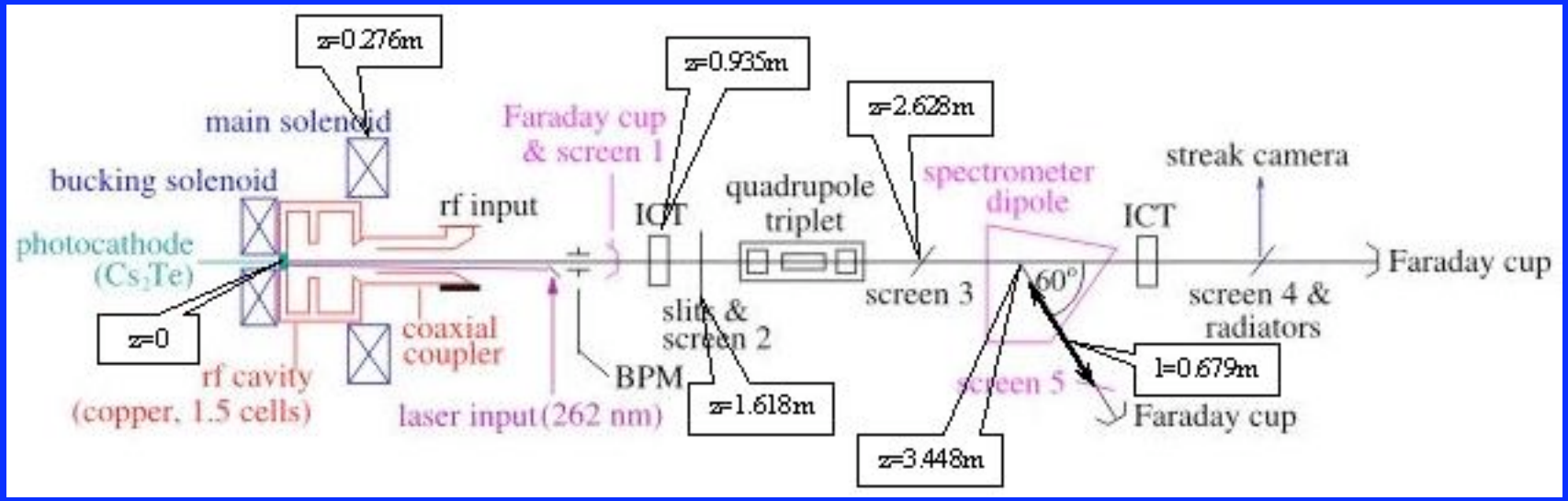
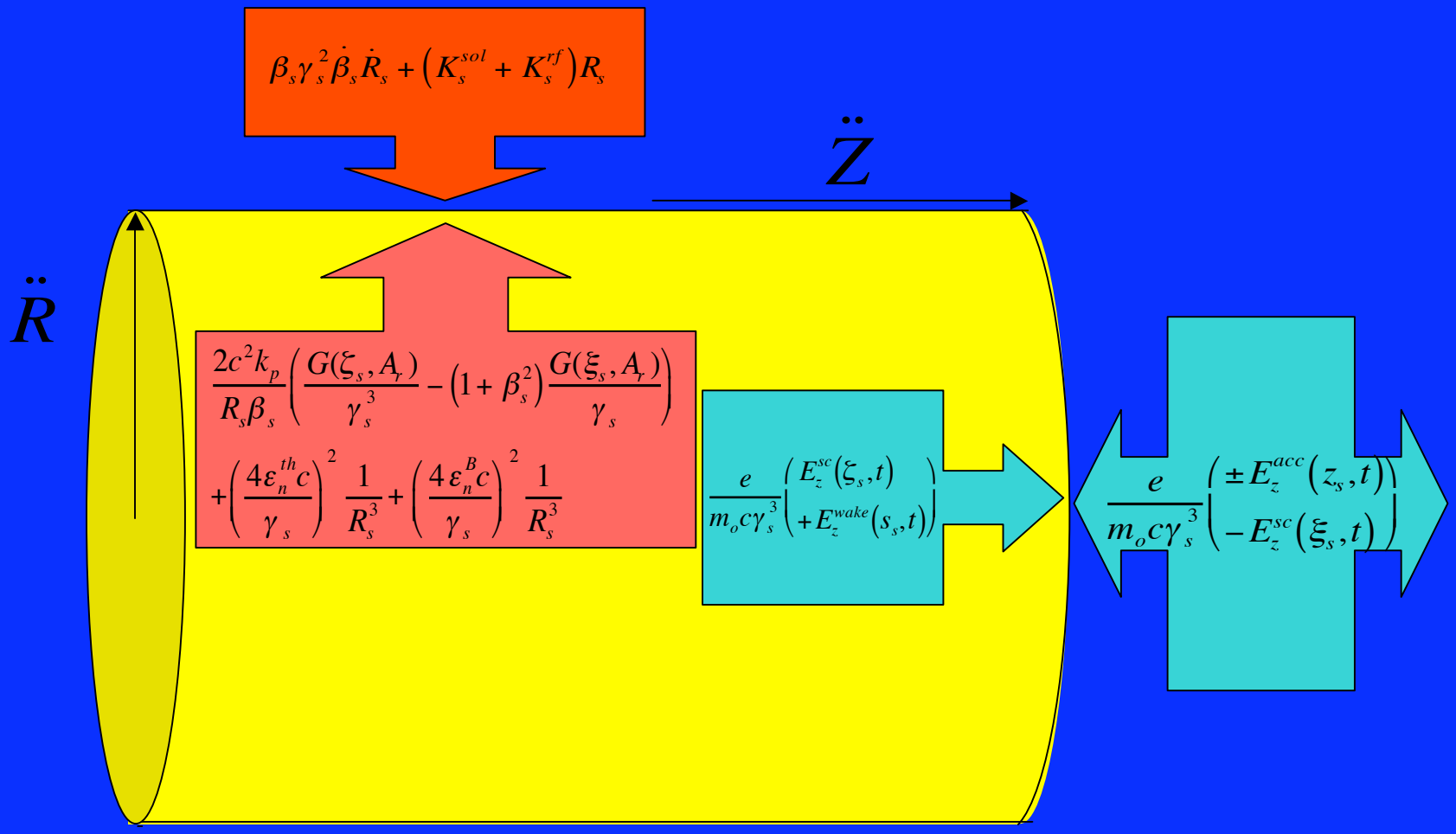
HOMDYN HOMEWORKS

Massimo Ferrario

INFN-LNF



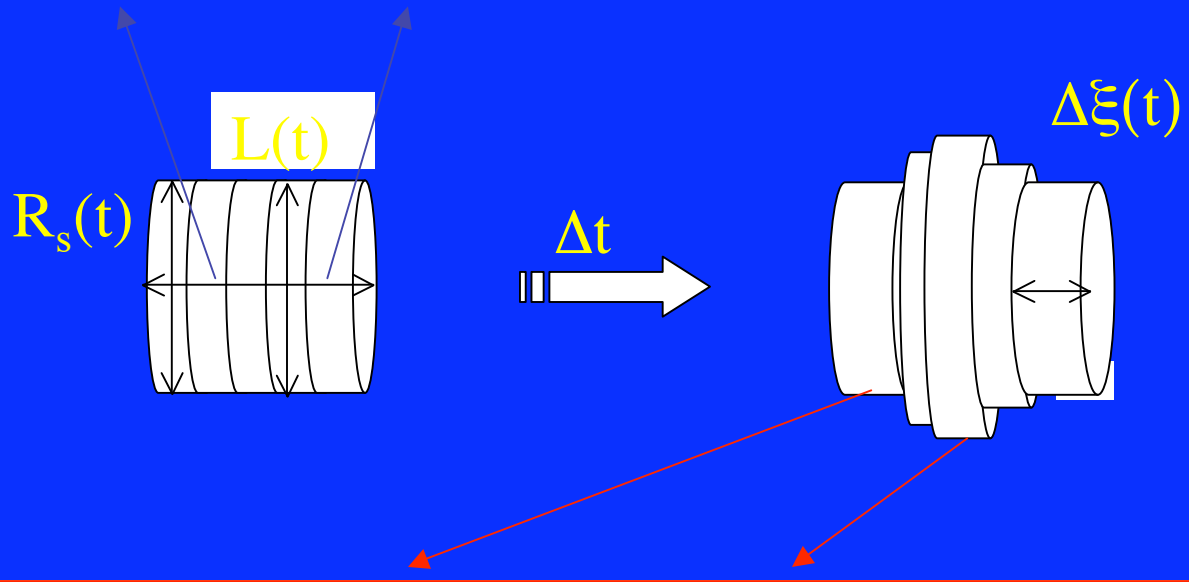
Artistic view of the Envelope Equations



Multi-Slice approximation and Envelope Equations:

