Argonne NATIONAL LABORATORY

Transverse-Longitudinal Phase Space Manipulations

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ANL and The University of Chicago

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Future Light Sources

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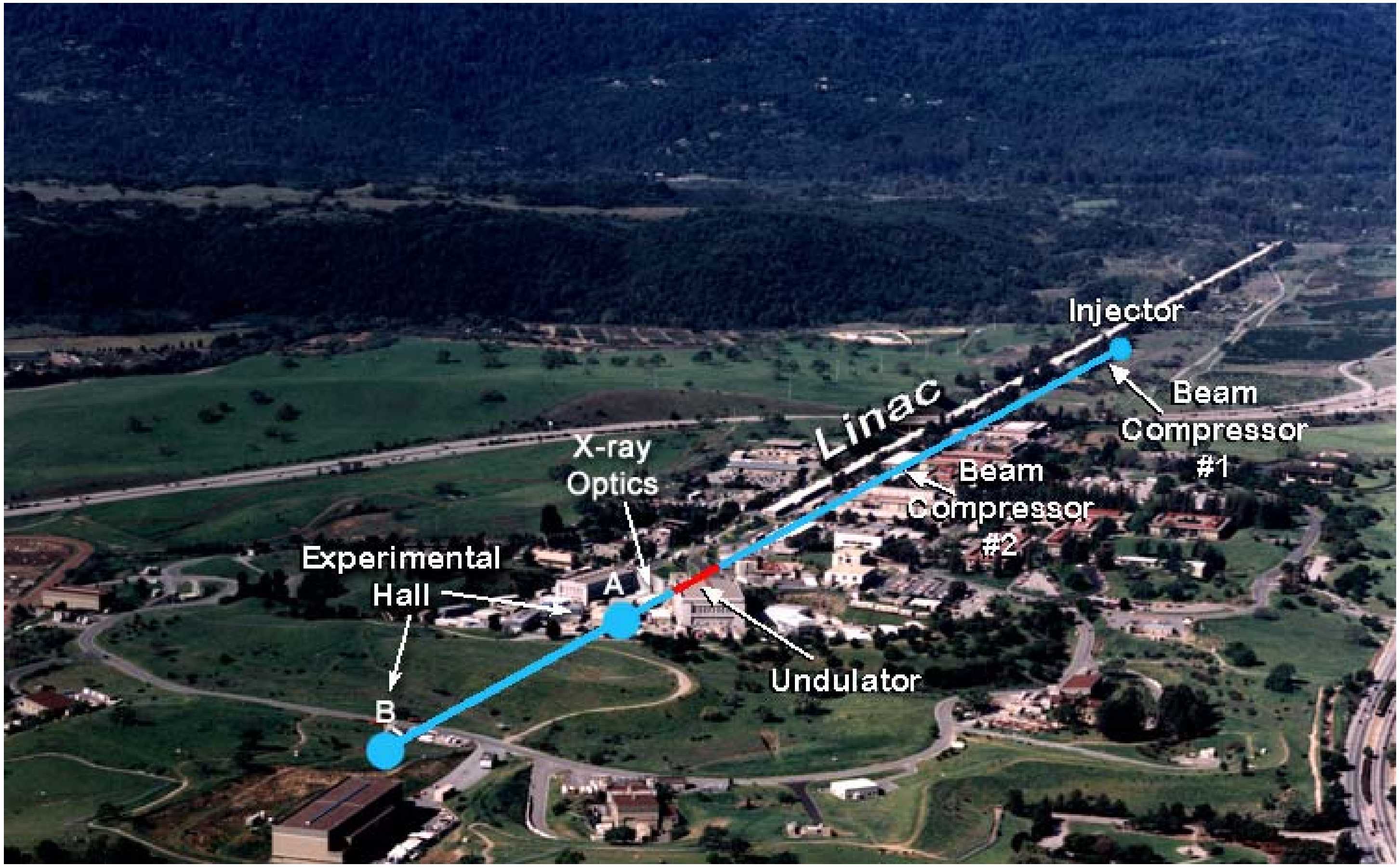
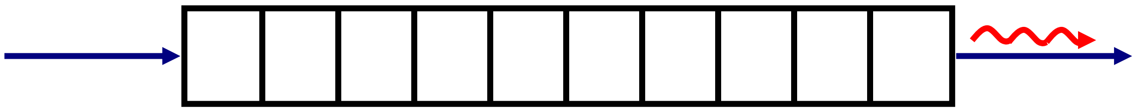
To Improve the Performance of X-Ray FELs



SASE FEL for 30 keV



- LCLS reference parameters:

  λ = 8 keV, λ_u = 3 cm, K = 3.7, I_p = 3.5 kA, E_e = 15 GeV,

  $\Delta E/E = 10^{-4}$, ε_n = 1.2 mm-mrad, L_{sat} = 100 m
- Vary K, ε_n, and E_e (Z.R. Huang)

K	**E_e (GeV)**	**ε_n (mm-mrad)**	**L sat (m)**	
3.7	**30**	**1.2**	**300**	
3.7	**30**	**0.5**	**130**	
3.7	**30**	**0.1**	**40**	← **shorter undulator**
1	**12**	**0.1**	**60**	← **shorter undulator and shorter linac**

- ***It pays to strive for an ultralow emittance e-beam***

Need for Phase Space Manipulation

- RF Photocathode Gun
  - Transverse emittance $\gamma\varepsilon_x, \gamma\varepsilon_y \sim 1 \times 10^{-6}$ m
  - Energy spread very small ~

    $\sigma_{\Delta E} \sim 1.5$ keV

$$\sigma_{\Delta E/E} = \frac{\sigma_{\Delta E}}{E} = 10^{-7} \text{ @ } \quad E \sim 15 \text{ GeV}$$

- FEL requires $\sigma_{\Delta E/E} < 10^{-4}$

- Can we do the transformation:

$$\left(\gamma\varepsilon_x, \gamma\varepsilon_y, \sigma_{\Delta E/E}\right) = \left(10^{-6} m, 10^{-6} m, 10^{-7}\right) \rightarrow \left(10^{-7} m, 10^{-7} m, 10^{-5}\right)?$$

Contents

- Some properties of Hamiltonian Transport
- Transverse-longitudinal *exchange* for x-ray FELs
- Implementation
  - Space charge effect
  - Flat beam technique
  - Optical system for exchange

Beam Transport and Manipulation

- **6D phase space: (x, x´, y, y´, z, δ)**
- **We will work mostly in 4D**
- **X** $= \begin{pmatrix} x \\ x' \\ z \\ \delta \end{pmatrix}$
- **Beam matrix:** $\sum = \langle x\tilde{x} \rangle = \begin{bmatrix} \langle x^2 \rangle & \langle xx' \rangle & \langle xz \rangle & \langle x\delta \rangle \\ \langle xx' \rangle & . & . & . \\ \langle xz \rangle & . & . & . \\ \langle x\delta \rangle & . & . & \langle \delta^2 \rangle \end{bmatrix}$
- **Transfer matrix: M**

  $X = M X_o$, $\Sigma = M \Sigma_o \tilde{M}$

Hamiltonian Transport

- **Unit symplectic matrix**

$$J = \begin{bmatrix} J_{2D} & 0 \\ 0 & J_{2D} \end{bmatrix}, \quad J_{2D} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

