Production of coherent X-rays with a free electron laser based on optical wiggler

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- Thomson back-scattering for highly relativistic electron beams is a source of incoherent radiation in the X-ray frequency range.

- The number of photons emitted is calculated by summing in the far region the e.m. energy fluxes produced by each single electron of the beam. All electrons of the beam are supposed to move only under the action of the laser fields.

- To increase the number of photons emitted in the process one has to increase the energy content of the laser pulse by increasing for instance the length of the pulse. The incoherent radiation intensity level inside the electron bunch increases up to the point when these em fields are able to influence the motion of the electrons.

In this new regime in which collective effects become important, the Thomson interaction between the electron beam and the laser pulse behaves like a free-electron laser based on an electromagnetic wiggler.

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B.G.Danly, G.Bekefi, R.C.Davidson, R.J.Temkin,T.M.Tran,J.S.Wurtele, IEEE Journ. of Quantum Electronics, QE-23,103(1987).
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In particular, if the time duration of the laser pulse ΔT_L is larger than a few gain times, i.e., if

$$\Delta T_L > (10)\, L_g/c$$

the electrons of the beam are bunched on the Thomson radiation wavelength and the FEL instability can develop.

The coherent radiation is expected to have a narrow spectrum bandwidth, much less than that of the incoherent Thomson radiation, a less broad angular distribution and a larger intensity (if saturation is reached).

3D equations

- **Slowly Varying Envelope Approximation, ("single mode" treatment), i.e., for the vector potential of the collective field**

$$\mathbf{A}(xyzt) = A(xyzt)e^{i(kz-\omega t)}\hat{\mathbf{e}} + cc + O(\lambda / L_T)$$

- In particular the envelope A must have a slow dependence on the transverse variables x and y, the transverse spatial scale $L_T >> \lambda$

- **Space charge effects neglected**

- **Relativistic equations for the electrons of the beam in the lab frame**

Laser system

Laser pulse characteristics:

Wavelength λ_L, power, time duration ,laser parameter a_{L0}

Circular polarization, focal spot diameter w_0

$z_0 = \pi w_0^2/4\lambda_L$ Rayleigh length

$$\mathbf{A}_L(\mathbf{r},t) = \frac{a_{L0}}{\sqrt{2}}(g(\mathbf{r},t)e^{-i(k_L z+\omega_L t)}\hat{\mathbf{e}} + cc) + O(\frac{\lambda_L}{w_0})$$

Guided pulse: g(**r**,t) step function

Gaussian pulse:

$$g(\mathbf{r},t) = \Phi(z+ct)\frac{1+i\frac{z}{z_0}}{1+\frac{z^2}{z_0^2}}\exp\left[-4\frac{x^2+y^2}{w_0^2(1+\frac{z^2}{z_0^2})} - 4i\frac{x^2+y^2}{w_0^2(\frac{z}{z_0}+\frac{z_0}{z})}\right]$$

Electron equations

Laser ponderomotive forces

Collective ponderomotive effects

Schroedinger-like radiation equation

$$\frac{d}{d\bar{t}}\bar{\mathbf{r}}_j(\bar{t}) = \rho\,\frac{\mathbf{P}_j(\bar{t})}{\bar{\gamma}_j(\bar{t})}$$

$$\frac{d}{d\bar{t}}P_{jz}(\bar{t}) = \boxed{-\frac{\bar{a}_{L0}^2}{2\rho\gamma_0^2}\frac{1}{\bar{\gamma}_j}\left[\frac{\partial}{\partial\bar{z}}|g|^2\right]_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j}} \boxed{-\frac{2}{\bar{\gamma}_j}\mathrm{Re}\,al\left[(g^*\bar{A})_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j}e^{i\theta_j(\bar{t})}\right]} + \ldots.$$

$$\frac{d}{d\bar{t}}\mathbf{P}_{j\perp}(\bar{t}) = \boxed{-\frac{\bar{a}_{L0}^2}{2\rho\gamma_0^2}\frac{1}{\bar{\gamma}_j}\left[\bar{\nabla}_\perp|g|^2\right]_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j}} \boxed{-\frac{4\eta}{1+\frac{k_L}{k}}\frac{1}{\bar{\gamma}_j}\mathrm{Im}\left[(\nabla_\perp(g^*\bar{A}))_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j}e^{i\theta_j(\bar{t})}\right]} + \ldots.$$

$$\boxed{\left(\frac{\partial}{\partial\bar{t}}+\frac{\partial}{\partial\bar{z}}\right)\bar{A}(\bar{\mathbf{x}},\bar{t}) - i\rho\frac{k_L}{k}\bar{\nabla}_\perp^2\bar{A} = b}$$

