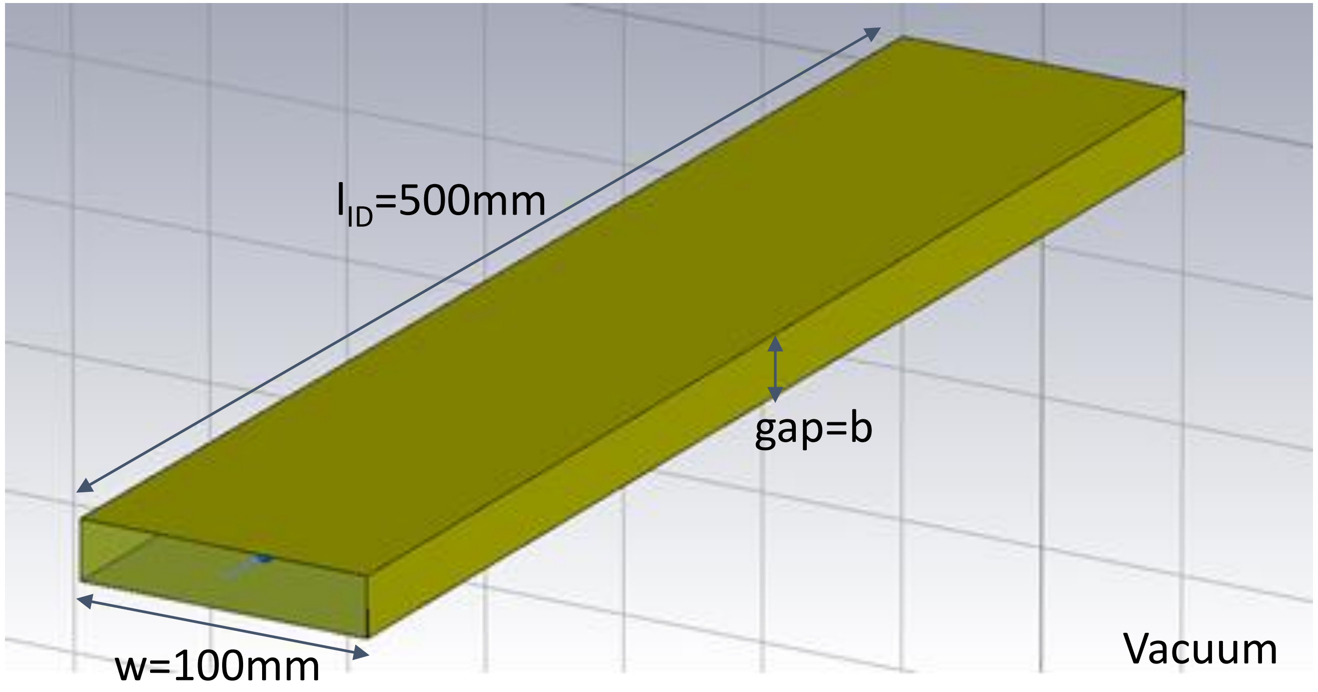
$$G(x) = x^2\sum_{m=0}^{\infty}(2m+1)\times\mathrm{sech}^2\left((2m+1)\frac{\pi x}{2}\right)\tanh\left((2m+1)\frac{\pi x}{2}\right).$$

$$b \ll w \ll l$$

2. Impedance Evaluation for PF IVU

Resistive-Wall Impedance of Undulator

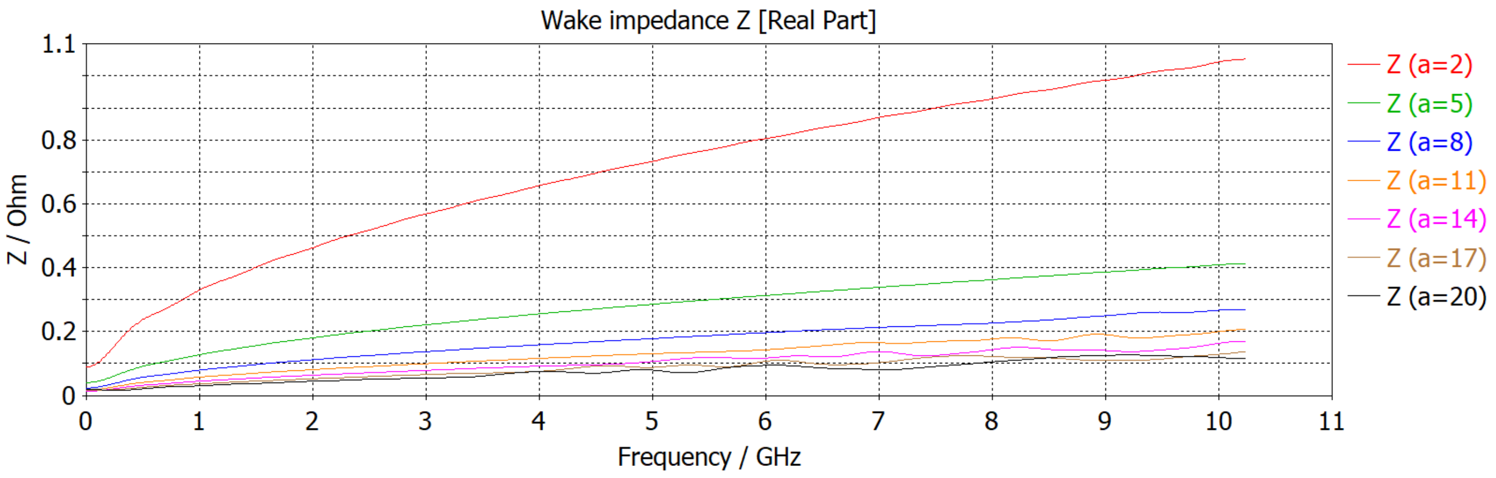
- By using the copper resistivity in CST, we can calculate the resistive impedance of the undulator with copper sheet