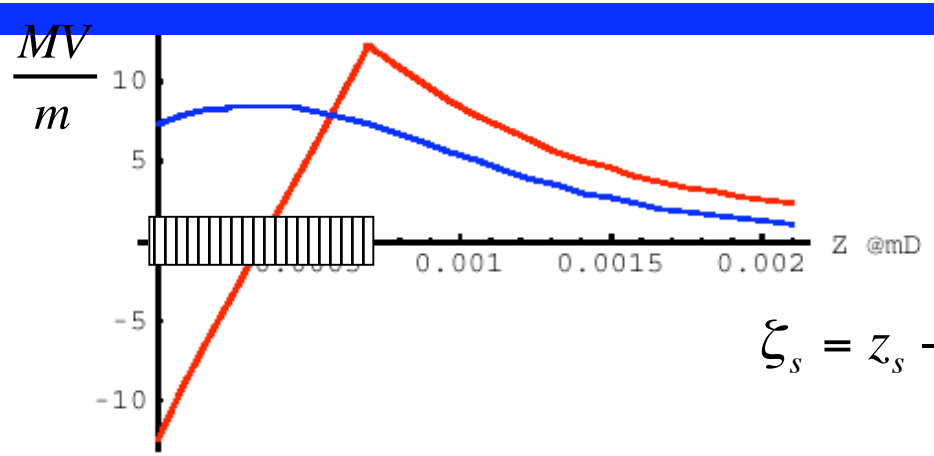
$$\dot{z}_s = c\beta_s$$

$$\dot{\beta}_s = \frac{e}{m_o c \gamma_s^3}\left(E_z^{acc}(z_s,t) + E_z^{wake}(s_s,t) + E_z^{sc}(\zeta_s,t) - E_z^{sc}(\xi_s,t)\right)$$



$$\ddot{R}_s + \beta_s\gamma_s^2\dot{\beta}_s\dot{R}_s + \left(K_s^{sol} + K_s^{rf}\right)R_s = \frac{2c^2k_p}{R_s\beta_s}\left(\frac{G(\zeta_s,A_r)}{\gamma_s^3} - \left(1+\beta_s^2\right)\frac{G(\xi_s,A_r)}{\gamma_s}\right) + \left(\frac{4\varepsilon_n^{th}c}{\gamma_s}\right)^2\frac{1}{R_s^3} + \left(\frac{4\varepsilon_n^{B}c}{\gamma_s}\right)^2\frac{1}{R_s^3}$$

Space Charge Field:



$$\zeta_s = z_s - z_t$$

FIG. 2: Longitudinal (red line) and Transverse(blue line) space charge field in a bunch just extracted from the cathode, image charges switched off.

Aspect Ratio: $A_{r,s} \equiv R_s / (\gamma_s L)$

Longitudinal Component (on axis)

$$E_z^{sc}(\zeta_s) = \frac{Q}{2\pi\varepsilon_o R_s^2}\left(\sqrt{(1-\zeta_s/L)^2 + A_{r,s}^2} - \sqrt{(\zeta_s/L)^2 + A_{r,s}^2} - |1-\zeta_s/L| + |\zeta_s/L|\right)$$

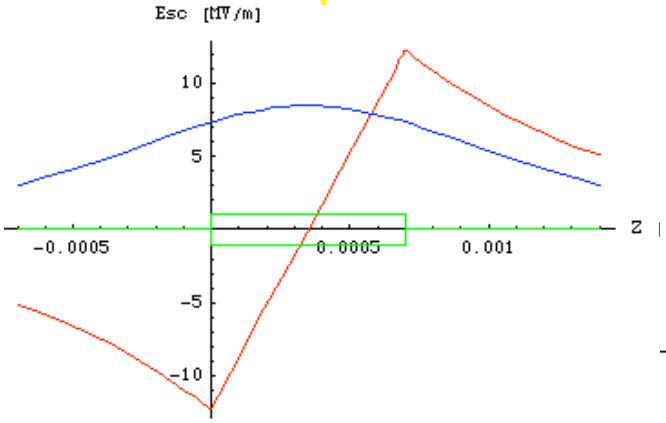
Radial Component (on envelope)

$$E_r^{sc}(r,z) = \left(\frac{\rho}{\varepsilon_o} - \frac{dE_z(r=0,z)}{dz}\right)\frac{R}{2}$$

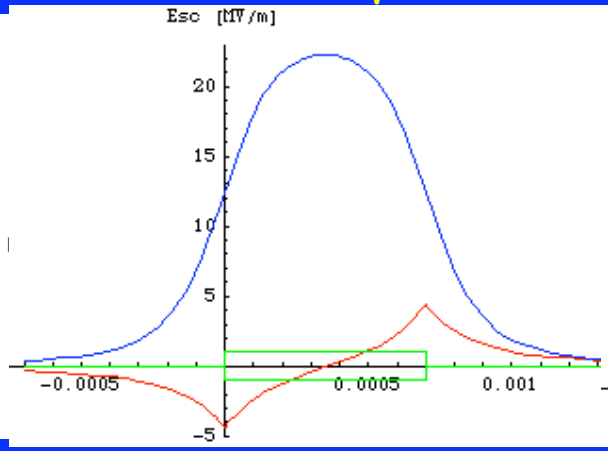
$$E_r^{sc}(\zeta_s) = \frac{Q}{4\pi\varepsilon_o R_s L}\left(\frac{1-\zeta_s/L}{\sqrt{(1-\zeta_s/L)^2 + A_{r,s}^2}} + \frac{\zeta_s/L}{\sqrt{(\zeta_s/L)^2 + A_{r,s}^2}}\right) = \frac{Q}{4\pi\varepsilon_o R_s L} G(\zeta_s, A_{r,s})$$

$$E_r^{sc}(\zeta_s) = \frac{Q}{4\pi\varepsilon_o R_s L}\left(\frac{1-\zeta_s/L}{\sqrt{(1-\zeta_s/L)^2 + A_{r,s}^2}} + \frac{\zeta_s/L}{\sqrt{(\zeta_s/L)^2 + A_{r,s}^2}}\right) = \frac{Q}{4\pi\varepsilon_o R_s L} G(\zeta_s, A_{r,s})$$