- **M is symplectic:** $\tilde{M}\, J\, M = J$

- ***Det M = 1***

- **Conserved quantities**

$$\varepsilon_{4D} = Det(\Sigma)$$

$$I^{(2)} = -\frac{1}{2} T_r(\Sigma\, J\, \Sigma)$$

- $\sum = \begin{bmatrix} \Sigma_x & \Sigma_c \\ \tilde{\Sigma}_c & \Sigma_z \end{bmatrix}$, $\Sigma_x, \ldots$ 2×2 submatrices

- Projected emittances

$$\varepsilon_x{}^2 = Det\,\Sigma_x, \quad \varepsilon_z{}^2 = Det(\Sigma_z)$$

- $I^{(2)} = \varepsilon_x{}^2 + \varepsilon_z{}^2 + 2\,Det\,(\Sigma_c)$

- Iff uncoupled; $\Sigma_c = 0$

$$Det(\Sigma) = \varepsilon_x{}^2 \varepsilon_z{}^2$$

$$I^{(2)} = \varepsilon_x{}^2 + \varepsilon_z{}^2$$

Emittance Exchange Theorem

(E. Courant, "Perspectives in Modern Physics, Essays in Honor of H.A. Bethe," Interscience Pub., 1966)

- **For transport from an uncoupled to another uncoupled system, ε_a and ε_b are uniquely determined up to switching**.

$$(\varepsilon_a, \varepsilon_b) \rightarrow (\varepsilon_a, \varepsilon_b) \text{ or } (\varepsilon_b, \varepsilon_a)$$

- **Proof**:

$$\varepsilon_{1a}^{2} + \varepsilon_{1b}^{2} = \varepsilon_{2a}^{2} + \varepsilon_{2b}^{2}$$

$$\varepsilon_{1a}^{2}\varepsilon_{1b}^{2} = \varepsilon_{2a}^{2}\varepsilon_{2b}^{2}$$

$$\therefore \quad QED$$

- **Can be generalized to higher dimensions**

An Emittance Switching Scheme for Improved X-Ray FEL Performance

(P. Emma, Z. Huang, P. Piot, and KJK, under preparation)

- **Flat beam technique (units in m-rad)**

$$\gamma\varepsilon_x \otimes \gamma\varepsilon_y : \quad \left(10^{-6}\right)^2 \rightarrow 10^{-5} \otimes 10^{-7}$$

- **Use short electron beam $\sigma_z = 33\ \mu$**

$$\gamma\varepsilon_z = \sigma_z\sigma_{\Delta\gamma} = 33\mu \otimes 3\times 10^{-3} = 10^{-7}\,m\ \left(\sigma_{\Delta\gamma} = 1.5\ \mathrm{keV/mc^2}\right)$$

Q = 33 pC, I = 100 A

- **Exchange (x $\leftrightarrow$ z)**

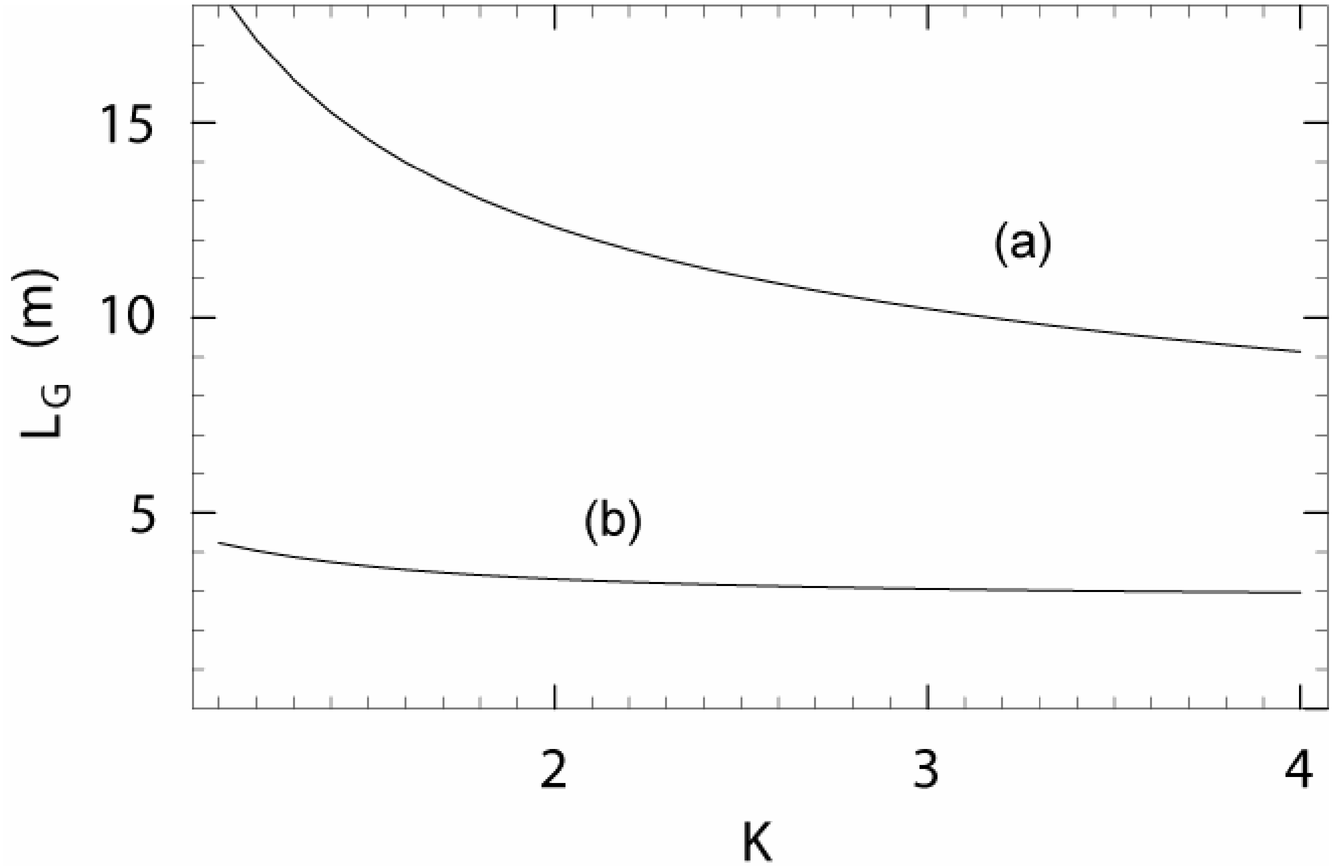
$$\gamma\varepsilon_x \otimes \gamma\varepsilon_y \otimes \gamma\varepsilon_z : \left(10^{-6},10^{-6},10^{-7}\right) \rightarrow \left(10^{-5},10^{-7},10^{-7}\right) \rightarrow \left(10^{-7},10^{-7},10^{-5}\right)$$

- **Final bunch length**

$$\gamma\varepsilon_z = \gamma\sigma_z\sigma_\delta = \gamma\sigma_z \times 10^{-4} = 10^{-5}, \quad \gamma = 3\times 10^4$$

$$\sigma_z = 3.3\ 10^{-6} \Rightarrow \textit{Compression}\ \text{by}\ 10$$

$$I = 100 \rightarrow 1000\ A$$



Power gain length *LG* of an x-ray FEL at 0.4 Å versus the undulator parameter *K* for (a) a beam with a normalized transverse emittance 1 × 10^{-6} m-r and a peak current 3.5 kA and (b) a beam with a normalized transverse emittance 1 × 10^{-7} m-r and a peak current 1 kA. The relative rms energy spread in both cases is 1 × 10^{-4} (courtesy of Z. Huang).