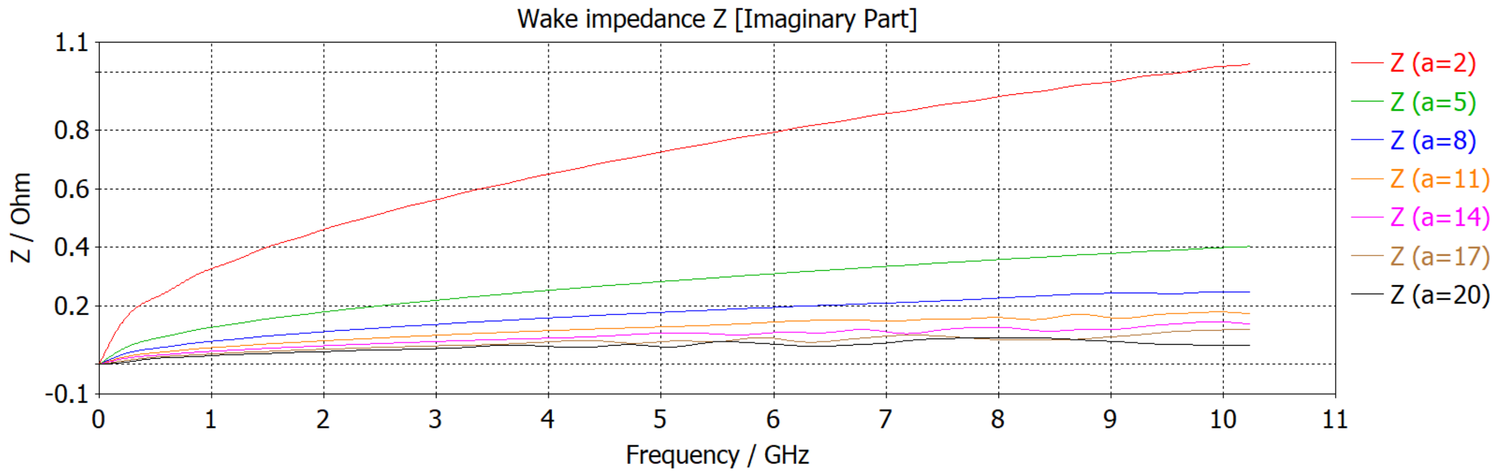


The electric conductivity of copper $\sigma_c = 5.9\times10^7 S/m$

2. Impedance Evaluation for PF IVU

Longitudinal Resistive-Wall Impedance of Undulator





- The real and the imaginary parts of longitudinal impedance are identical as the theory shows:

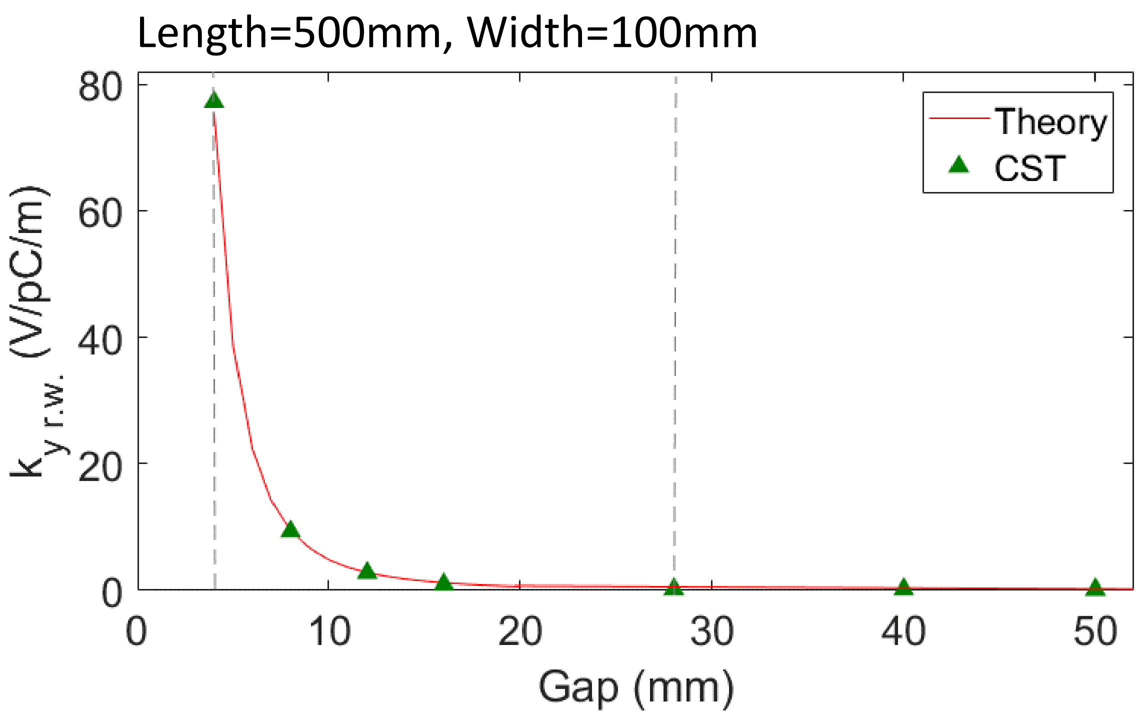
$$Z_{l\ R.W.} = \frac{1 \pm i}{\pi b} \sqrt{\frac{\mu_r |\omega|}{2\sigma_c}} \left[1 + \frac{\pi y}{b} \tan\left(\frac{\pi y}{b} \right) \right]$$

A. Piwinski, Report No. DESY-94-068, 1994

It demonstrates that our CST Studio simulations are very accurate!

