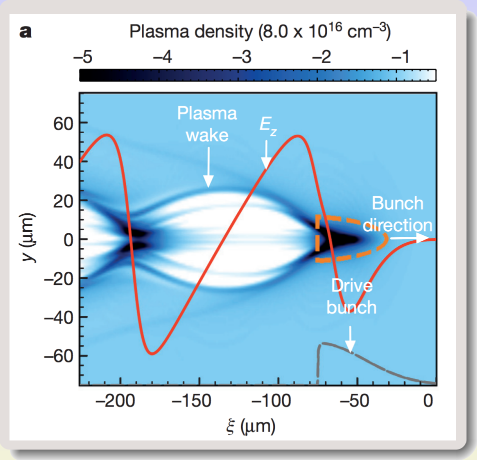
Generation of a Wakefield Undulator in Plasma With Transverse Density Gradient

Gennady Stupakov
SLAC National Accelerator Laboratory, Menlo Park, CA 94025

Future Light Sources 2018
Shanghai, March 5-9, 2018

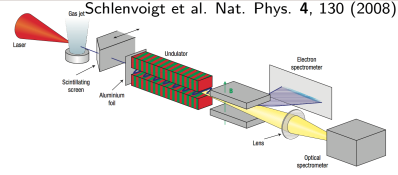
PWFA and LPA



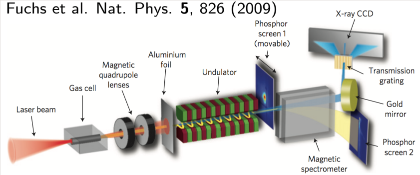
From S. Corde et al. Nature, **524**, 442 (2015).

LPA FEL programs toward compact x-ray sources

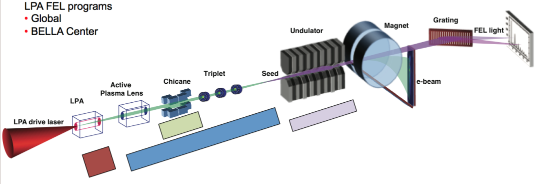
There is an intensive effort to use the plasma-assisted acceleration for compact x-ray FELs. Radiation sources based on a combination of LWFA and magnetic undulators have been demonstrated for visible light and soft x-ray emission.



Schlenvoigt et al. Nat. Phys. **4**, 130 (2008)



Fuchs et al. Nat. Phys. **5**, 826 (2009)

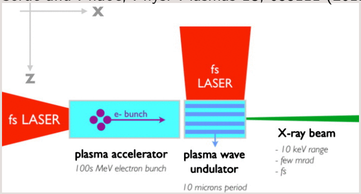


There is ongoing activity at BELLA facility at LBNL (talk by J. van Tilborg on Tuesday) .

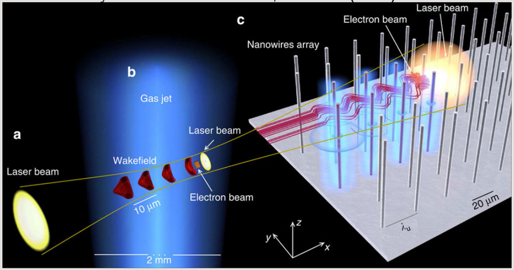
Ultracompact undulators

Ultracompact undulators were proposed in several papers[1].

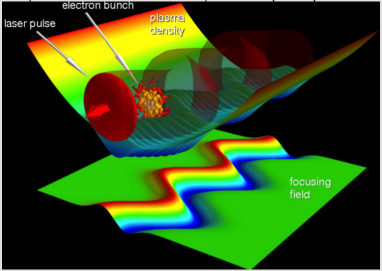
Corde and Phuoc, Phys. Plasmas **18**, 033111 (2011)



Andriyash et al. Nat. Com. **5**, 4736 EP (2014)



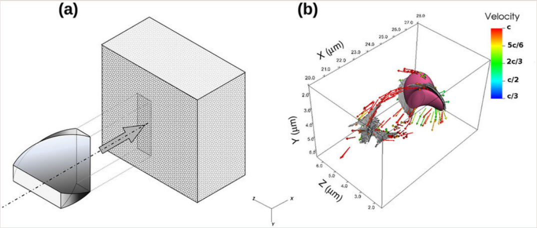
Rykovanov et al. PRL **114**, 145003 (2015)



[1] C. Joshi et al., IEEE Journ. Quant. Electronics **23**, 1571 (1987).