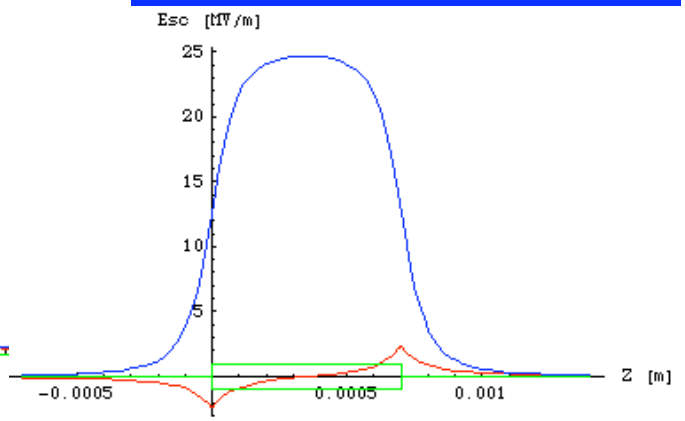
γ= 1



γ = 5

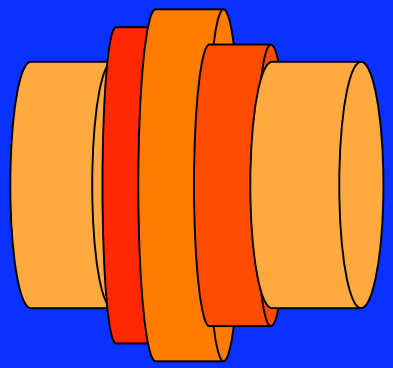


γ = 10

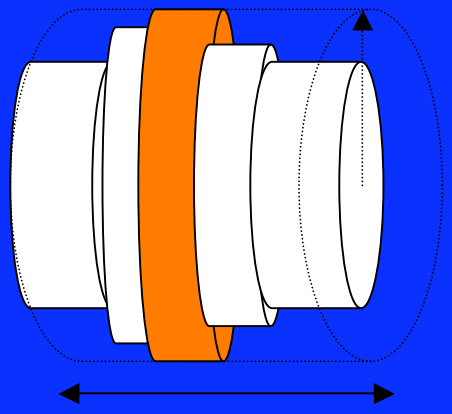


$$A_{r,s} \equiv R_s/(\gamma_s L)$$

Rs

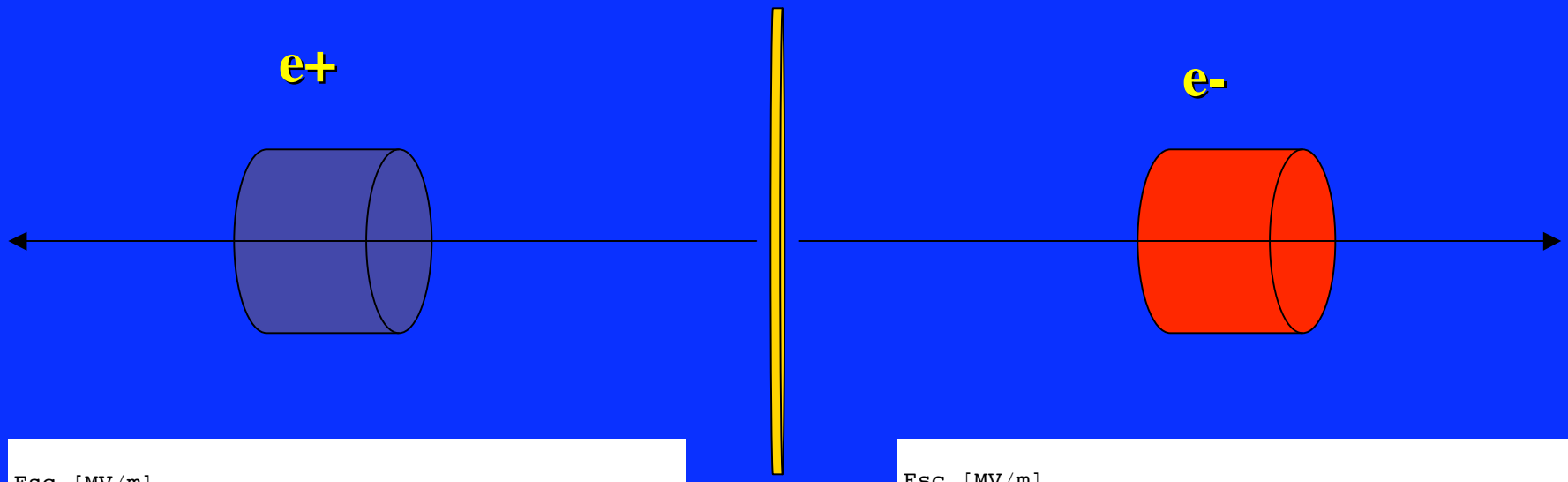


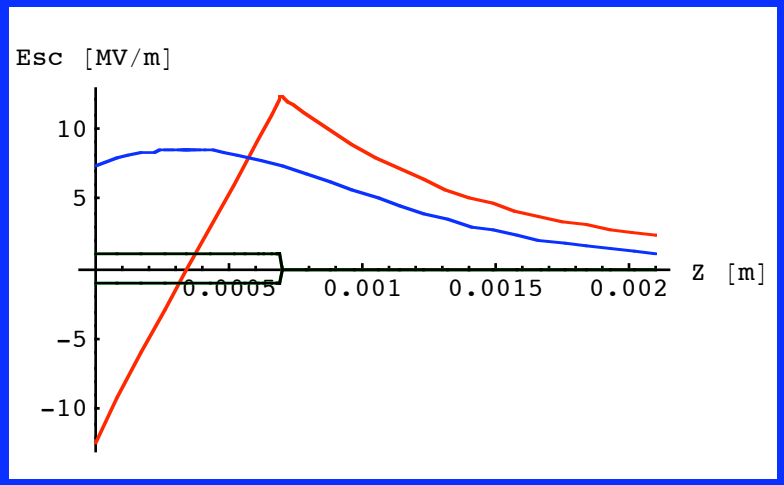
Rs



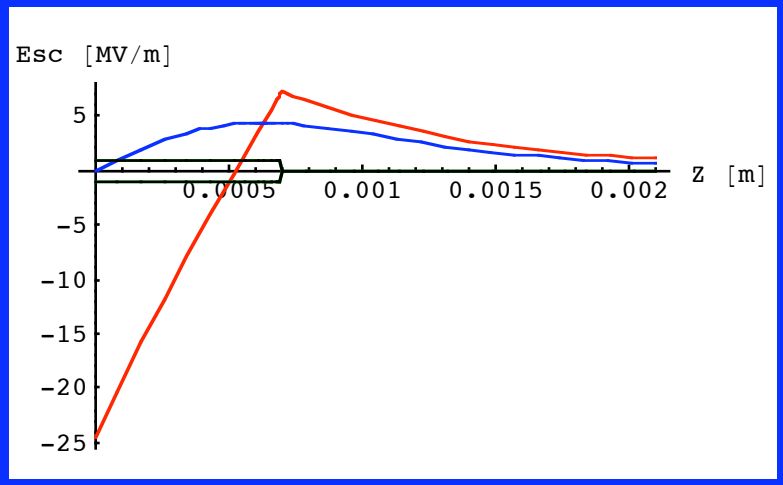
L

Beam Generation and Cathode Image Charges:

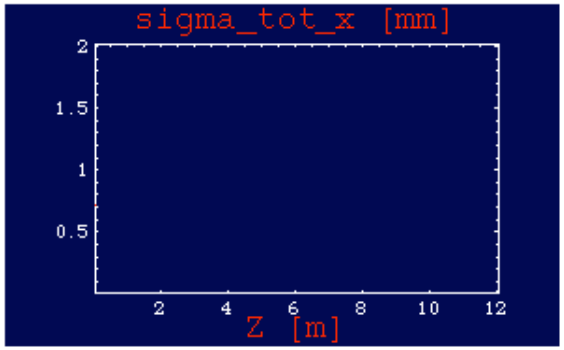


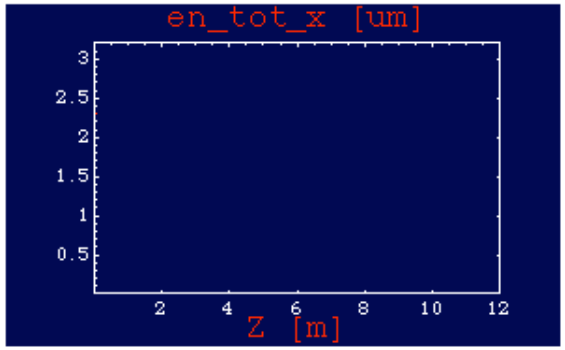


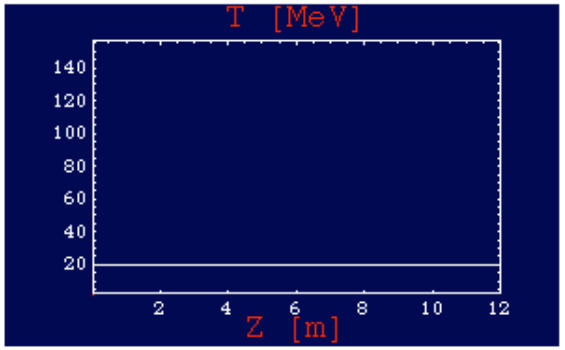
No Image Charges

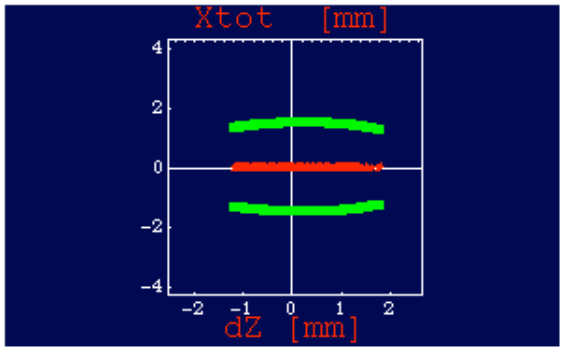


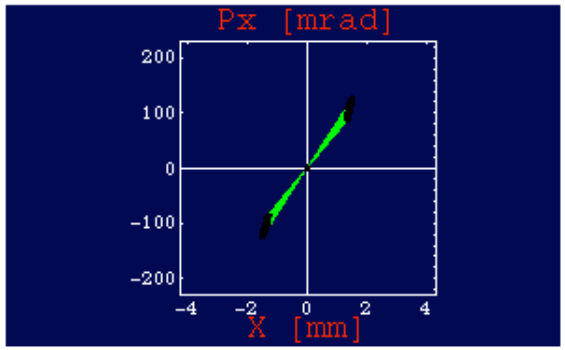
With Images Charges

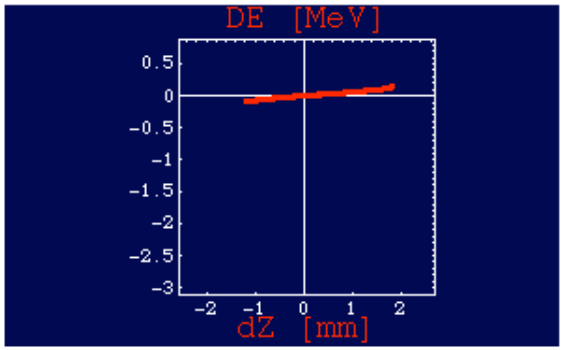

sigma_tot_x [mm]
Z [m]