2. Impedance Evaluation for PF IVU

Transverse Resistive-Wall Impedance of Undulator



O. Frasciello + V. Smaluk

$$k_{yR.W.} = \frac{cL}{8b^3}\sqrt{\frac{2Z_0}{\sigma_z \sigma_c}}\,\Gamma\left(\frac{5}{4}\right)$$

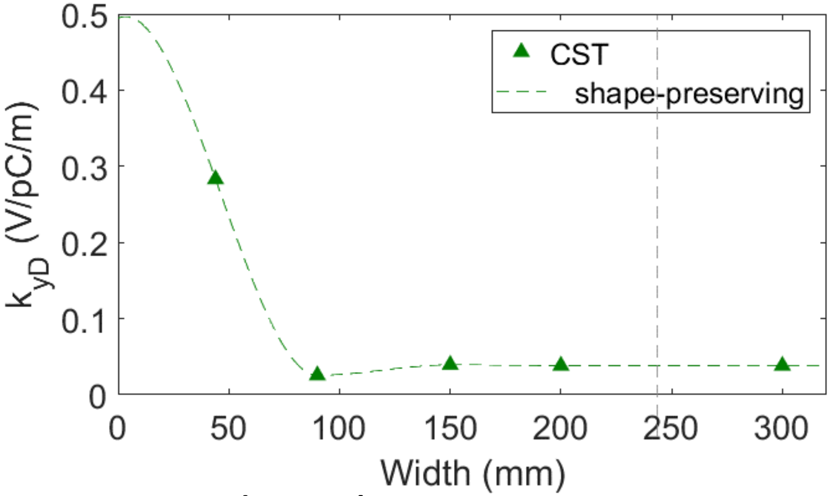
L=Length of undulator

$\Gamma\left(\frac{5}{4}\right)$=0.9064

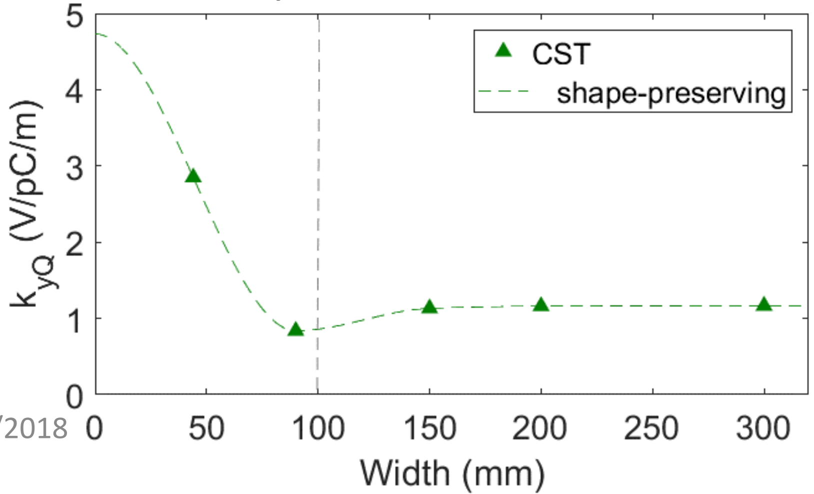
2. Impedance Evaluation for PF IVU

Geometrical Impedance of Step Transition

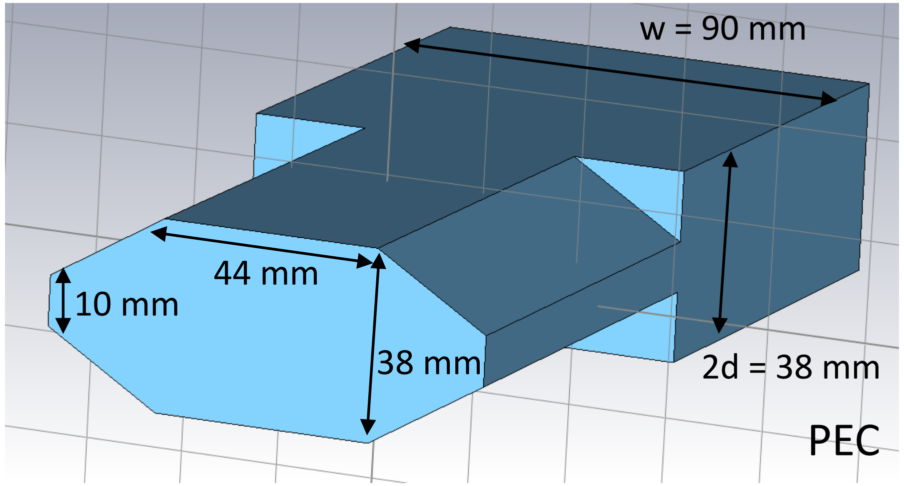
Dipolar



Quadrupolar



CST model of the step transition



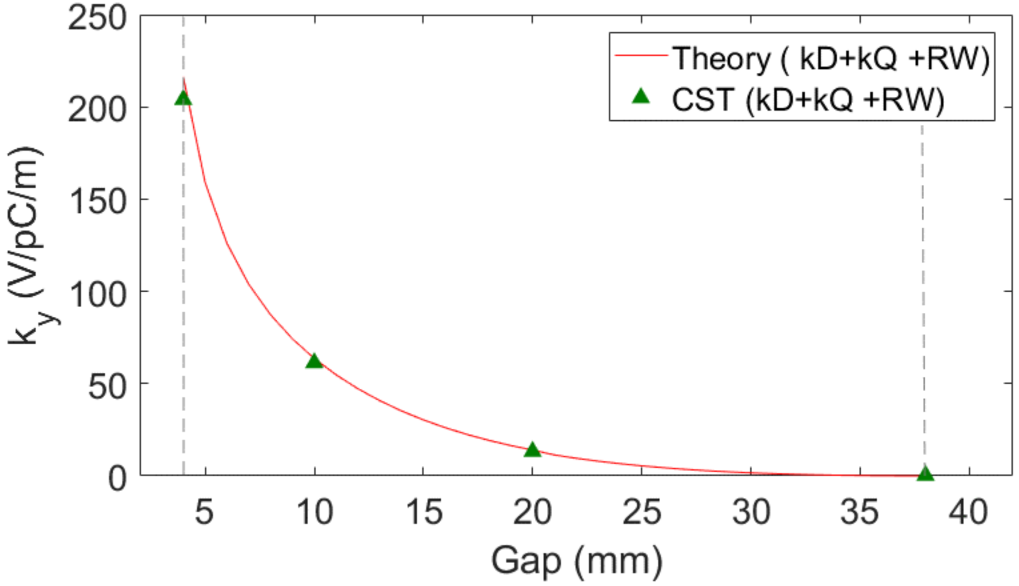
They have very small contributions to the total vertical kick factor and saturate at width = 150 mm

2. Impedance Evaluation for PF IVU

Total Transverse Impedance of the IVU

- The total vertical kick factor due to 1 IVU is

Total Vertical Kick Factor of PF IVU



Vertical kick factor per 1 IVU		CST PS	Theory
Taper vertical kick factor, V/pC/m	Dipolar	110.47	116.13
	Quadrupolar	16.64	24.61
Undulator vertical kick factor, V/pC/m	Dipolar	50.80 +	75.57
	Quadrupolar	26.40	
Total vertical kick factor, V/pC/m		**204.31**	**216.31**

- Impact of the step transition is three orders less, therefore is negligible

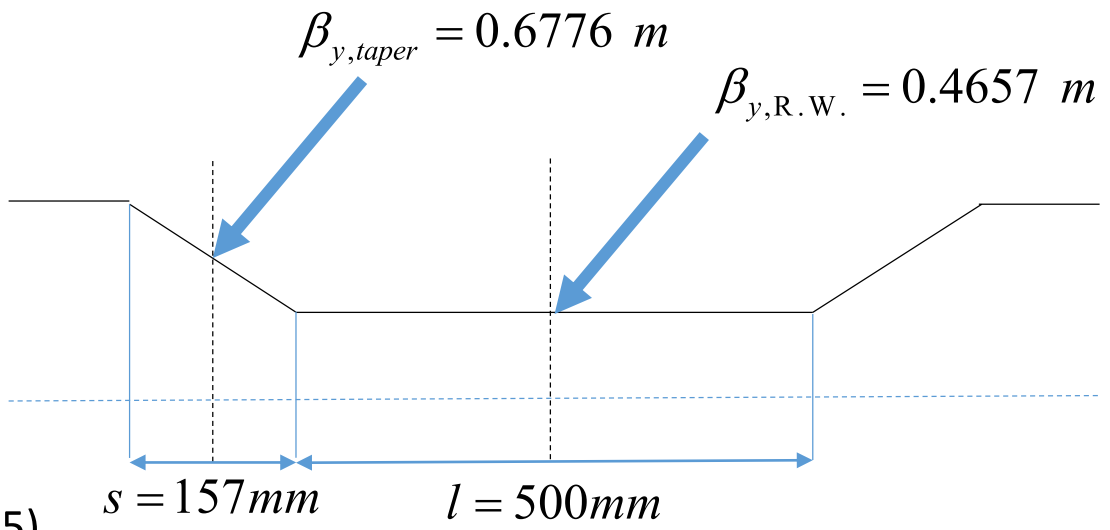