Plasma can sustain strong magnetic fields

Plasma is capable to sustain ultrahigh electric fields. The breakdown electric field for the plasma with density n_0 is estimated as $E \sim ek_p/r_e$, where $k_p = \sqrt{4\pi n_0 r_e}$ is the inverse plasma skin depth and $r_e = e^2/mc^2$ is the classical electron radius. For $n_0 = 10^{17}\ \mathrm{cm}^{-3}$ this estimate gives $E \sim 30$ GV/m, with the corresponding magnetic field $B \sim 100$ T. A solenoidal magnetic field of even much higher strength (~GigaGauss) can be generated through the interaction of a screw-shaped laser pulse with under-dense plasma[2].



[2] Zs. Lécz et al. "GigaGauss solenoidal magnetic field inside bubbles excited in under-dense plasma", Sci. Rep. **6**, 36139 EP (2016).

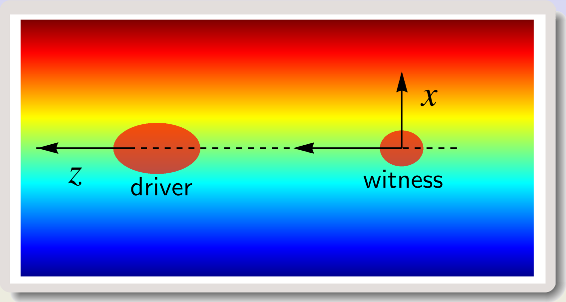
The concept

A fraction of the field $B \sim ek_p/r_e$ can be arranged in such a way that a witness bunch executes wiggling motion as it travels through the plasma behind the driver[3]. This is achieved by introducing a transverse density gradient that periodically changes direction and oscillates along the path of the beam. The analysis is carried out for the PWFA case when the driver is an electron bunch. It can also be extended for the laser driver by replacing the electromagnetic force from the electron bunch by the ponderomotive force of the laser field.

[3] G. Stupakov, Physics of Plasmas, **24**, 113110 (2017).

Setup

Consider a plasma that has a transverse density gradient in x-direction independent of z and y.



The plasma density in the $y - z$-plane is constant, n_0.

An ultra-relativistic electron driver bunch is moving with $v = c$ along the z-axis. The electron beam distribution is assumed axisymmetric,

$$n_b(r, z, t) = n_{b0} f(r) g(\xi)$$

where $\xi = t - z$, $r = \sqrt{x^2 + y^2}$ (we use the standard plasma dimensionless units).

Method

To make the problem tractable analytically, we will make two important approximations.

First, we will assume that the beam density n_{b0} is small enough that one can treat plasma perturbations and the fields in the plasma in linear approximation, neglecting terms of the second and higher order (this is not a blowout regime).

The equilibrium dimensionless plasma density is approximated by the linear profile

$$n_p(x) = 1 + \alpha x$$

where α is the dimensionless density gradient. The parameter α is treated as small, and we will solve our equations using perturbation theory that neglects terms of the second and higher order in α. In physical units, our assumption means that the plasma density changes in x direction on the scale much larger than k_p^{-1}.