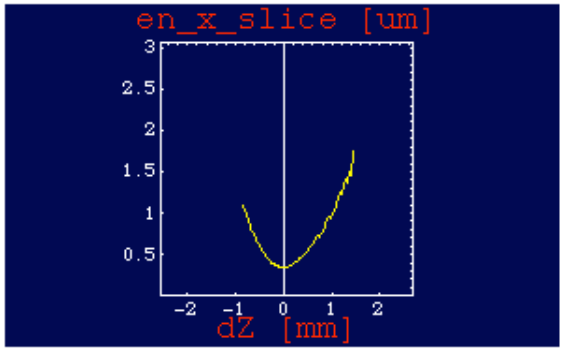

en_tot_x [um]
Z [m]

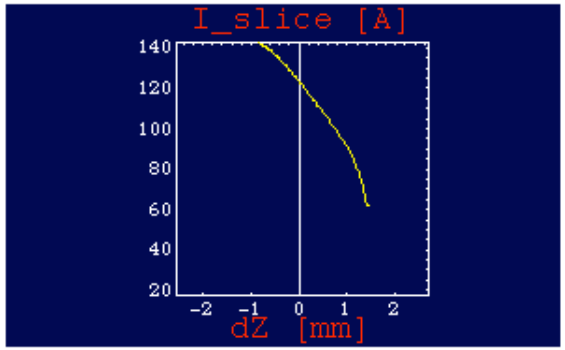

T [MeV]
Z [m]

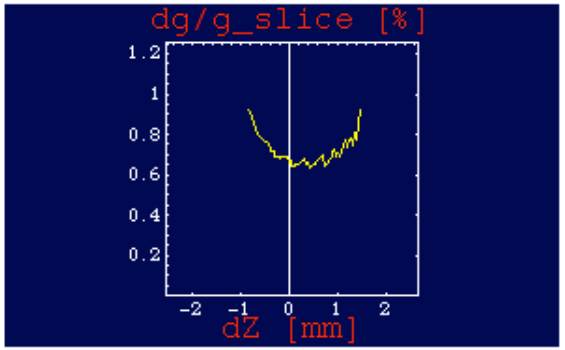

Xtot [mm]
dZ [mm]


Px [mrad]
X [mm]

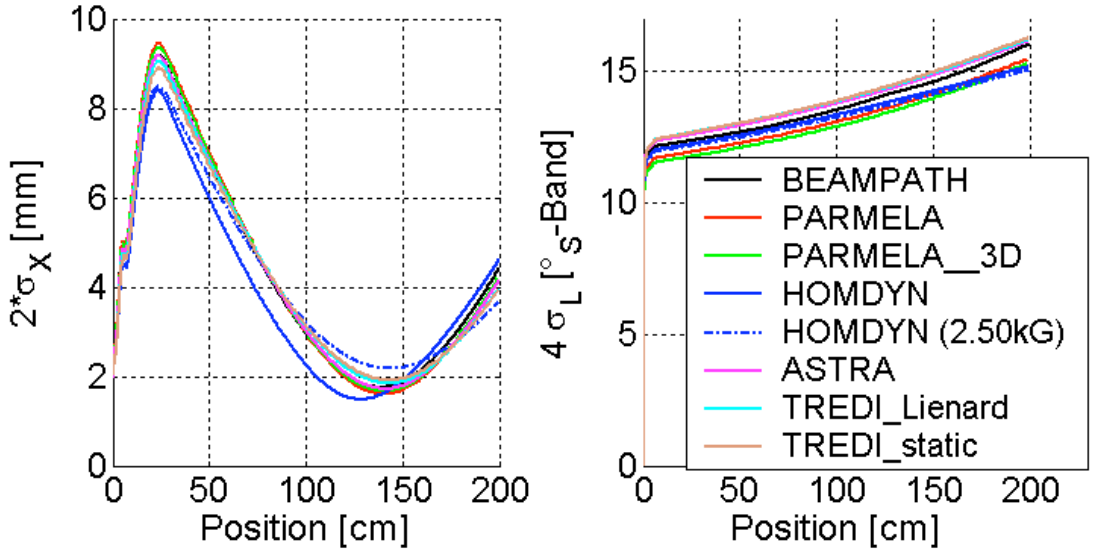

DE [MeV]
dZ [mm]

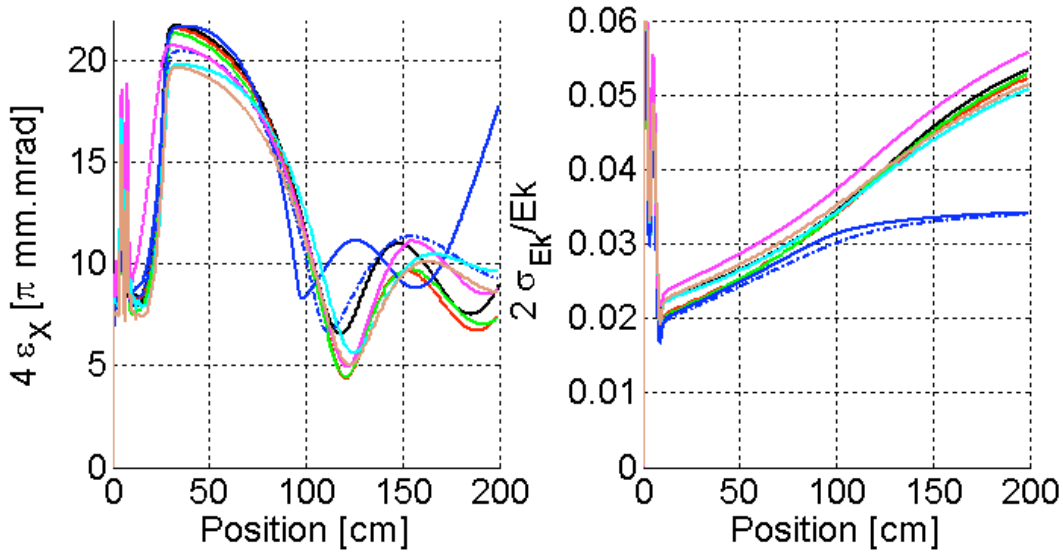

en_x_slice [um]
dZ [mm]


I_slice [A]
dZ [mm]

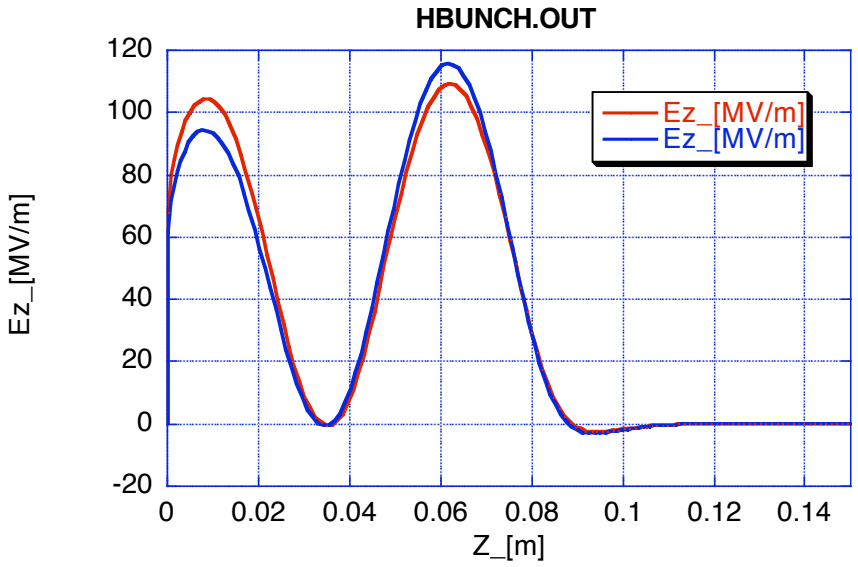

dg/g_slice [%]
dZ [mm]