2. Impedance Evaluation for PF IVU

Additional Tune Shift by 4 IVU at PF (I)

- Tune shift per unit of bunch current caused by the IVU impedance can be estimated using formula:

$$\frac{\Delta\nu_y}{I_b} = -\frac{1}{4\pi(E/e)f_0}\sum_j \beta_{y,j}\left[k_{y,j}^{(1)} + k_{y,j}^{(2)}\right]$$

S. Sakanaka, et. al. Phys. Rev. ST Accel. Beams **8,** 042801 (2005)



- Average betatron function in the center of the undulator:

$<\beta_{y,RW}>=\beta_{y0}+(1+\alpha_{y0}^2)l^2/12/\beta_{y0}=0.415+(1+0.099^2)\text{x}0.5^2/12/0.415=0.4657\text{ m}$

- Average betatron function in the center of the taper:

$<\beta_{y,taper}>=\beta_{y0}+(1+\alpha_{y0}^2)(s/2+l/2)^2/\beta_{y0}=0.415+(1+0.099^2)\text{x}0.3285^2/0.415=0.6776\text{ m}$

2. Impedance Evaluation for PF IVU

Additional Tune Shift by 4 IVU at PF (II)

- According to the CST simulations,

$$\frac{\Delta\nu}{I_b} = -4\times\langle\beta\times k_y\rangle/(4\pi f_0(E/e)) = -\ 0.488\text{x}\ 10^{15}/(4\pi \times 1.6 \times 10^6 \times 2.5 \times 10^9) =$$

$$= -9.713\times10^{-6}\ (\text{mA}^{-1})$$

- According to the theoretical formulas,

$$\frac{\Delta\nu}{I_b} = -4\times\langle\beta\times k_y\rangle/(4\pi f_0(E/e)) = -\ 0.522\text{x}\ 10^{15}/(4\pi \times 1.6 \times 10^6 \times 2.5 \times 10^9) =$$