Theory

All fields and perturbations depend on z and t through the combination $\xi = t - z$. From the expressions for the fields in terms of the scalar potential ϕ and vector potential $\boldsymbol{A}$, $\boldsymbol{E} = -\nabla\phi - \partial_\xi \boldsymbol{A}$, $\boldsymbol{B} = \nabla \times \boldsymbol{A}$, follows

$$\boldsymbol{E}_\perp = -\nabla_\perp \psi - \hat{\boldsymbol{z}} \times \boldsymbol{B}_\perp, \qquad E_z = \partial_\xi \psi$$

where $\psi = \phi - A_z$, $\nabla = (\partial_x, \partial_y, -\partial_\xi)$, $\hat{\boldsymbol{z}}$ is the unit vector in the z-direction, and the subscript $_\perp$ refers to the vector components perpendicular to the z axis.
Manipulations with linearized plasma and Maxwell's equations yield

$$\partial_{\xi\xi} n_1 + n_p n_1 = -n_p n_b + \boldsymbol{E}_\perp \cdot (\nabla_\perp n_p)$$
$$\Delta_\perp \psi - n_p \psi = n_1$$
$$\Delta_\perp \boldsymbol{B}_\perp = \hat{\boldsymbol{z}} \times \nabla_\perp (n_p \psi - n_b) - \hat{\boldsymbol{z}} \times \partial_\xi (n_p \boldsymbol{v}_\perp)$$

where n_1 is the plasma density perturbation, n_b is the beam density. They constitute a full set of equations for the three unknowns n_1, ψ, and $\boldsymbol{B}_\perp$.

Zeroth-order approximation in α

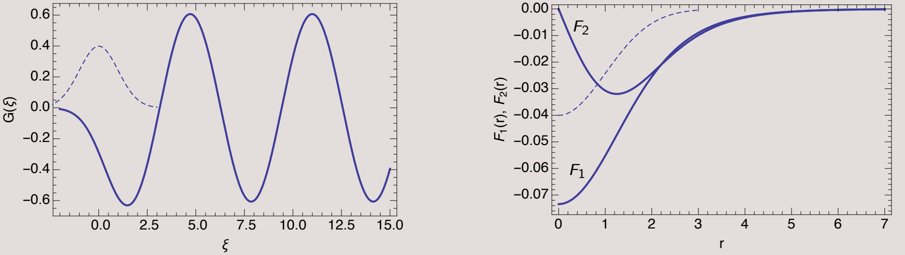
This corresponds to $\alpha = 0$, which means a uniform plasma. This is the problem formulated and solved in the original pioneering papers on PWFA[4]. In this approximation, there is no transverse force acting on a relativistic particle on the axis of the system. The magnetic field has the only component B_θ that together with n_1 and ψ depend on r and ξ. The solution is given by the following equations,

$$n_1^{(0)} = n_{b0} f(r) G(\xi),$$
$$\psi^{(0)}(r,\xi) = n_{b0} F_1(r) G(\xi),$$
$$B_\theta^{(0)}(r,\xi) = n_{b0} F_2(r) f(\xi),$$

[4] P. Chen et al, PRL **54**, 693 (1985); P. Chen et al, Plasma Science, IEEE Transactions on **15**, 218 (1987).

Zeroth-order approximation

Plots of functions G, F_1 and F_2 for a Gaussian driver with $\sigma_z = \sigma_r = 1$ ($\sigma_r = \sigma_z = k_p^{-1}$ in dimensional units).



First order approximation in gradient

In the next, linear approximation in α, the density perturbation n_1 and ψ is a sum of the zero-order terms found above and corrections δn_1 and $\delta\psi$ due to the nonuniformity of the plasma density:

$$n_1 = n_1^{(0)} + \delta n_1, \qquad \psi = \psi^{(0)} + \delta\psi$$

The transverse magnetic force on-axis $\boldsymbol{F}_\perp = \nabla_\perp \delta\psi|_{r=0}$ is directed along $\hat{\boldsymbol{x}}$. Calculations give

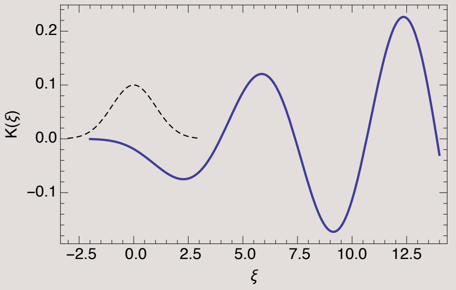
$$F_{\perp x} = n_{b0} \alpha K(\xi)$$

Transverse force behind the driver

Gaussian driver with $\sigma_z = \sigma_r = 1$,

$$n_b(r,\xi) = \frac{n_{b0}}{(2\pi)^{3/2}} \exp\left(-\frac{\xi^2 + r^2}{2}\right)$$

Function $K(\xi)$ oscillates around zero with the oscillation amplitude linearly growing with ξ.