Gun + solenoid + drift : comparison of codes with space charge





π and π/3 modes excitation



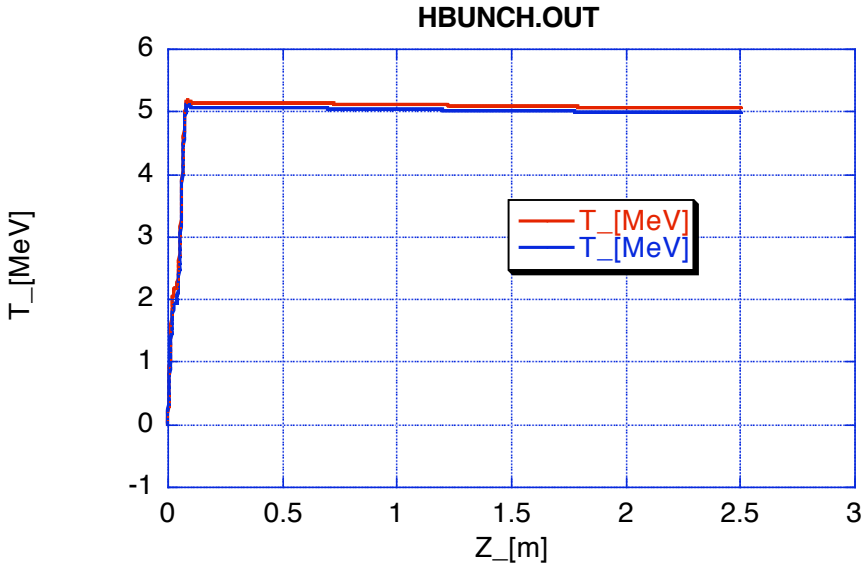
Q = 1 nC $\varepsilon_{th} = 0.6\ \mu m$

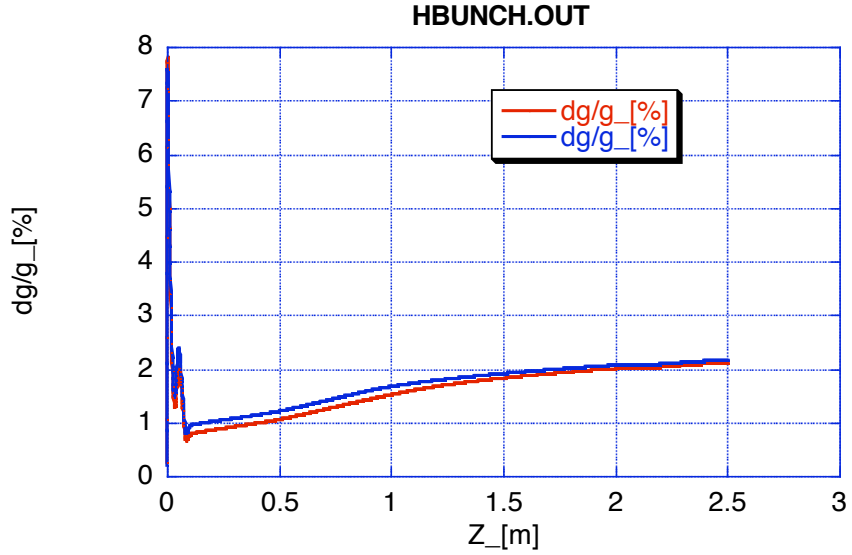
$E_{acc} = 110$ MV/m $\phi = 30^{o}$ ν=2856.0 MHz

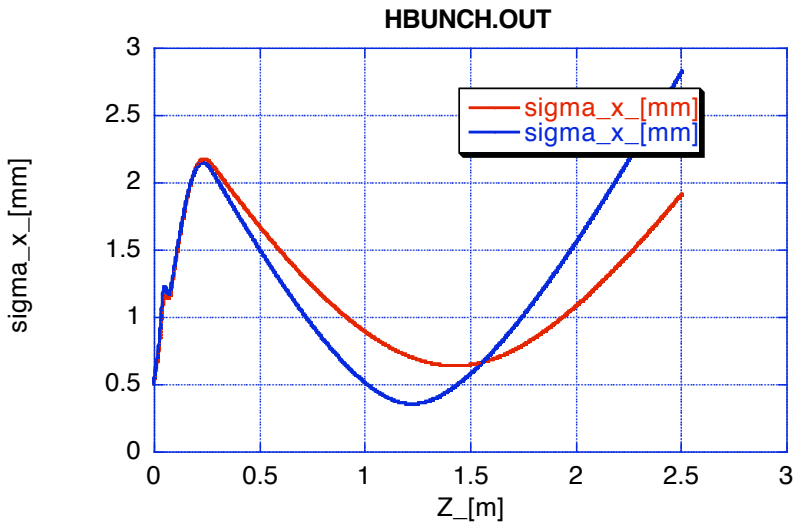
$E_{acc} = 11$ MV/m $\phi = -60^{o}$ ν=2856.0 MHz