$$= -10.39 \times10^{-6}\ (\text{mA}^{-1})$$

2. Impedance Evaluation for PF IVU

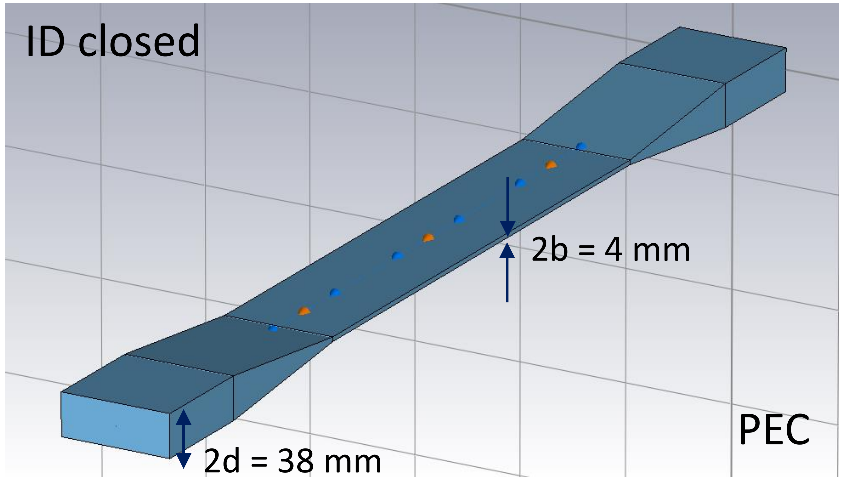
Summary

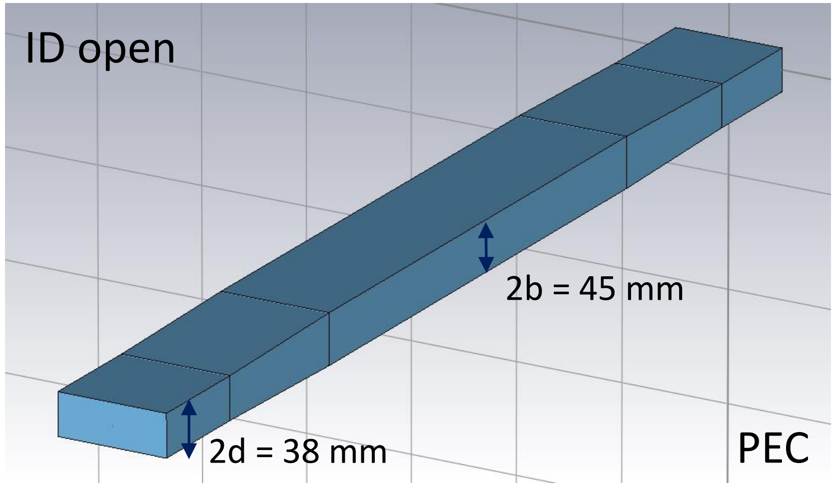
- Excellent agreements between the theoretical predictions and CST Studio simulations for PF IVU
- Therefore, the new impedance evaluations of PF IVU are accurate enough in the framework of the theory and the simulation codes
- We can use these calculation results and computation resources and techniques for future impedance measurements, for the design of a new IVUs, and even for the impedance budget of the components of any new accelerator

3. Measurements of Kick Factors

3. Measurements of Kick Factors

Tune Shift Measurement Method





- This additional tune shift corresponds to a difference of the vertical tune shifts for ID open (gap=45mm) and ID closed (gap=4mm) cases
- According to the CST simulations,

$$\frac{\Delta\nu}{I_b} = -4\times\langle\beta\times k_y\rangle/(4\pi f_0(E/e))$$

$$=-0.488 \times 10^{15}/(4\pi \times 1.6 \times 10^{6} \times 2.5 \times 10^{9})$$

$$= -\ 9.713 \times 10^{-6}\ (mA^{-1})$$

- According to the theoretical formulas,

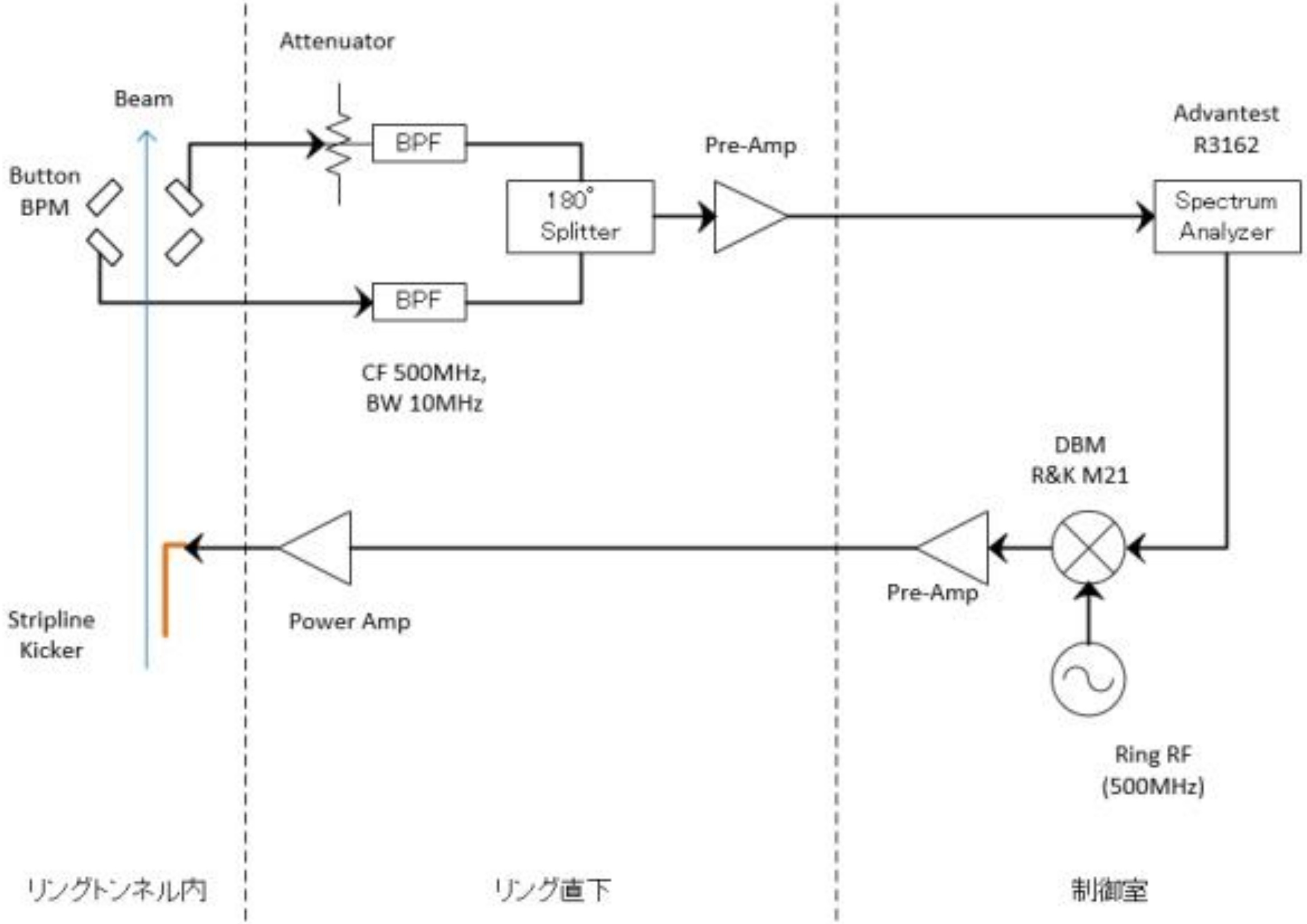
$$\frac{\Delta\nu}{I_b} = -4\times\langle\beta\times k_y\rangle/(4\pi f_0(E/e))$$