The physical mechanism behind this growth is the resonant excitation of the angular dependent density perturbation δn_1 by the axisymmetric component of the plasma waves in the wake through the density gradient coupling. In our numerical estimate we take $K \approx -0.17$ at the second minimum at $\xi \approx 9.1$ as a representative estimate of K,

$$|F_{\perp x}| \approx 0.17 n_{b0} \alpha$$

Estimate of the transverse field

The transverse force is proportional to the beam density n_{b0}. Large beam density, however, would lead to a nonlinear plasma flow with relativistic velocities, which is beyond the range of applicability of our linear theory that assumes $n_b \ll 1$.

For a numerical estimate, choose $\alpha = 0.3$ and $n_{b0} = 3$ (the latter corresponds to the maximum beam density $n_b^{max} = 0.2$) which gives $F_{\perp x} = 0.15$. In dimensional units, this force corresponds to the effective magnetic field

$$B = \frac{F_{\perp x}}{e} = 0.15 \frac{mc\omega_p}{e^2}$$

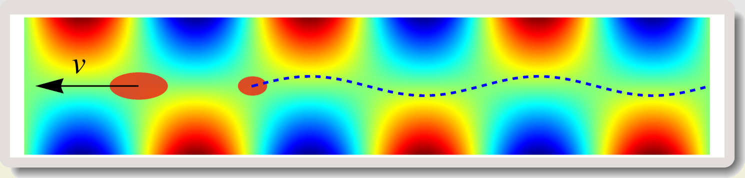
Choosing the plasma density $n_0 = 10^{17}\ \mathrm{cm}^{-3}$ (for which $k_p^{-1} = 17\ \mu\mathrm{m}$) we find $B = 15.5$ T. This fields scales with the plasma density as $\propto \omega_p \propto \sqrt{n_0}$, so the field is larger in a more dense plasma.

Note that

$$\frac{1}{n_p}\frac{dn_p}{dx} = \alpha k_p \approx \frac{1}{50\ \mu\mathrm{m}}$$

Undulator

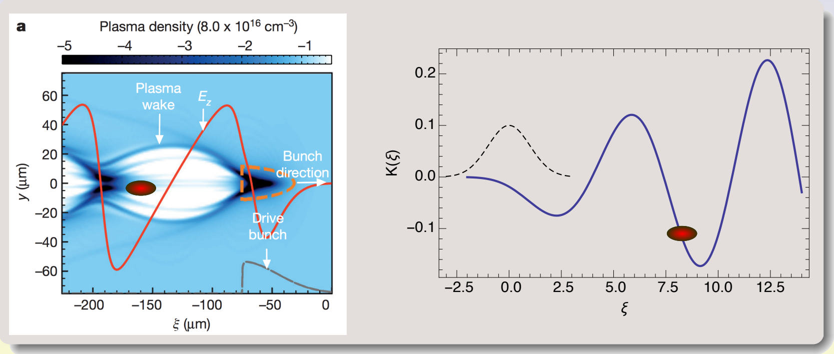
Our results are also valid when α slowly varies along z on the scale that is large compared to k_p^{-1}. For a sinusoidally varying $\alpha(z) = \alpha_0 \sin k_u z$, with the period $\lambda_u = 2\pi/k_u$, we obtain an undulator with linear polarization. Such an undulator field with a period of $\lambda_u = 1$ mm and the estimated above the magnetic field of 15.5 T has the undulator parameter $K = eB/k_u mc^2$ equal to 1.45.