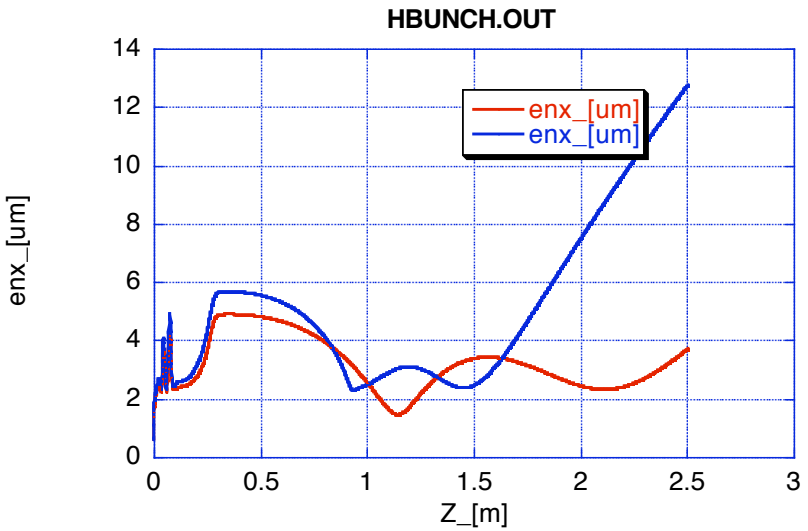
B=0.25 T

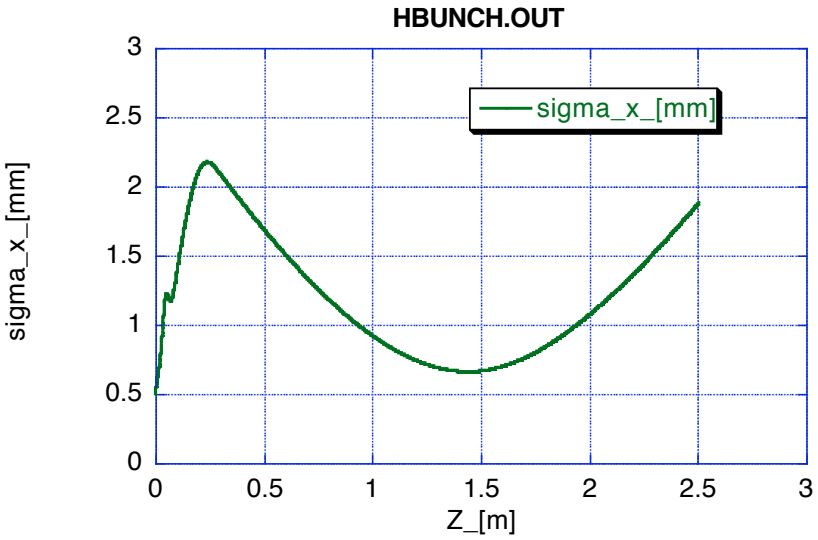
$\sigma_x = 0.5$ mm $\sigma_z = 2.9$ ps

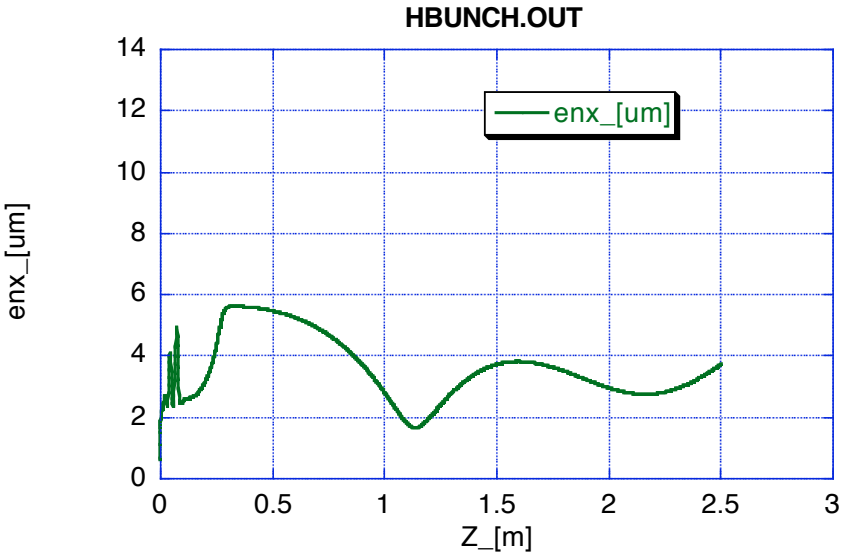




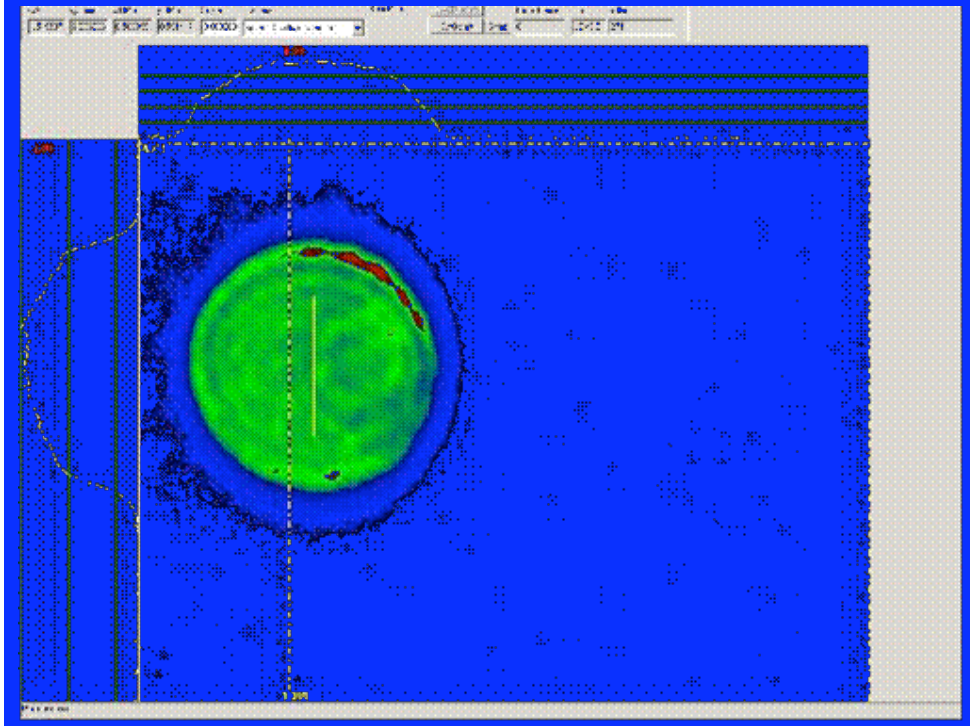


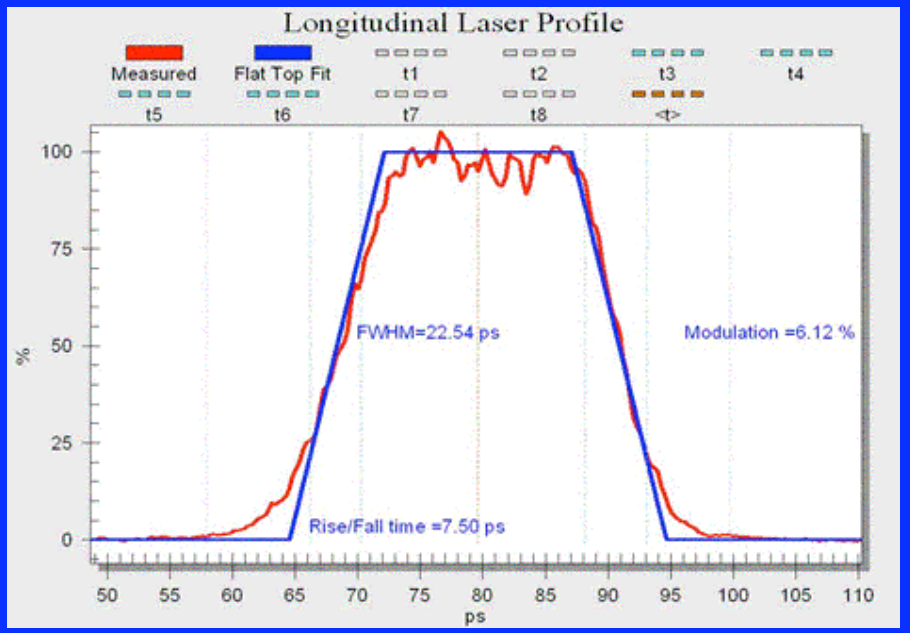






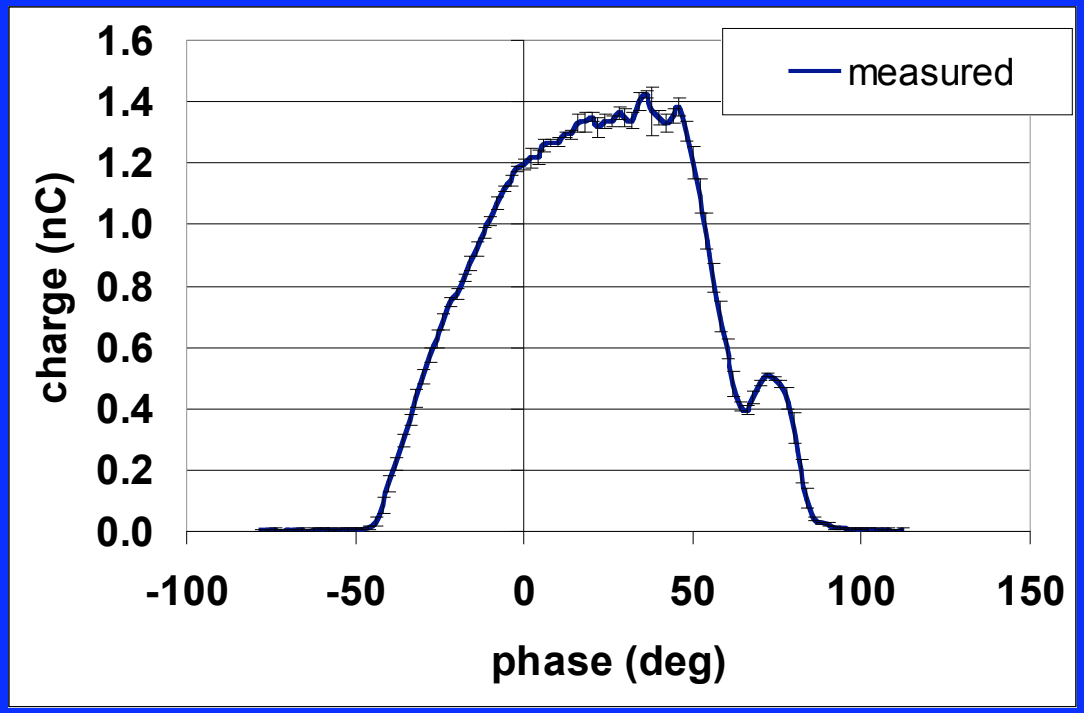
equivalent to B = 0.243 T

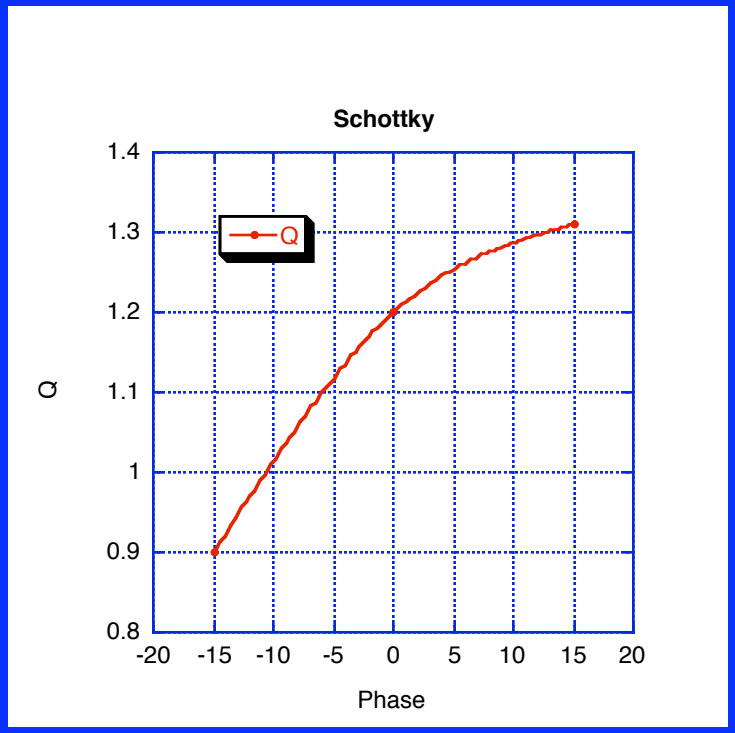


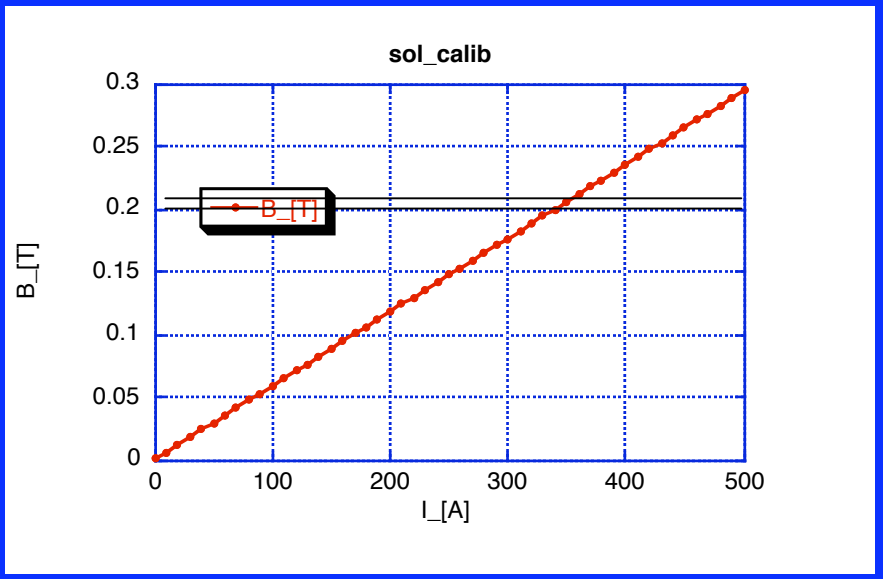


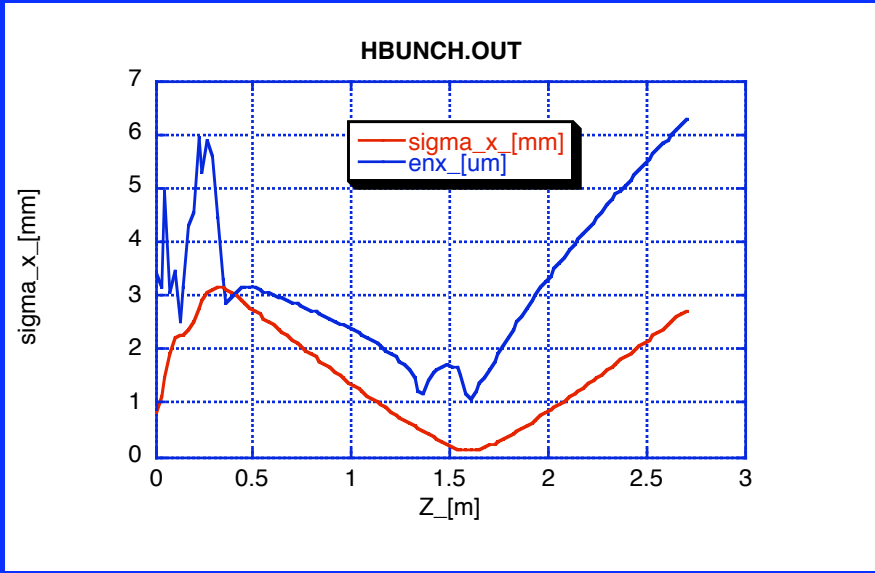
L = 23 ps