$$=-0.522 \times 10^{15}/(4\pi \times 1.6 \times 10^{6} \times 2.5 \times 10^{9})$$

$$= -\ 10.39 \times 10^{-6}\ (mA^{-1})$$

3. Measurements of Kick Factors
RF Knock-Out

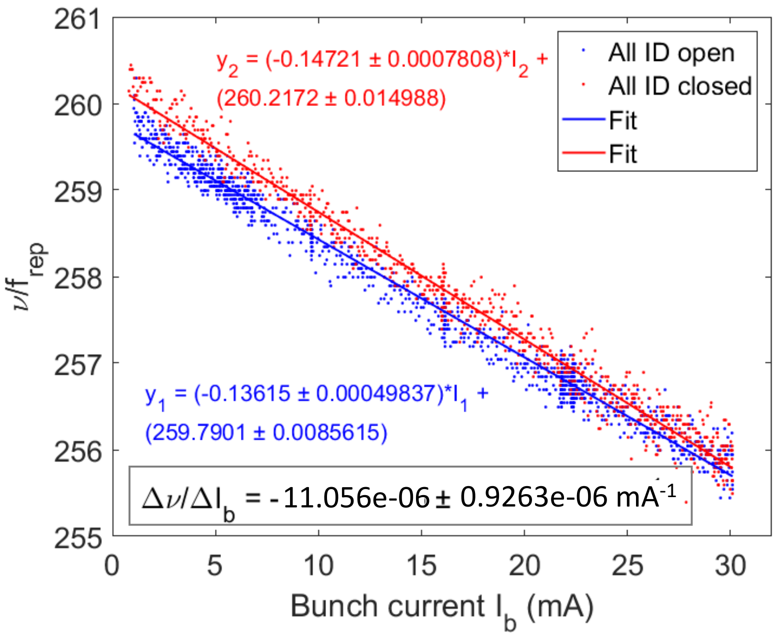
- Single bunch
- Feedback OFF
- The responses of the stripline kicker oscillations were measured by sweeping the bunch current (equal to changing the betatron frequency) using a spectrum analyzer equipped with a tracking generator.