Is $(10^{-6}, 10^{-6}, 10^{-7})$m consistent with the space charge degradation?

- $$\gamma\varepsilon_i^{SC} = \frac{\pi}{4}\frac{1}{(\sin\phi_o)}\frac{2mc^2}{E_o}\mu_i(A), \quad A = \frac{\sigma_x}{\sigma_z}$$

  (KJK, NIM, A275, 201 (1989))

  $$\mu_{x,y} = \frac{1}{3A+5}, \quad \mu_z = \frac{1.1}{1+4.5A+2.9A^2}$$

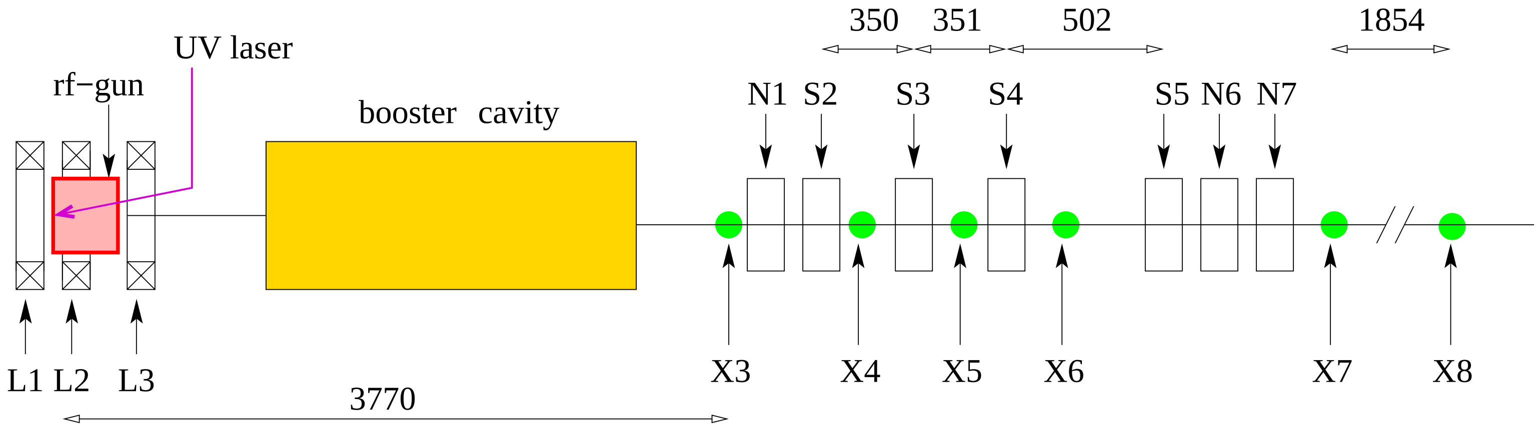
- Use spot size σ_x = 0.5 mm, σ_z = 33 μm, A ~ 18

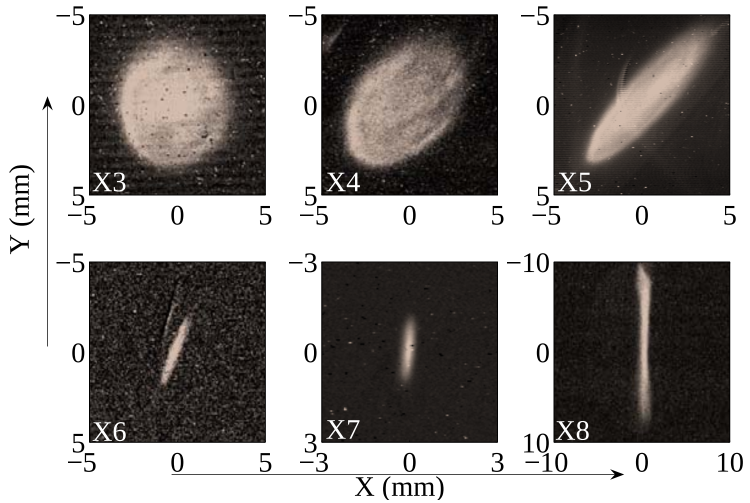
  E_o = 120 MV/m, I = 100 A, $\pi/4 \sin\phi_o \sim 1$

  $\rightarrow (10^{-6}, 10^{-6}, 10^{-7})$!

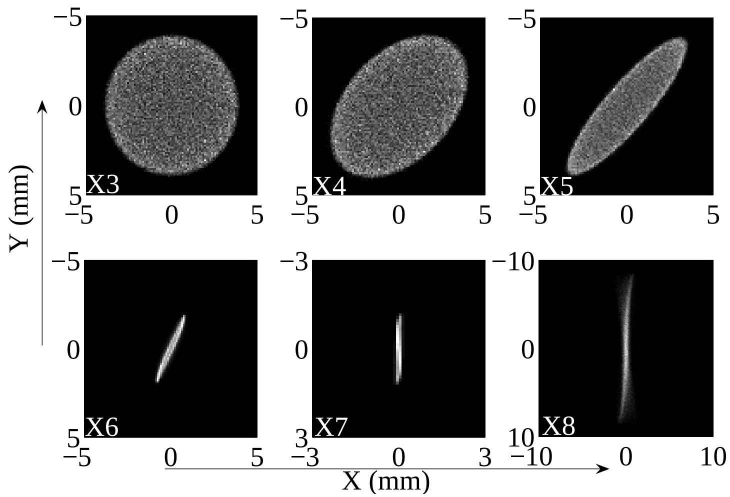
Generating a Flat Beam with Angular Mom. Dominated Beam

(Y. Derbenev), (R. Brinkmann, Y. Derbenev, K. Flöttmann), (D. Edwards …), (Y.-e Sun)



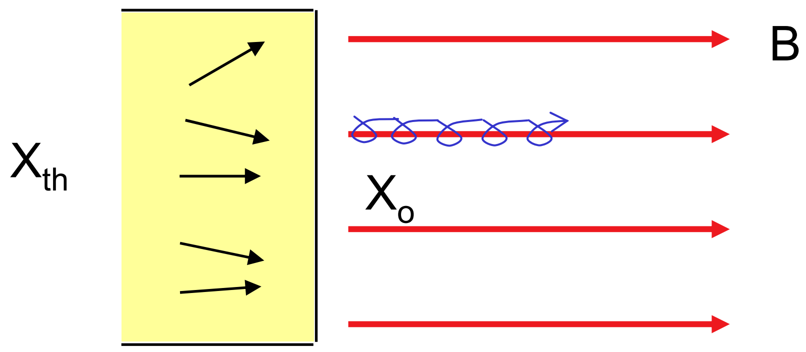


experiment



simulation

Electron Emission into Axial Magnetic Field



$$X_{th} = \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} \qquad X_o = \begin{pmatrix} x \\ x' - \kappa y \\ y \\ y' + \kappa x \end{pmatrix}, \kappa = \frac{qB}{2P_s}$$

Does Flat Beam Technique Violate the EET?