Bunching factor

$$b = \frac{1}{N_s}\frac{V_b(0)}{V_b(\bar{t})}\sum_s \frac{g(\mathbf{r}_s(\bar{t}),\bar{t})}{\bar{\gamma}_s(\bar{t})} e^{-i\theta_s(\bar{t})}$$

$$\theta_j(\bar{t}) = \frac{k}{2\rho k_L}\left(\left(1+\frac{k_L}{k}\right)\bar{z}_j(\bar{t}) + \left(\frac{k_L}{k}-1\right)\bar{t}\right)$$

Normalization

$$\rho = \frac{1}{\gamma_0}\left(\frac{\omega_b^2 \bar{a}_{L0}^2\left(1+\frac{k_L}{k}\right)}{16\omega_L^2}\right)^{\frac{1}{3}}$$

$$\bar{\mathbf{x}} = 2\rho k_L \mathbf{x}$$

$$\bar{t} = 2\rho\omega_L t$$

$$\frac{eA}{mc^2} = -i\left(\frac{\omega_b\sqrt{\gamma_0\rho}}{\sqrt{2}\omega_R}\right)\bar{A}$$

$$\bar{\gamma}_j = \gamma_j/\gamma_0 \qquad \mathbf{P}_j = \mathbf{p}_j/\gamma_0\rho \qquad \bar{a}_{L0} = \frac{e}{mc^2}a_{L0}$$

$$\gamma_j^2 = 1 + \gamma_0^2\rho^2 P_{jz}^2 + \bar{a}_{L0}^2(|g|^2)_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j(\bar{t})} + \ldots$$

Resonance condition

$$\omega_R = \frac{4\gamma_0^2\omega_L}{1+\bar{a}_{L0}^2}$$

3D code

- Fourth order RKG for the particles trajectories. Electrons modeled as macroparticles
- Explicit finite difference scheme for the Schroedinger-like radiation equation. The collective vector potential is averaged over a conveniently small average volume.
- Start-up from noise

First example

Laser pulse time duration up to 100 ps, power 40-100 GW, w_0=100μm, λ_L=10 μm (CO_2 laser),total laser energy 4-10J, a_{L0}=0.3, guided

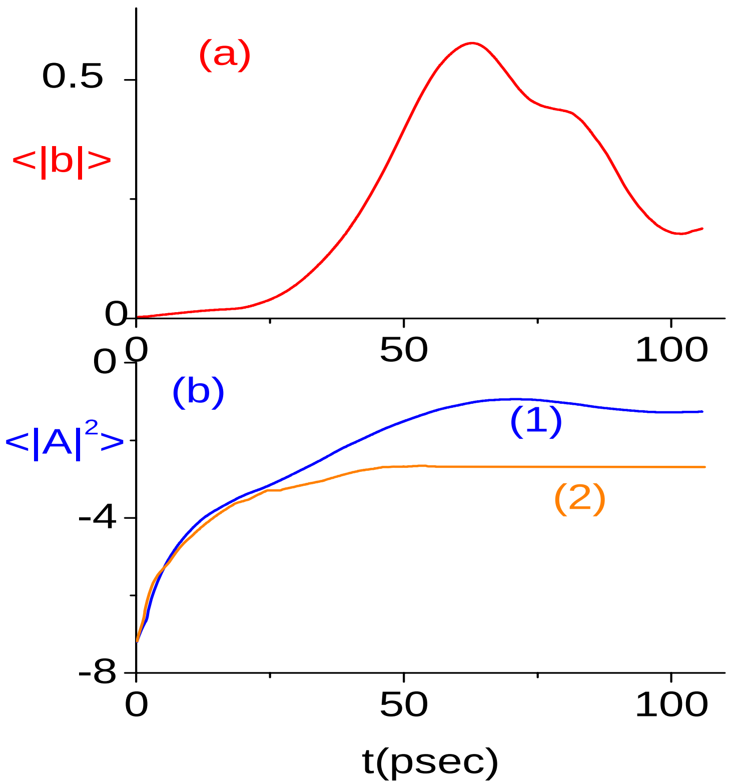
Electron beam Q=1-5nC, L_b=1mm, focal radius σ_0=25 μm, I=0.3-1.5 kA, energy=30 MeV (γ_0=60) , transverse normalized emittance up to 1 mm mrad, $\delta\gamma/\gamma=10^{-4}$.