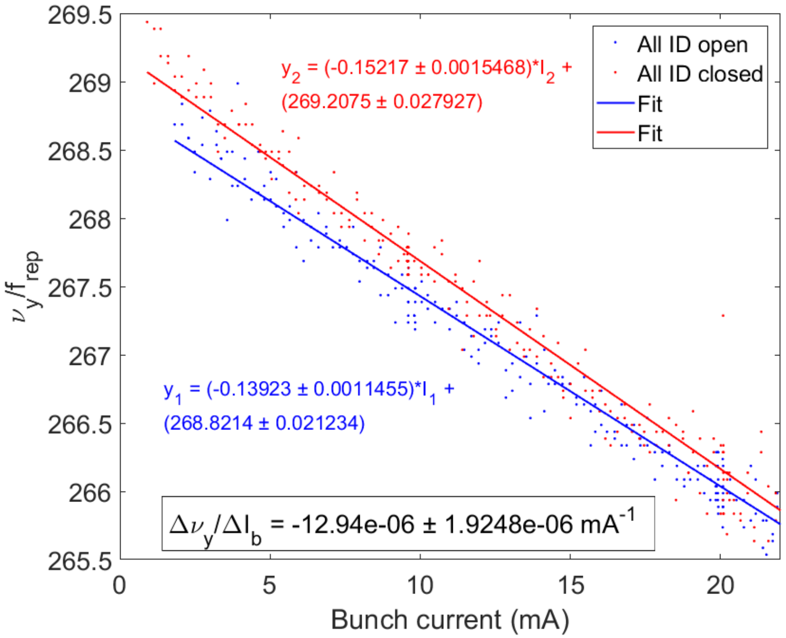
3. Measurements of Kick Factors

Tune Shift Measurement Result

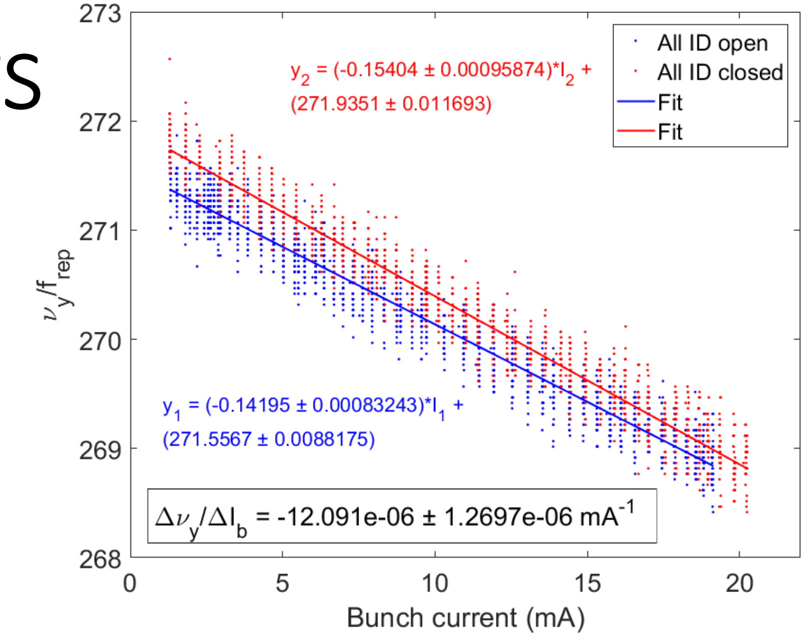
Measurement #1



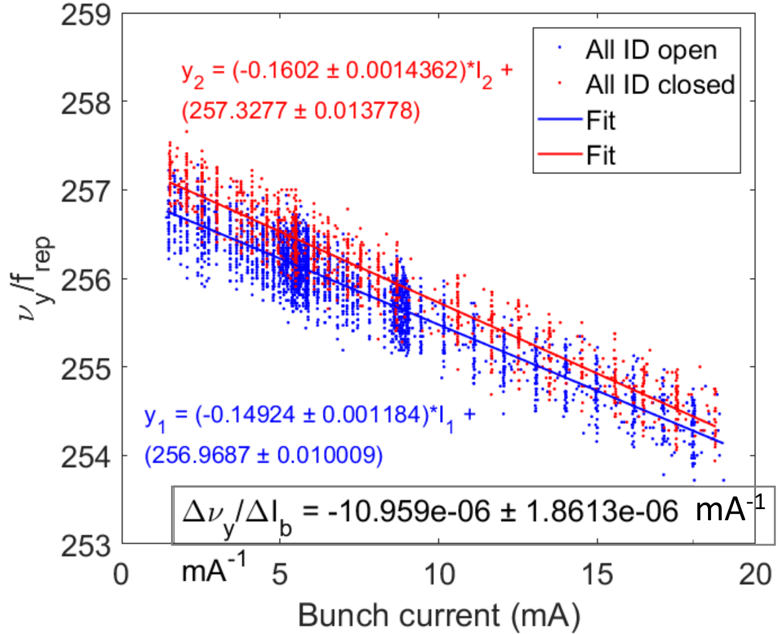
Measurement #2



Measurement #3



Measurement #4



	Theory	CST PS	Measurement (average)
Tune shift per bunch current (mA^{-1})	-10.39×10^{-6}	-9.713×10^{-6}	$(-11.7615 \pm 1.4955)\times10^{-6}$

Good agreement with our evaluations!

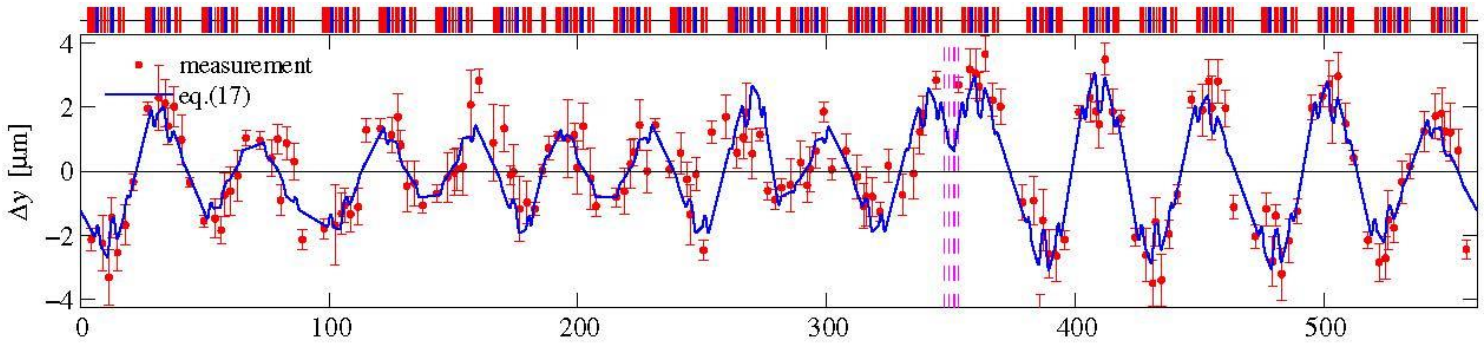
3. Measurements of Kick Factors

Orbit Bump Measurement Method (I)

- Create an orbit bump at a location including IVU
- This orbit bump (y_0) creates orbit deviations proportional to the kick factor of IVU along the ring:

$$\Delta y(s) = \frac{\Delta q}{E/e} k_y y_0 \frac{\sqrt{\beta(s)\beta(s_0)}}{2\sin(\pi\nu)} \cos\left[\left|\mu(s)-\mu(s_0)\right| - \pi\nu\right],$$