- $(\gamma\varepsilon_x, \gamma\varepsilon_y)$: $(10^{-6}, 10^{-6}) \rightarrow (10^{-5}, 10^{-7})$
- Thermal distribution before emission

$$\Sigma_{th} = \left\langle X_{th}\tilde{X}_{th} \right\rangle = \begin{bmatrix} \varepsilon_{th}T_{th,} & 0 \\ 0, & \varepsilon_{th}T_{th,} \end{bmatrix}, \varepsilon_{th} = \sigma_x\sigma_{x'}, T_{th} = \begin{bmatrix} \beta_{th} & 0 \\ 0 & 1/\beta_{th} \end{bmatrix}$$

- Distribution after emission

$$\Sigma_o = \left\langle X_o\tilde{X}_o \right\rangle = \begin{bmatrix} \varepsilon_{eff}T_o & \mathrm{L}J \\ -\mathrm{L}J, & \varepsilon_{eff}T_o \end{bmatrix}$$

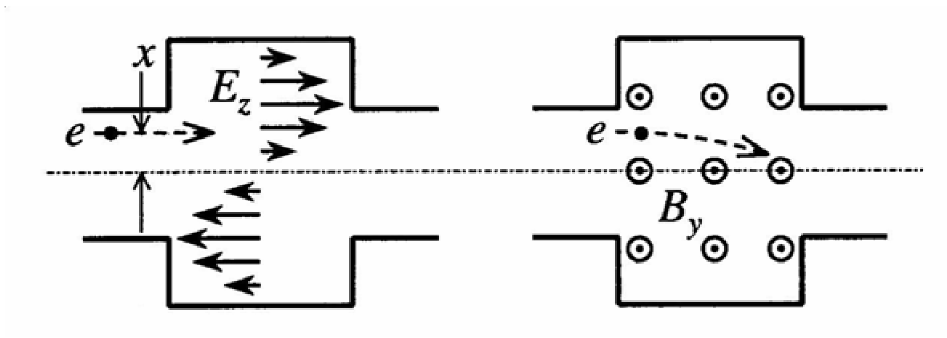
$$\kappa = \frac{qB}{2P_s}, \quad \varepsilon_{eff} = \sqrt{\varepsilon_{th}^{\;2} + \mathrm{L}^2}, T_o = \begin{bmatrix} \beta_o & 0 \\ 0 & 1/\beta_o \end{bmatrix}, \mathrm{L} = \kappa_o\sigma_x^{\;2}$$

- $X_{th} \rightarrow X_o$ non-symplectic
- Σ_o: coupled

Optical System Producing the x-z Exchange

$(\gamma\varepsilon_x, \gamma\varepsilon_z) = (10^{-5}, 10^{-7}) \rightarrow (10^{-7}, 10^{-5})$

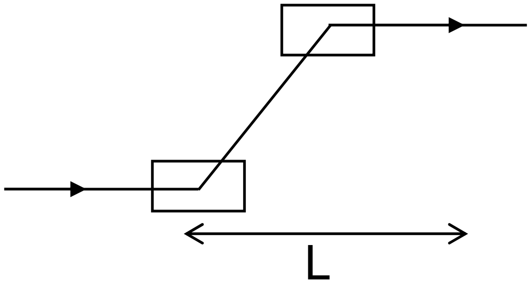
- Dipole mode cavity:



$$\Delta\delta = kx,\ \Delta x' = kz;\ k = eV_o/eE$$

$$M_C(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{bmatrix}$$

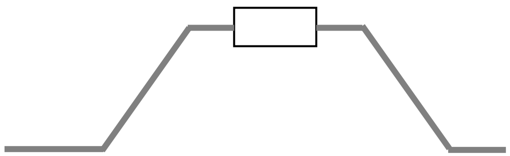
- Dog leg



$$M_D(\eta,\xi,L) = \begin{bmatrix} 1, & L, & 0, & +\eta \\ 0 & 1 & 0 & 0 \\ 0 & \eta & 1 & \xi \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

An Approximate Scheme for (x,z) Exchange

M. Cornacchia and P. Emma, PRSTAB, 5, 084001 (2002)



- M = M_D (-η,ξ,L) M_C(k) M_D (η,ξ,L)
- Choose ηk = 1 then

$$\varepsilon_x = \sqrt{\varepsilon_{zo}{}^2 + 4\sigma_{x'}{}^2\sigma_\delta{}^2\eta^2}$$

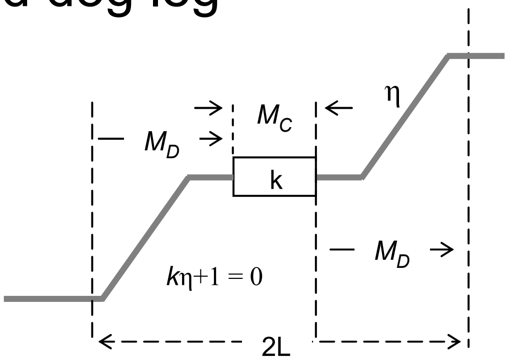
$$\varepsilon_z = \sqrt{\varepsilon_{xo}{}^2 + 4\sigma_{x'}{}^2\sigma_\delta{}^2\eta^2}$$

- Works if $\frac{2\sigma_{x'}\sigma_\delta\eta}{\varepsilon_{zo}} << 1.$

η cannot be reduced arbitrarily small due to second order aberration

An Exact Scheme for Emittance Exchange (KJK)

- Reverse second dog leg



- Choose $k\eta = -1$

- Then $$M = \begin{bmatrix} 0 & B \\ C & 0 \end{bmatrix}$$

- $$\begin{bmatrix} \Sigma_x & 0 \\ 0 & \Sigma_z \end{bmatrix} \rightarrow \begin{bmatrix} B\Sigma_z\tilde{B} & 0 \\ 0 & C\Sigma_x\tilde{C} \end{bmatrix}$$

Technical Detail-1/ Gun-Flat Beam Design (Phillipe Piot)

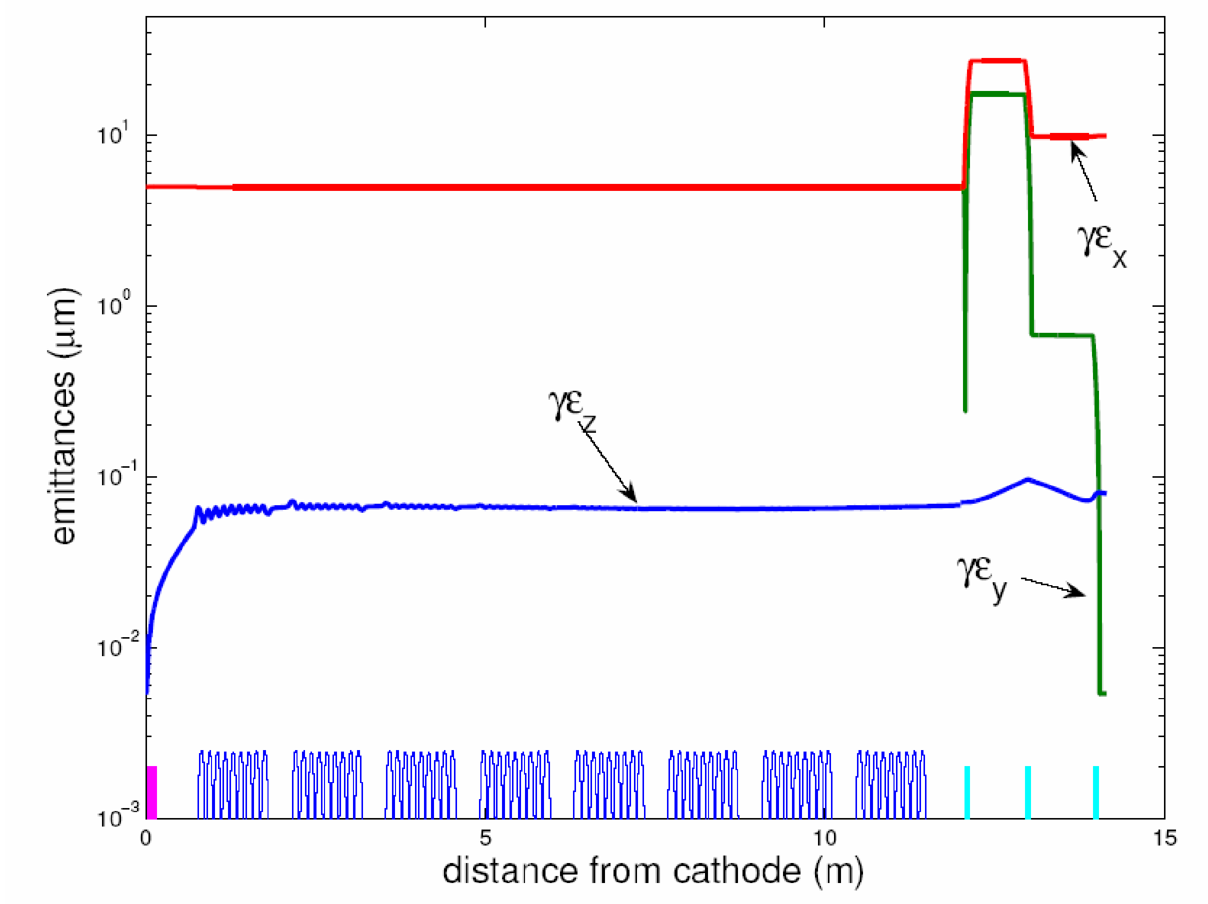
- High aspect ratio is difficult to maintain
- Phillipe Piot will solve the problem