$\rho=2.8\ 10^{-4}$ gain length L_g= 2.83 mm

Radiation λ_R=7.56 Angstrom Z_R=2.5 m

$\bar{\rho} = 41.32\rho\gamma_0\lambda_R(\mathring{A})$ = 5.25 $\Longrightarrow$ no appreciable quantum effects

ε_n=0.6 mmmrad at t=0, guided laser pulse

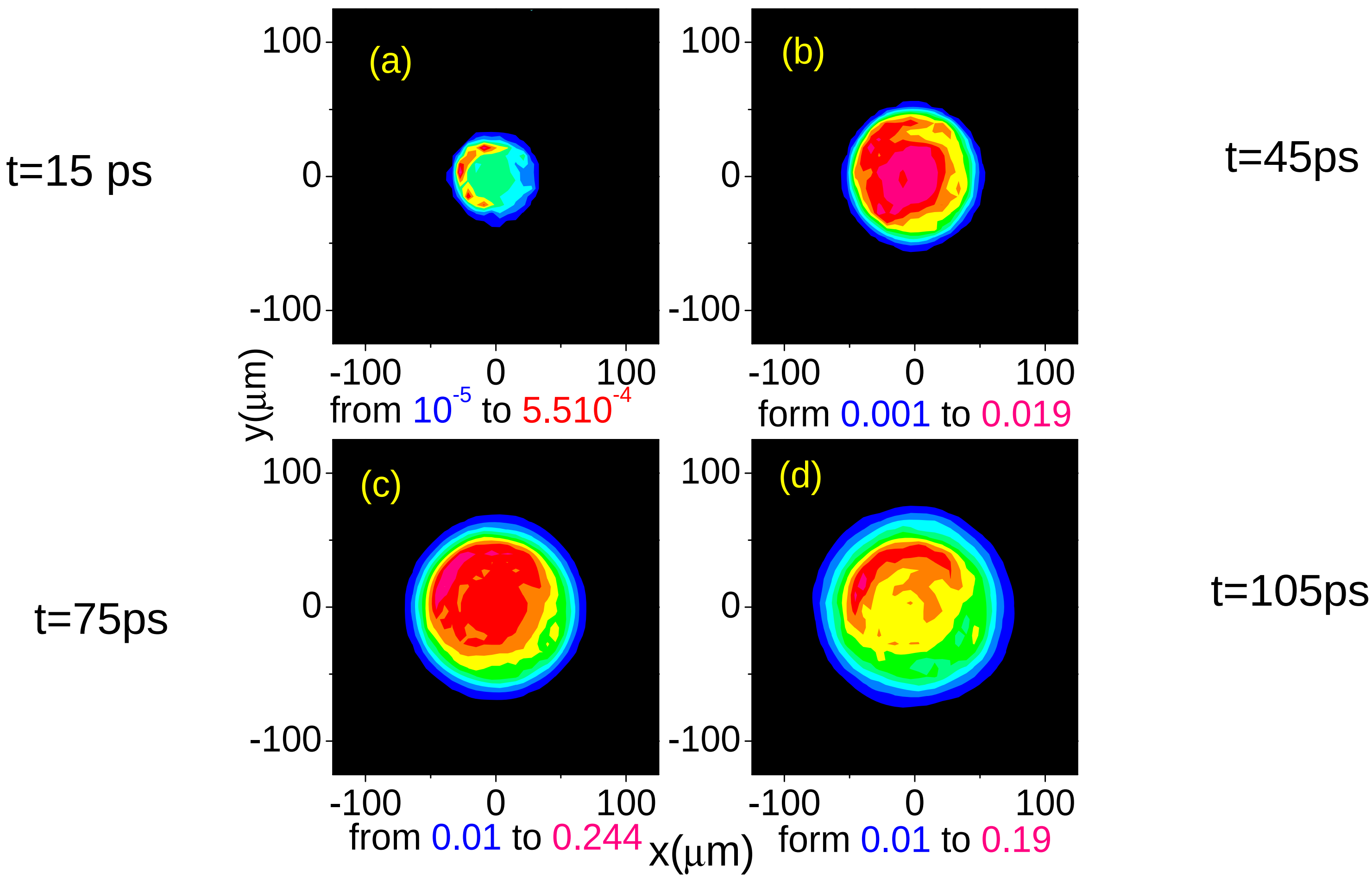


(1) Collective radiation
$2\ 10^{10}$ Photons
(1) Incoherent radiation
$2 10^{8}$ photons

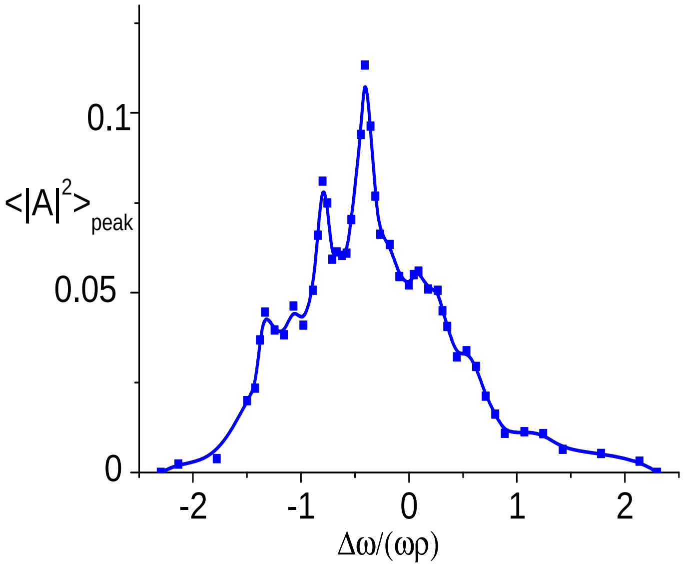
$|A|^2_{sat}$=0.11, saturation length about 7 L_g (70 ps) $2{,}36 10^{10}$ photons