V. Smaluk, Phys. Rev. ST Accel. Beams 17, 074402 (2014)

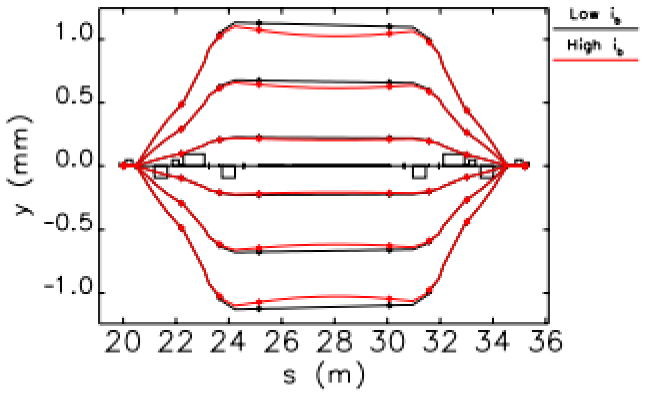


3. Measurements of Kick Factors

Orbit Bump Measurement Method (II)

- Measure the orbit deviations at many BPM positions to reduce statistical errors
- Repeat the above procedure for different orbit bumps and bunch charges to eliminate systematic errors caused by intensity dependent behavior of BPM electronics
- Using the analytical formula and the Twiss parameters of the ring, we can identify the kick factor of IVU
- The measurement is scheduled in April

L. Emery, APS



Summary

- We have identified the major impedance contributors of PF IVU and successfully evaluated their impedance using theoretical formulas, CST Studio simulations and measurements
- The three evaluations show very good agreements
- The established methods and procedure will greatly help the design of future IVU for further reduction of impedance

Thank you for your attention!

Backup

Possible reasons of the difference in estimated and measured tune shift values

1. **Size of the gap between two copper shields (3.83 mm vs 4 mm)**
2. **Thickness of the copper shield is not enough**
3. Difference in present and model values of betatron function
4. Difference in values of betatron function when ID gap is opened/closed
5. Reliability of CST code
6. Accuracy of the tune shift measurement

Courtesy of N. Nakamura

1. Size of the gap between two copper shields

- The smallest ID gap *g* = 2*b* = 4 mm (*b* = 2 mm)
- The smallest ID gap *g* = 2*b* = 4 mm (*b* = 2 mm)
- Thickness of the shield *t* = 60 mm (Cu) + 25 mm (Ni) = 85 mm
- Real size of the gap g_s = $2b_s$ = 4 - 0.085 x 2 = 3.83 mm (b_s = 1.915 mm)
- Resistive-wall impedance(imaginary part) & kick factor

$$\mathrm{Im}\, Z_y(f) \approx -\frac{\pi Z_0 L}{16 b_s^3}\sqrt{\frac{1}{\pi |f| \mu_0 \sigma_{Cu}}}$$

$$k_y = f_0 \sum_{p=-\infty}^{\infty} \mathrm{Im}\, Z_{Dy}(pf_0 + f_\beta) h(pf_0 + f_\beta)$$

$$\left(h(\omega) = \exp\left\{ -\left(\omega \sigma_z / c \right)^2 \right\} \right)$$

$$k_y = 85.97\ \mathrm{V/pC/m}\quad (f = -10 \sim +10\ \mathrm{GHz})$$

$$cf.\ \ k_y(b = 4\mathrm{mm}) = 75.46\ \mathrm{V/pC/m}$$

Influence of about 14%

Courtesy of N. Nakamura

2. Thickness of the copper shield is not enough

- Frequency at which copper skin depth and copper sheet thickness are the same (δ_{Cu}=d_{Cu}) $\quad \delta_{Cu} = d_{Cu} \quad \left(\delta_{Cu} = \sqrt{\frac{1}{\pi \sigma_{Cu} \mu_0 |f|}} \right) \rightarrow f_\delta = \frac{1}{\pi \sigma_{Cu} \mu_0 d_{Cu}^2}$
- Resistive-wall impedance formula (switched by frequency)

$$|f| \ge f_\delta = \frac{1}{\pi \sigma_{Cu} \mu_0 d_{Cu}^{\ 2}} = 1.19\ \mathrm{MHz}\left(d_{Cu} = 60\ \mu\mathrm{m}\right) \rightarrow \ \mathrm{Im}\, Z_y(f) \approx -\frac{\pi Z_0 L}{16 b^3}\sqrt{\frac{1}{\pi |f| \mu_0 \sigma_{Cu}}}$$

$$|f| < f_\delta \ (f = \Delta \nu_\beta f_0,\ f = -(1-\Delta \nu_\beta) f_0) \rightarrow \ \mathrm{Im}\, Z_y(f) \approx -\frac{\pi Z_0 L}{16 b^3}\sqrt{\frac{1}{\pi |f| \mu_0 \sigma_{NdFeB}}}$$

$$\left(\sigma_{NdFeB} = 0.6 \times 10^6\ \Omega^{-1}\mathrm{m}^{-1}, \sigma_{Ni} = 14 \times 10^6\ \Omega^{-1}\mathrm{m}^{-1}\right)$$

$$k_y = f_0 \sum_{p=-\infty}^{\infty} \mathrm{Im}\, Z_y(p f_0 + f_\beta) h(p f_0 + f_\beta)$$

$$k_y = 86.04\ \mathrm{V/pC/m}\ \ (f = -10 \sim +10\ \mathrm{GHz}) \quad cf.\ \ k_y(Cu) = 75.46\ \mathrm{V/pC/m}$$

Courtesy of N. Nakamura

Influence of about 14%