Gun-Flat beam design

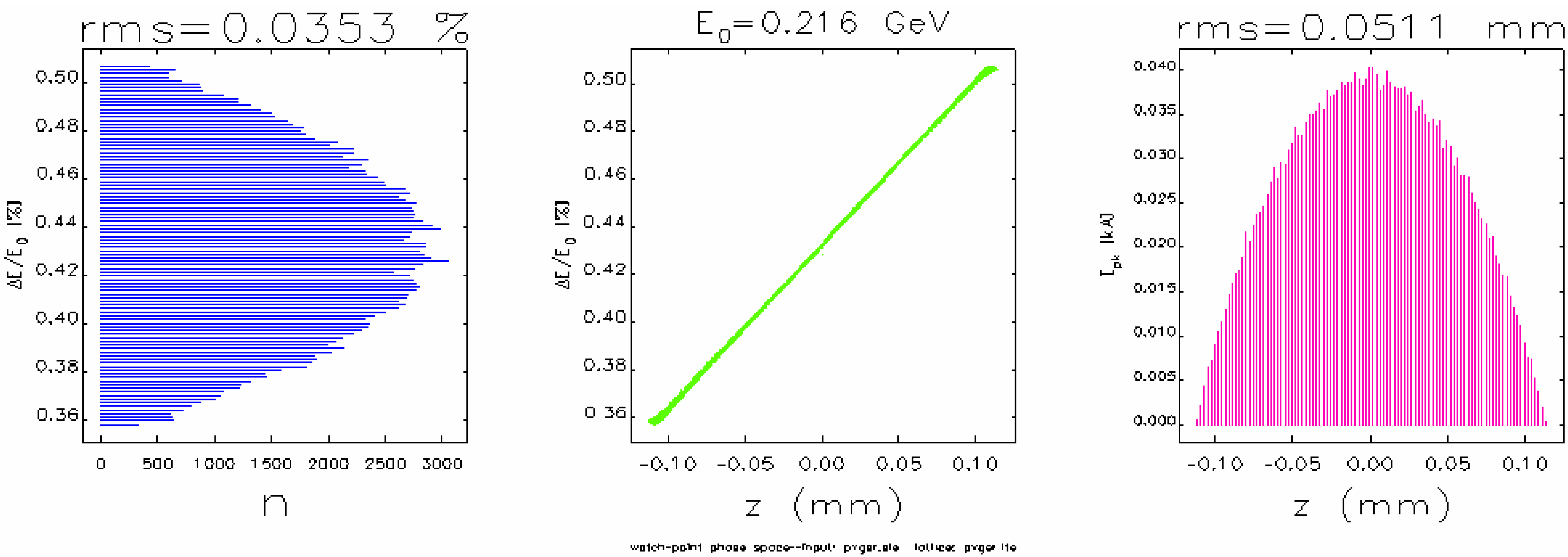
Phillipe Piot

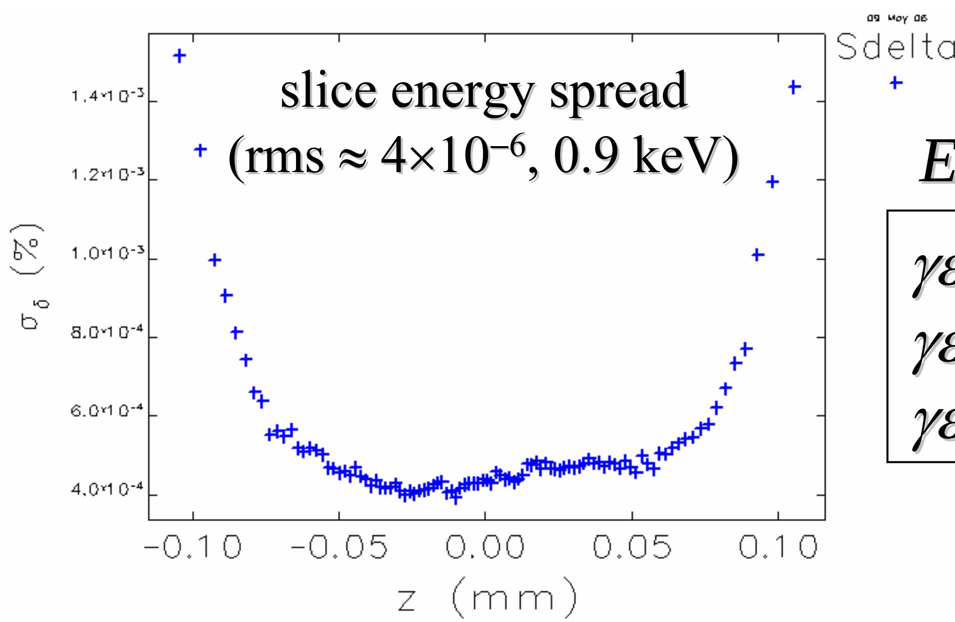
Operating Parameter	**Value**	**Units**
Bunch charge	20	pC
Laser rms spot size	300	μm
Laser rms pulse duration	80	fs
Peak E-field in rf-gun	138	MV/m
Launch phase	45	deg
Peak E-field in TESLA cavities	36	MV/m
B-field on photocathode	0.191	T
Cavity off-crest phase	4	deg
Beam Parameter	**Value**	**Units**
Before flat beam transformation		
Transverse emittances	4.96	μm
$\varepsilon_{4D} = \gamma\sqrt{\langle (X,Y)(X,Y)^T \rangle}$	0.23	μm
Longitudinal emittance	0.071	μm
Kinetic energy	215.4	MeV
After round-to-flat-beam transformer		
Emittance $\gamma\varepsilon_x$	9.923	μm
Emittance $\gamma\varepsilon_y$	0.005	μm
Longitudinal emittance	0.080	μm
$\gamma\sqrt{\varepsilon_x \varepsilon_y}$	0.23	μm