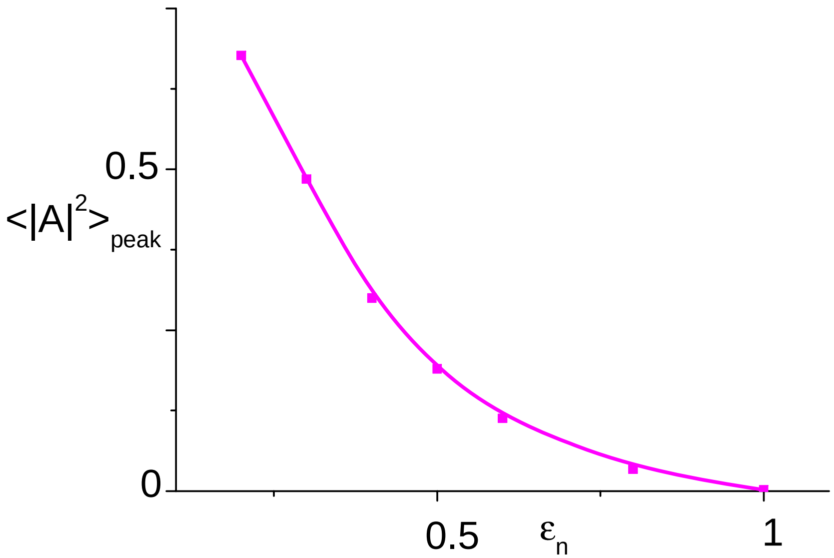
(a) averaged bunching factor <|b|> in the middle of the bunch vs time, (b) logarithmic plot of $<|A|^2>$ vs time in both coherent (1) and incoherent (2) cases. w_0=50 mm with a flat laser profile, a_{L0}=0.3, Q=3 nC, I= 0.9 kA,$<\gamma>$=60, $\Delta p_z/p_z=10^{-4}$, ε_n=0.6 , $\Delta\omega/\omega$= -10^{-4}.

Transverse radiation intensity
normalized emittance ε_n=0.6 mm mrad



Level curves of $|A|^2$ in the (x,y) plane for(a) t= 15 ps, (b) t= 45 ps, (c) t=75 ps and (d) t= 105 ps.