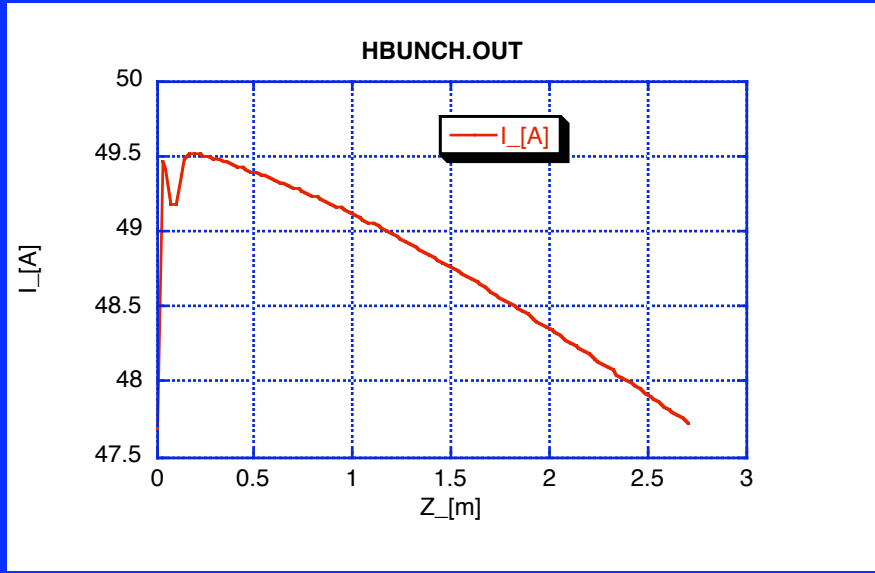
σ_x=0.55 mm ==> X=1.1 mm

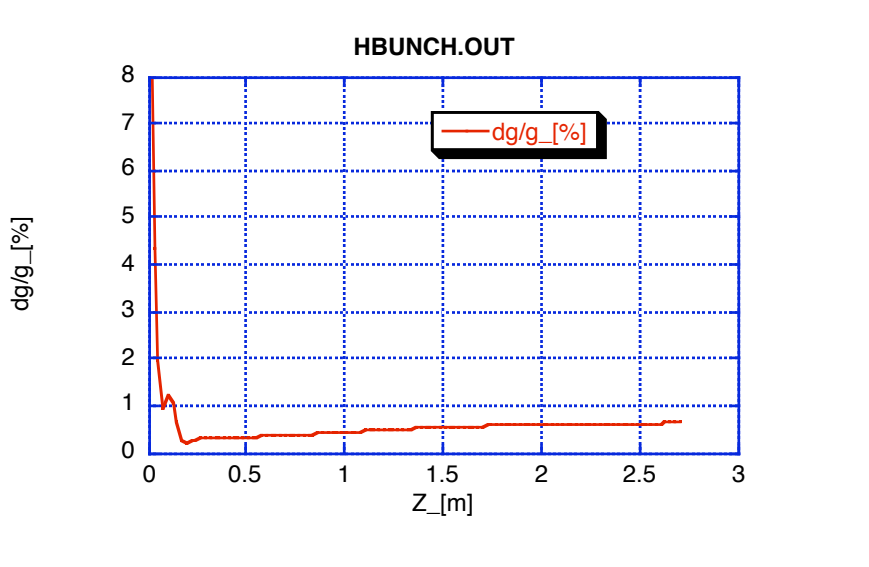

measured
charge (nC)
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-100
-50
0
50
100
150
phase (deg)


Schottky
Q
1.4
1.3
1.2
1.1
1
0.9
0.8
-20
-15
-10
-5
0
5
10
15
20
Phase


sol_calib
B_[T]
0.3
0.25
0.2
0.15
0.1
0.05
0
0
100
200
300
400
500
I_[A]

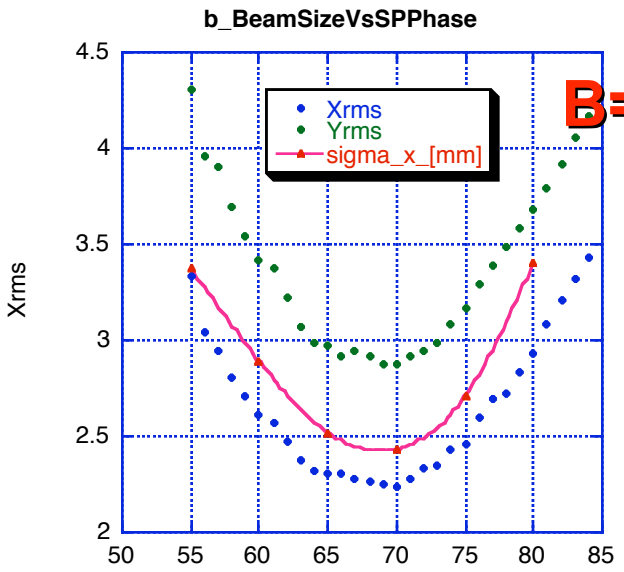

HBUNCH.OUT
sigma_x_[mm]
enx_[um]
sigma_x_[mm]
Z_[m]
0
1
2
3
4
5
6
7
0
0.5
1
1.5
2
2.5
3