Gun-Flat Beam Design by Phillipe Piot



Philippe's Original File (longitudinal distributions)





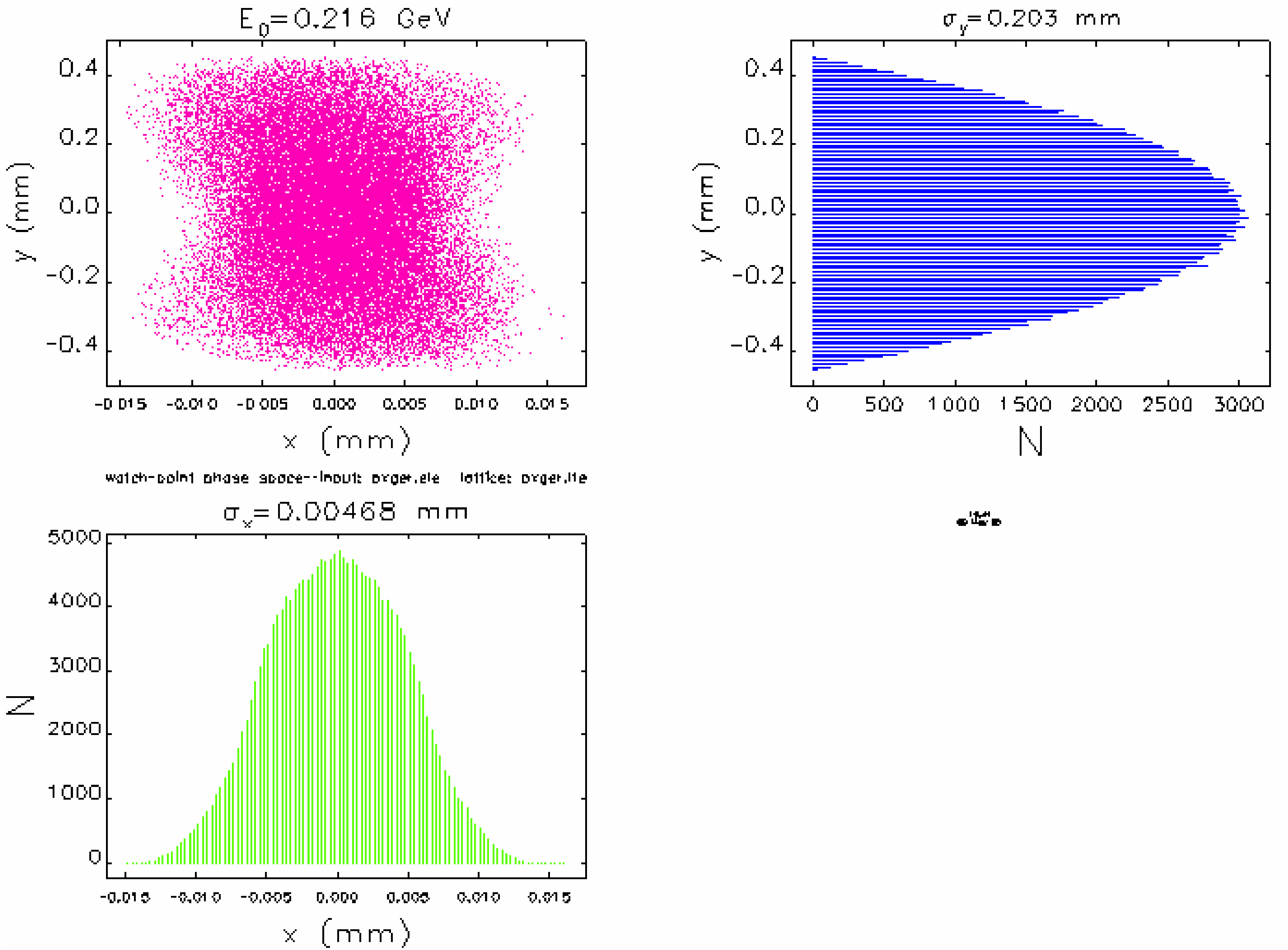
$E_0 = 215.9$ MeV

$$\gamma\varepsilon_z = 0.080\ \mu\text{m}$$
$$\gamma\varepsilon_x = 0.0054\ \mu\text{m}$$
$$\gamma\varepsilon_y = 9.92\ \mu\text{m}$$

Technical Detail-2/ Exchanger Paul Emma

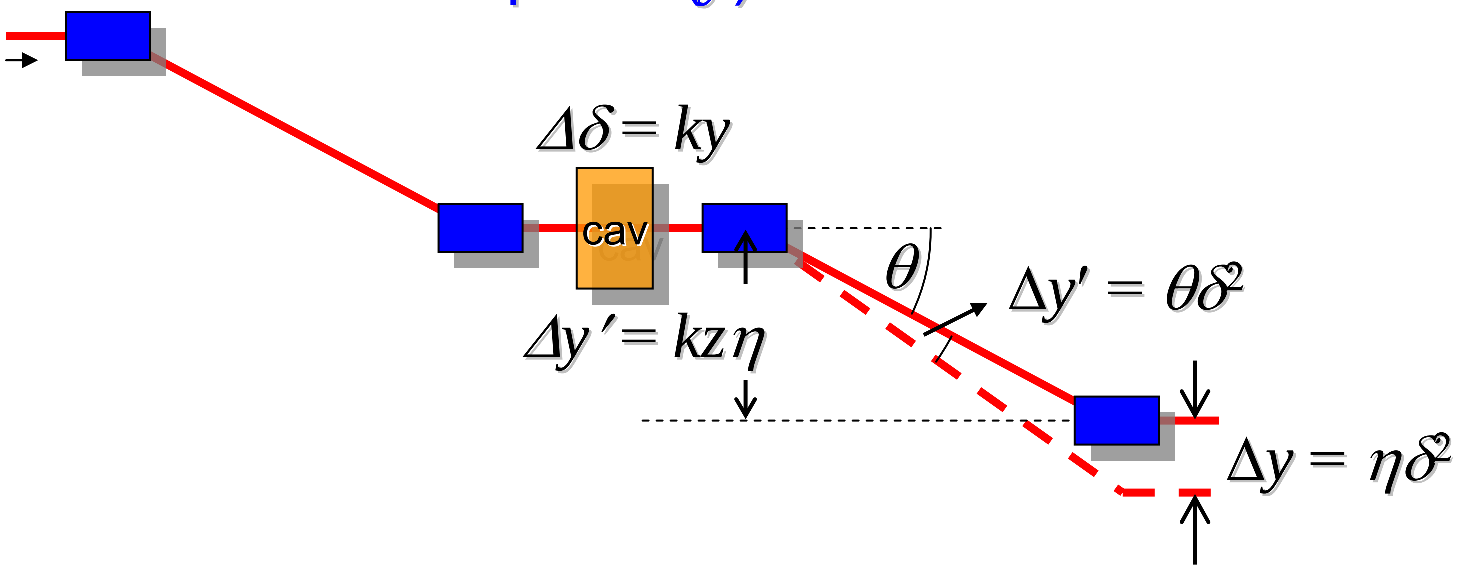
- Energy spread in the dipole cavity→ second order dispersion→ emittance growth
- Minimize emittance growth with suitable initial chirp
- Phillipe's out put turns out to have almost the right chirp!!