Saturation peak value of $<|A|^2>$ versus $\Delta\omega/(\omega\rho)$

Saturation peak value of $<|A|^2>$ versus ε_n (μm) and with $\Delta\omega/\omega = -10^{-4}$, $w_0=50\mu m$, $a_{L0}=0.3$, $\Delta\gamma/\gamma=10^{-4}$

Optimization of the radiation intensity by shifting the focus of the beam at 8 mm

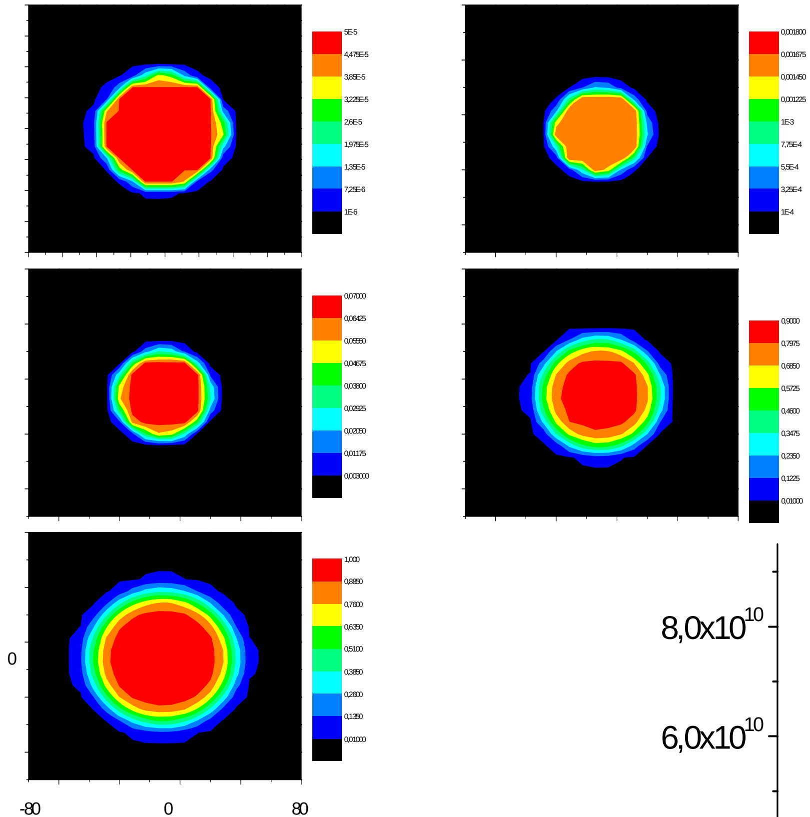
t=0 ps

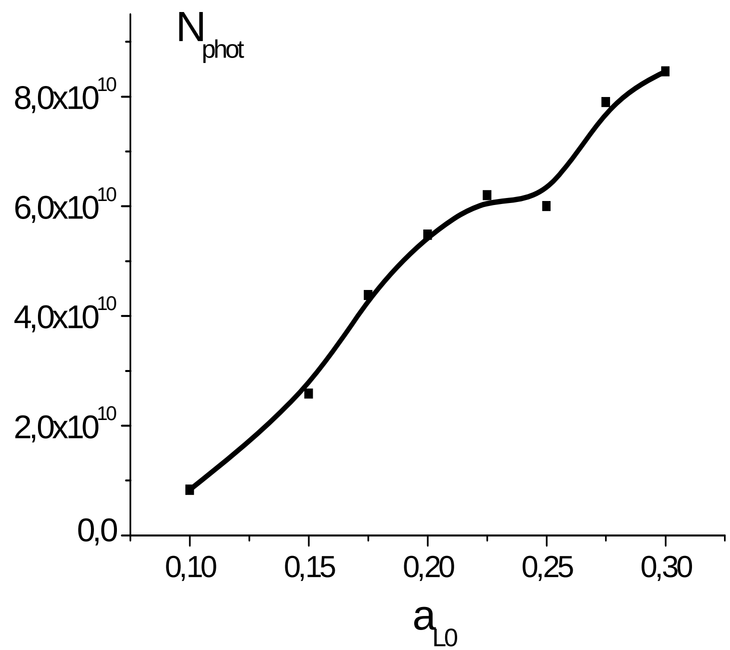
t=14 ps

t=28 ps

t=42 ps

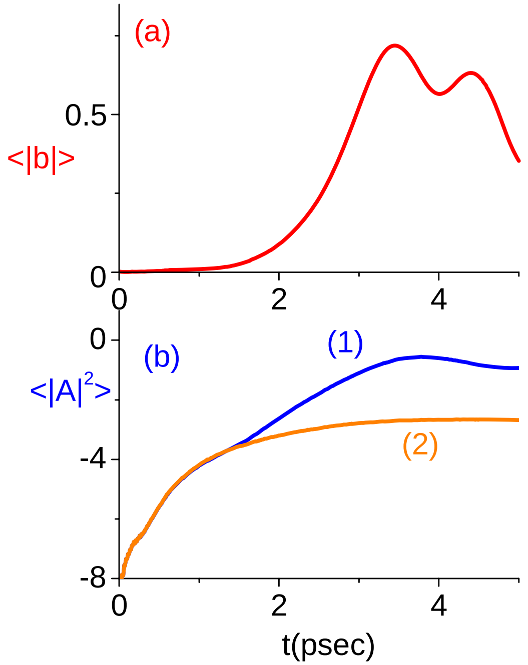
t=56 ps

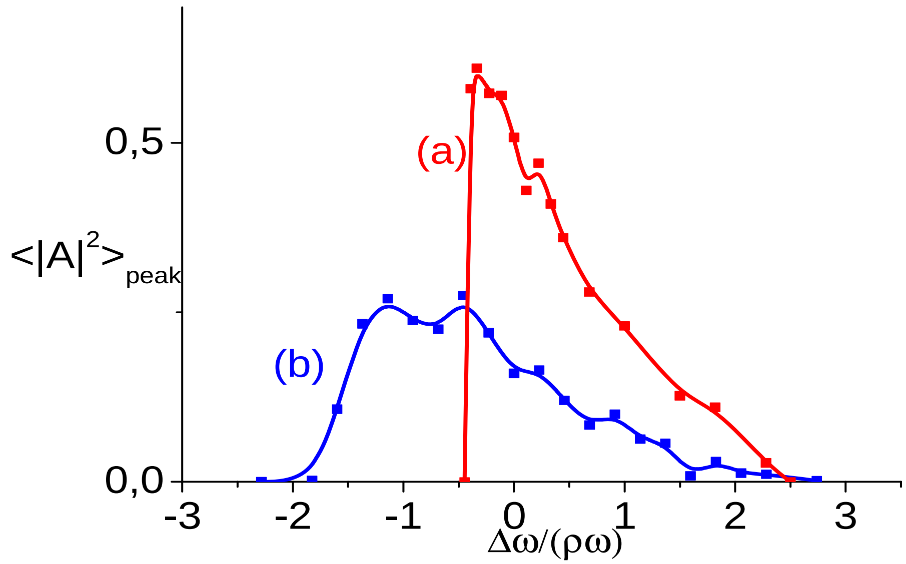




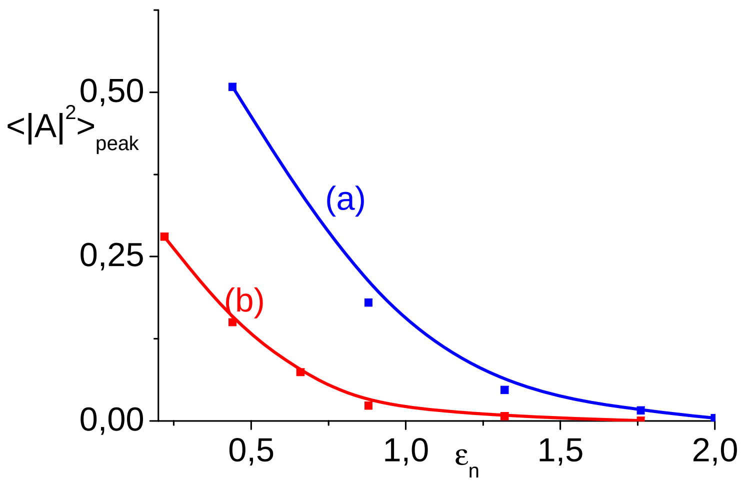
Second example

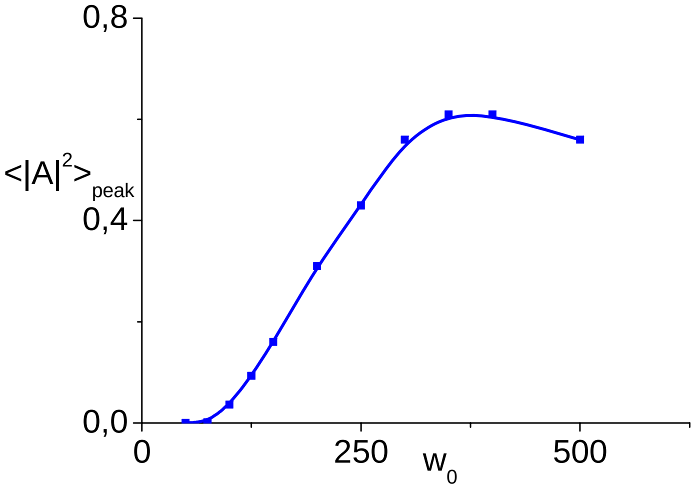
γ_0= 30, beam mean radius σ_0=10 μm, total charge Q=1 nC,
bunch length L_b= 200 μm, beam current of 1.5 kA.
The laser pulse has a wavelength λ_L=0.8 μm, a focal spot diameter w_0 of 100 μm ,
a_{L0}=0.8 , λ_R= 3.64 Angstrom, ρ = 4.38 10^{-4} .L_g=145 μm,
quantum parameter $\bar{\rho}$ =2





$\langle|A|^2\rangle_{peak}$ versus $\Delta\omega/(\omega\rho)$ for (a) ε_n=0.44 μm and (b) ε_n=0.88 μm.