With a three dimensional control of the plasma gradient one can generate a helical undulator: $\boldsymbol{\alpha} = \alpha_0(\hat{\boldsymbol{x}} \sin k_u z + \hat{\boldsymbol{y}} \cos k_u z)$.

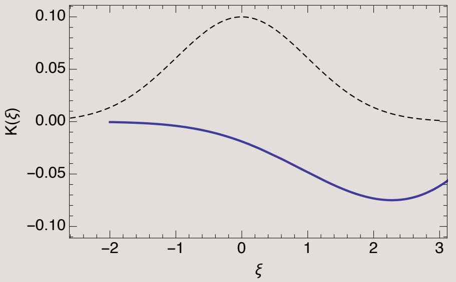
Discussion

In general, particles in the head and the tail of the witness bunch will experience different undulator fields. For an FEL application, this property of the plasma undulator can be utilized for the compensation of the energy chirp in the witness bunch which is typical for plasma acceleration (a chirp in γ in the beam is compensated by a chirp in K of the undulator).



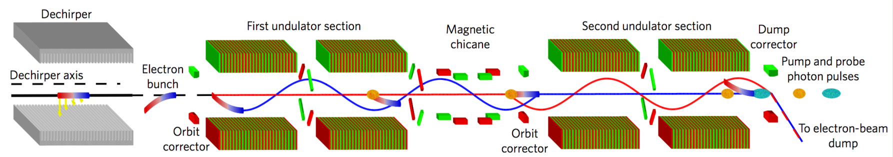
Effective transverse wake

An electron beam propagating through a plasma with transverse gradient will have its tail particles deflected. This acts as an effective transverse wake.

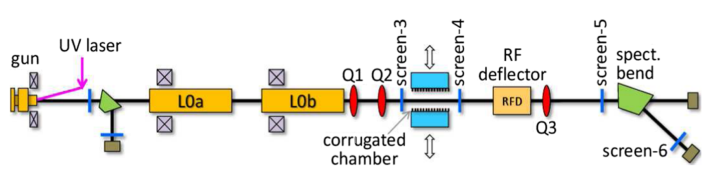


Plasma as transverse wake deflector?

A transverse wakefield deflector (based on a corrugated structure—“dechirper”) was used at LCLS to generate two-color x-rays[5].



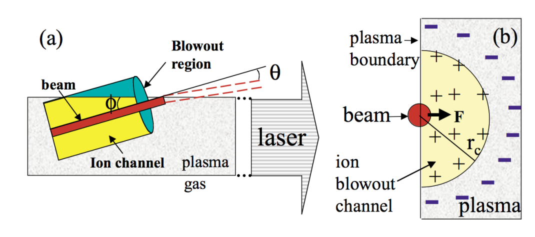
Transverse wake can also be used for bunch length measurement[6]



[5] A. Lutman et al. Nature Photonics **10**, 745 (2016)
[6] Jimin Seok et al. PRAB, **21**, 022801 (2018).

FACET experiment

An effect of similar nature has been observed in the experiment[7], where a 28.5 GeV electron beam was refracted/reflected from the boundary of the plasma channel. In our interpretation, such a boundary can be considered as an extreme case of the density gradient, in which the plasma radial profile can be approximated by a step function.



[7] P. Muggli et al. PRSTAB **4**, 091301 (2001).

Summary/Issues

Summary

- Plasma has a potential to support strong magnetic fields, $B \sim 100$ T (for $n = 10^{17}\ \mathrm{cm}^{-3}$).
- A considerable fraction of this field can be generated in the transverse direction in a plasma with a transverse density gradient. With an alternating direction of the gradient this makes a short-period plasma undulator.
- The effective transverse wake inside such plasma can also find other applications in modern FELS replacing multi-meter long transverse deflectors based on corrugated structures.

Issues:

- How to generate an alternating transverse gradient? A combination of nozzles and lateral jet-blades?
- Further analysis is needed of complicated 3D beam dynamics of two bunches propagating in such plasma with account of longitudinal and transverse fields.