Philippe's Original File (transverse distributions)



Control of Second-Order Dispersive Aberration

- Energy spread is induced in T-cav due to transverse beam extent ($\delta = ky$)
- Second-order dispersion is generated in last two bends, which dilutes bend plane (y) emittance



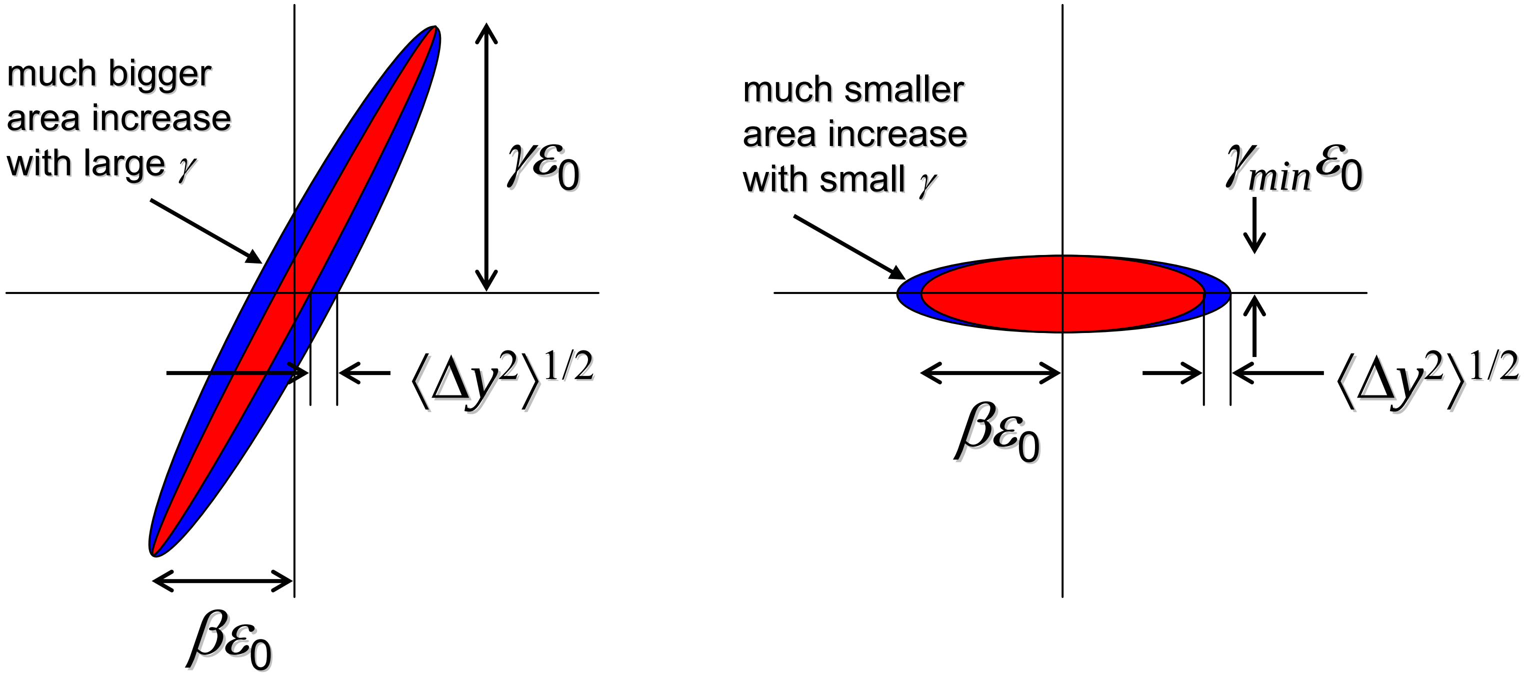
- The right initial energy chirp minimizes the divergence, γ, after the last bend, which minimizes emittance growth

$$\varepsilon^2 = \langle (y+\Delta y)^2 \rangle \langle y'^2 \rangle - \langle (y+\Delta y) y' \rangle^2 = \varepsilon_0^2 + \gamma \varepsilon_0 \langle \Delta y^2 \rangle$$

$$\frac{\varepsilon}{\varepsilon_0} = \sqrt{1 + \frac{\gamma}{\varepsilon_0} \langle \Delta y^2 \rangle}$$

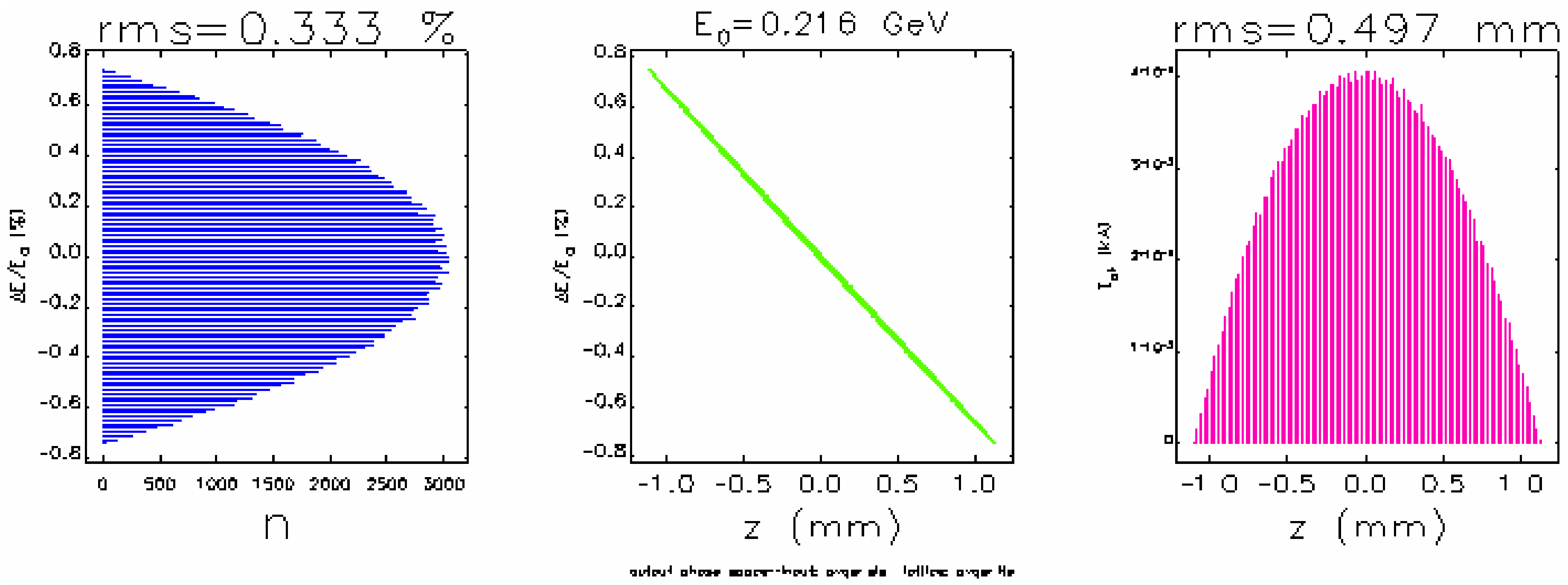
The Effect of Initial Chirp → Small γ at System Exit