$<|A|^2>_{peak}$ versus ε_n with $\Delta\omega/\omega=0$ and for flat laser profile with $w_0=50$ µm (curve (a)) and $a_{L0}=0.8$ and for a Gaussian laser profile (curve (b)) with $a_{L0}=0.8$ and $\sigma_L=106$ µm.

$<|A|^2>_{peak}$ versus w_0 for the Gaussian laser profile with $\varepsilon_n=0{,}44$ µm, $\Delta\omega/\omega= -1.10^{-4}$, $a_{L0}=0.8$

- The line width in a situation dominated by the emittance can be written as

$$\frac{\Delta\lambda}{\lambda_R} \approx \frac{\gamma_0^2 (\delta\theta)^2}{1+a_{L0}^2} \approx \frac{\varepsilon_{n,x}^2}{\sigma_0^{\ 2}}$$

On the other hand, in order to have considerable emission we must assume that the following inequality is satisfied

$$\Delta\lambda/\lambda_R < \alpha\rho,$$

with α a numerical factor of the order of 1. We can then write for the normalized transverse emittance

$$\varepsilon_{n,x} \le \sqrt{\alpha\rho}\ \sigma_0$$

Through the definition of the gain length $L_g = \lambda_L/(4\pi\rho)$ and the radiation Rayleigh length $Z_R = 2\pi\sigma_R^2/\lambda_R$, we can express the factor ρ in terms of the ratio Z_R/L_g, obtaining

$$\rho = \frac{Z_R}{L_g}\frac{\lambda_R \lambda_L}{8\pi^2\sigma_R^2}$$

Finally, if the electron beam and the radiation overlap, i.e., $\sigma_0 = \sigma_R$, remembering the resonance relation in its simpler form $\lambda_L = 4\gamma_0^2\lambda_R$, we obtain for an optical undulator

$$\varepsilon_{n,x} \le \sqrt{\frac{\alpha Z_R}{2\pi^2 L_g}\gamma_0\lambda_R}$$

Conclusions

At the present state of the analysis we may say that the growth of collective effects during the back scattering Thomson process is possible provided that:

i) a low-energy , high-brigthness electron beam is available (normalized transverse emittance at t=0 preferably less than 1)

ii) the optical laser pulse is long enough to allow the electron bunching by the spontaneous (incoherent) radiation and the consequent FEL instability

iii) the laser envelope has rather “flat” transverse and longitudinal profiles