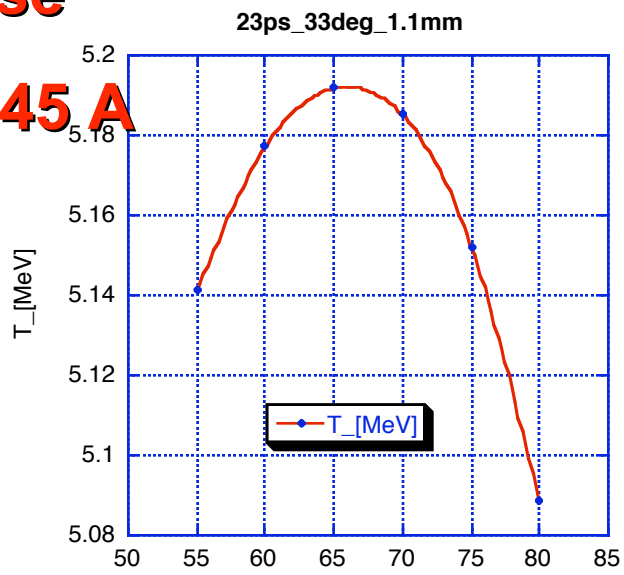

HBUNCH.OUT
I_[A]
I_[A]
Z_[m]
47.5
48
48.5
49
49.5
50
0
0.5
1
1.5
2
2.5
3

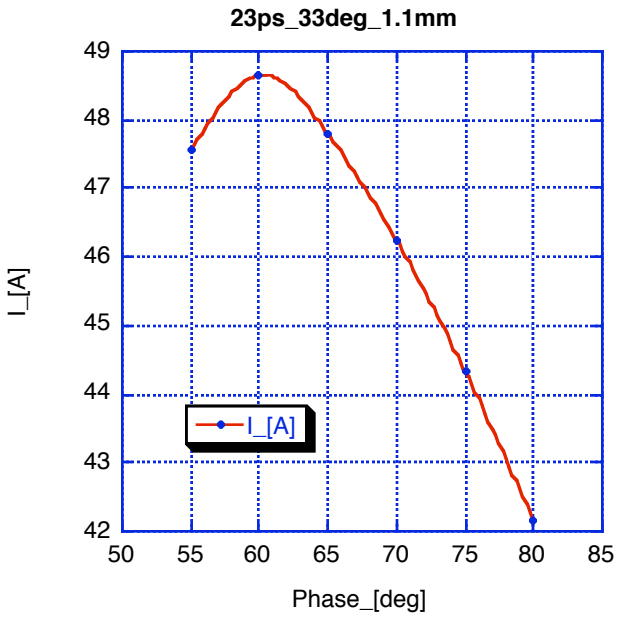

HBUNCH.OUT
dg/g_[%]
dg/g_[%]
Z_[m]
0
1
2
3
4
5
6
7
8
0
0.5
1
1.5
2
2.5
3

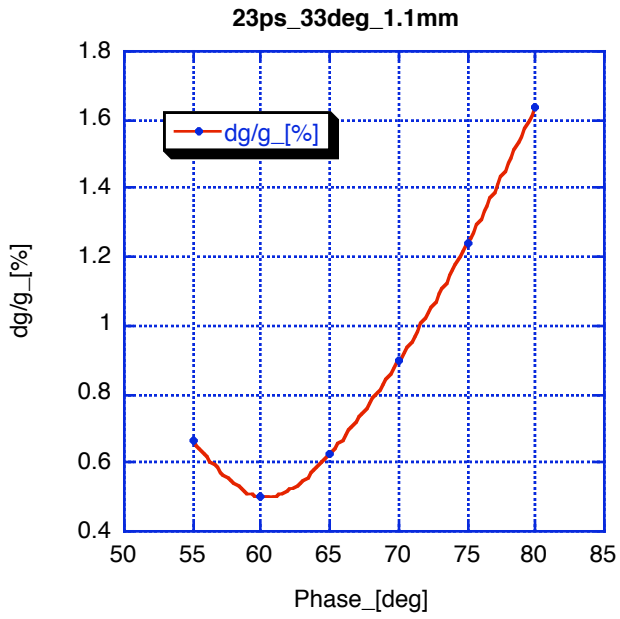
Reference Phase

B=0.2057 T ==> 345 A

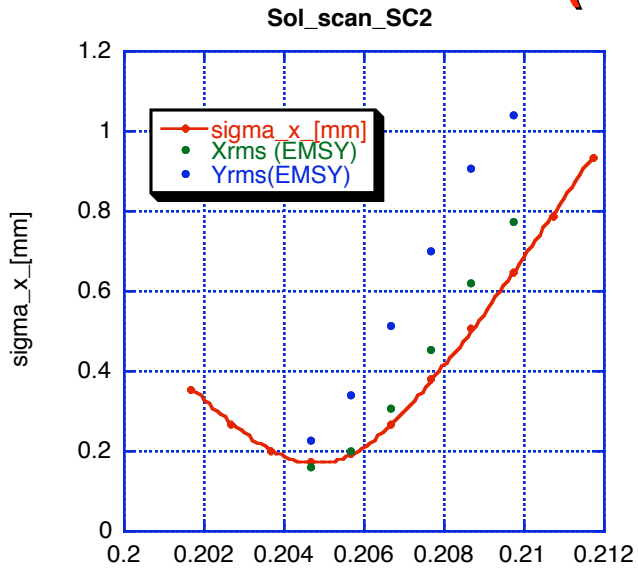


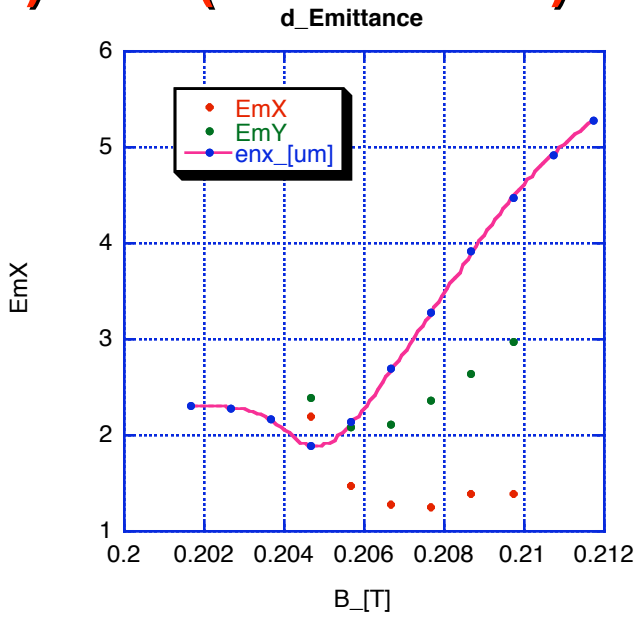


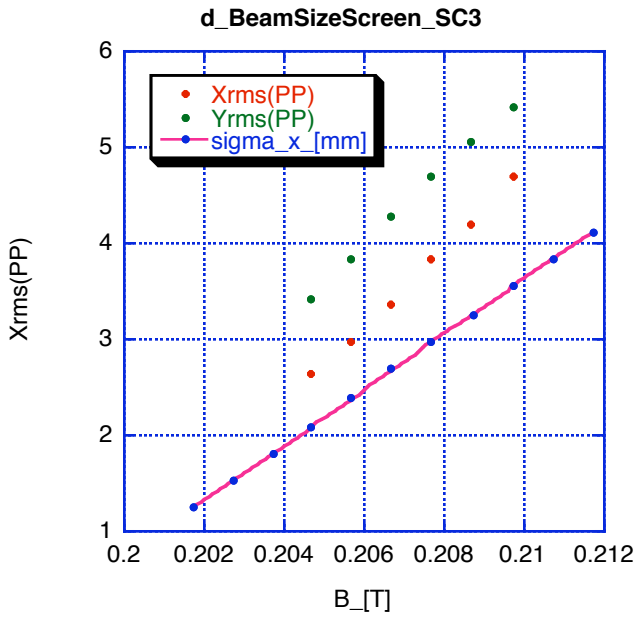




Solenoid scan (0.201 - 0.212 T) ==> (340 - 360 A)







Solenoid longitudinal displacement + 5mm