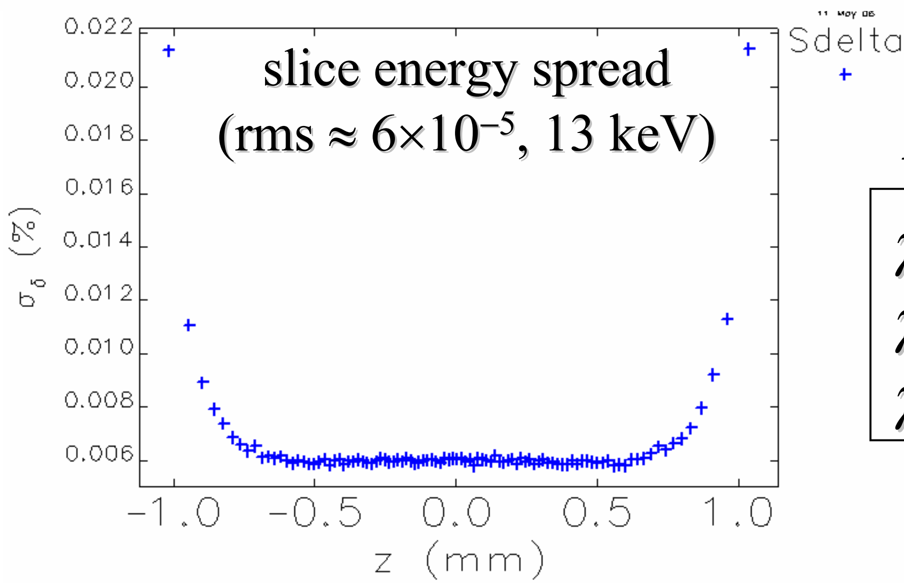
- The final divergence, γ, is decreased by the initial chirp → shorter bunch in cavity → less $y' = kz$ after cavity...



- For large γ (left) and small γ (right), the same Δy increase produces much larger area increase (emittance growth) when γ is large

After Emittance Exchanger (longitudinal distributions, no CSR)





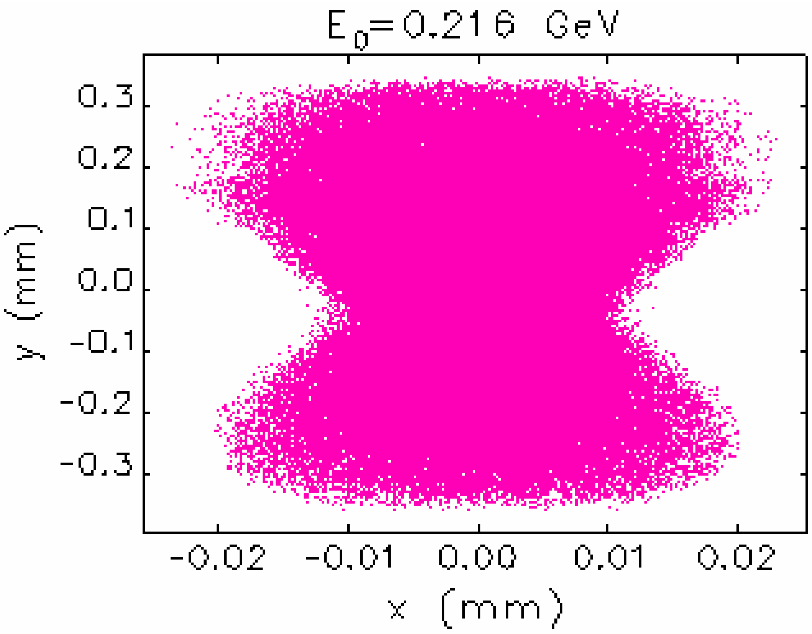
$E_0 = 215.9$ MeV

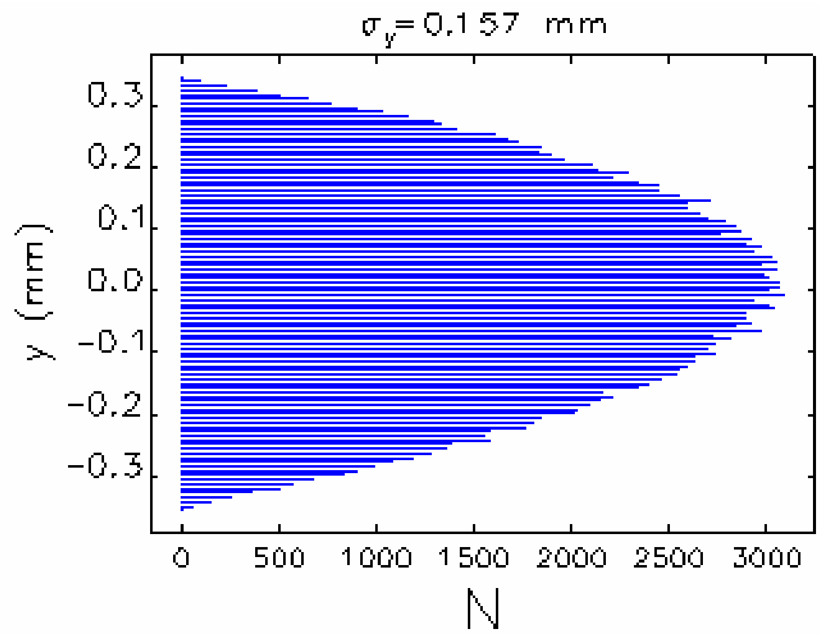
$\gamma\varepsilon_z = 9.92\ \mu\text{m}$

$\gamma\varepsilon_x = 0.0054\ \mu\text{m}$

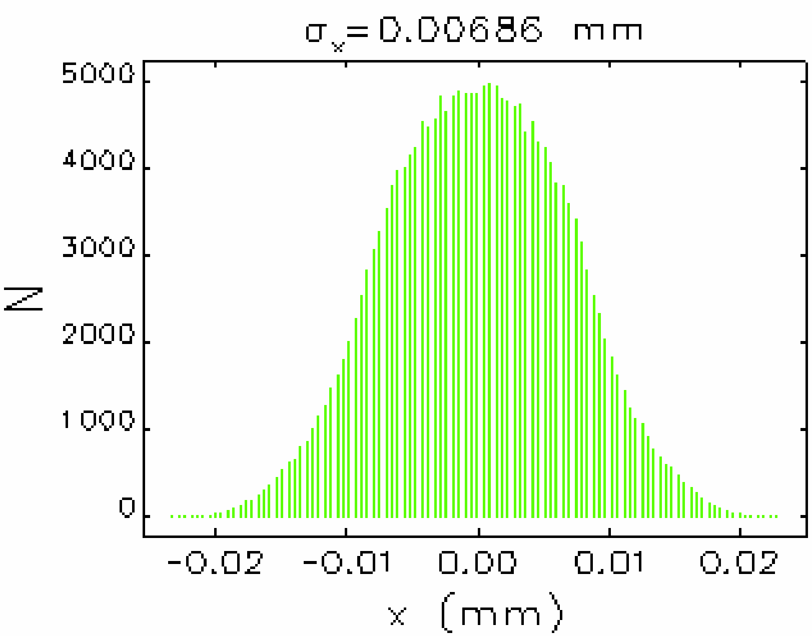
$\gamma\varepsilon_y = 0.084\ \mu\text{m}$

After Emittance Exchanger (transverse distributions, no CSR)





output phase space--input: pxger.ele lattice: pxger.lte



Before and After Emittance Exchanger (1st order – no CSR)