A Smith-Purcell BWO for Intense Terahertz Radiation

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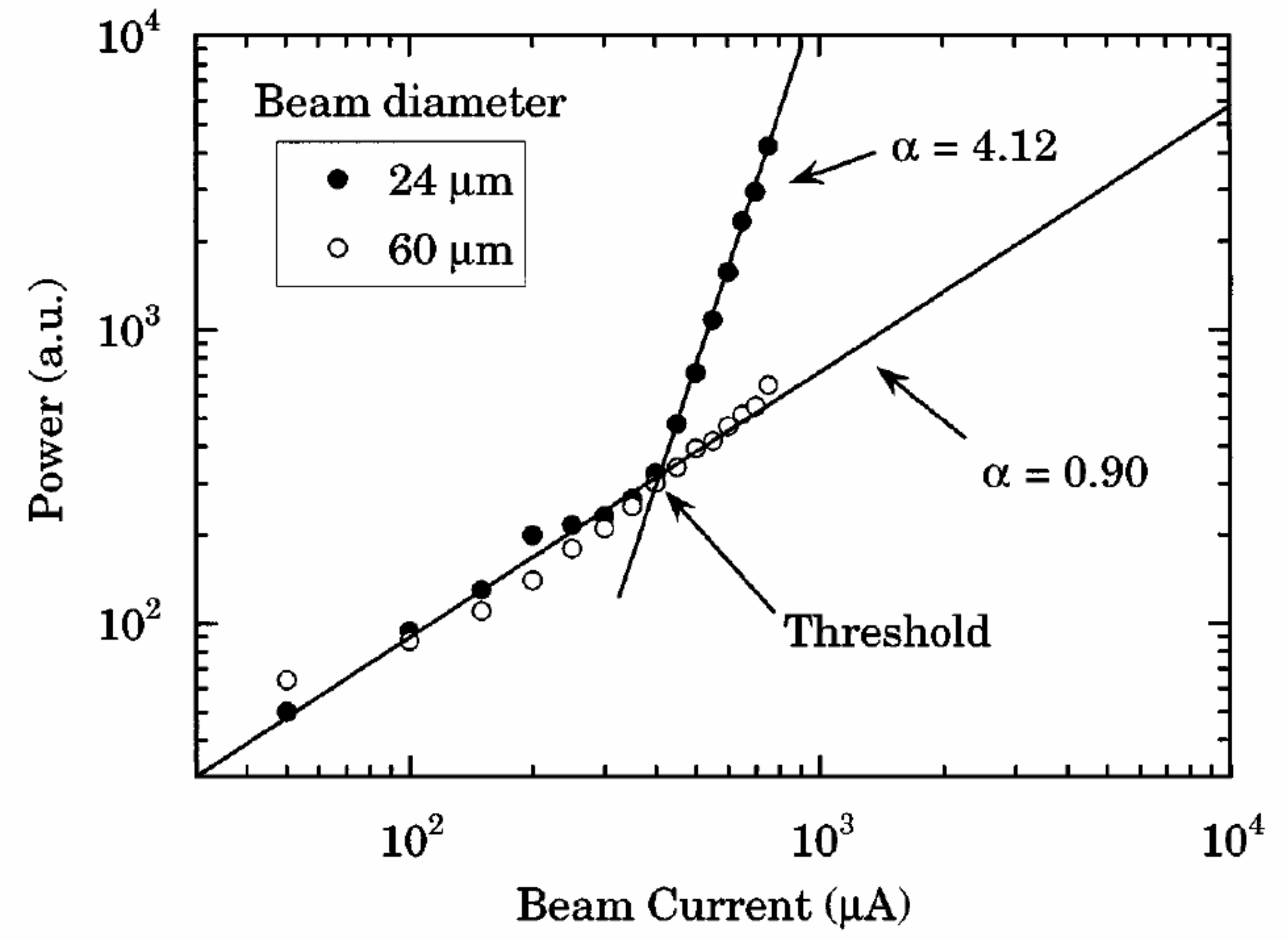
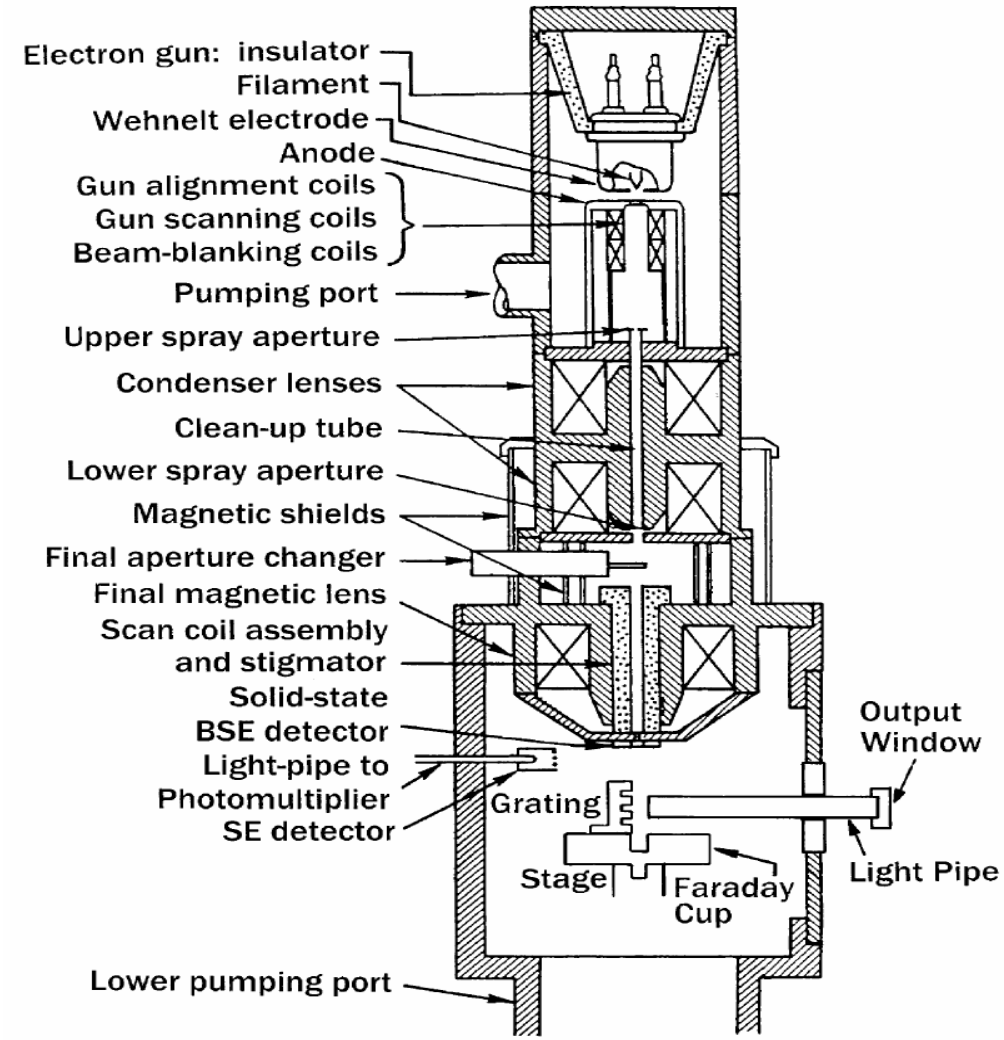
Non-Linear Behavior in Smith-Purcell Radiation ? *(J. Urata et al., PRL 80 (1998) 516-519)*



SEM-Based Smith-Purcell Radiator



β= 0.35 (35 keV)

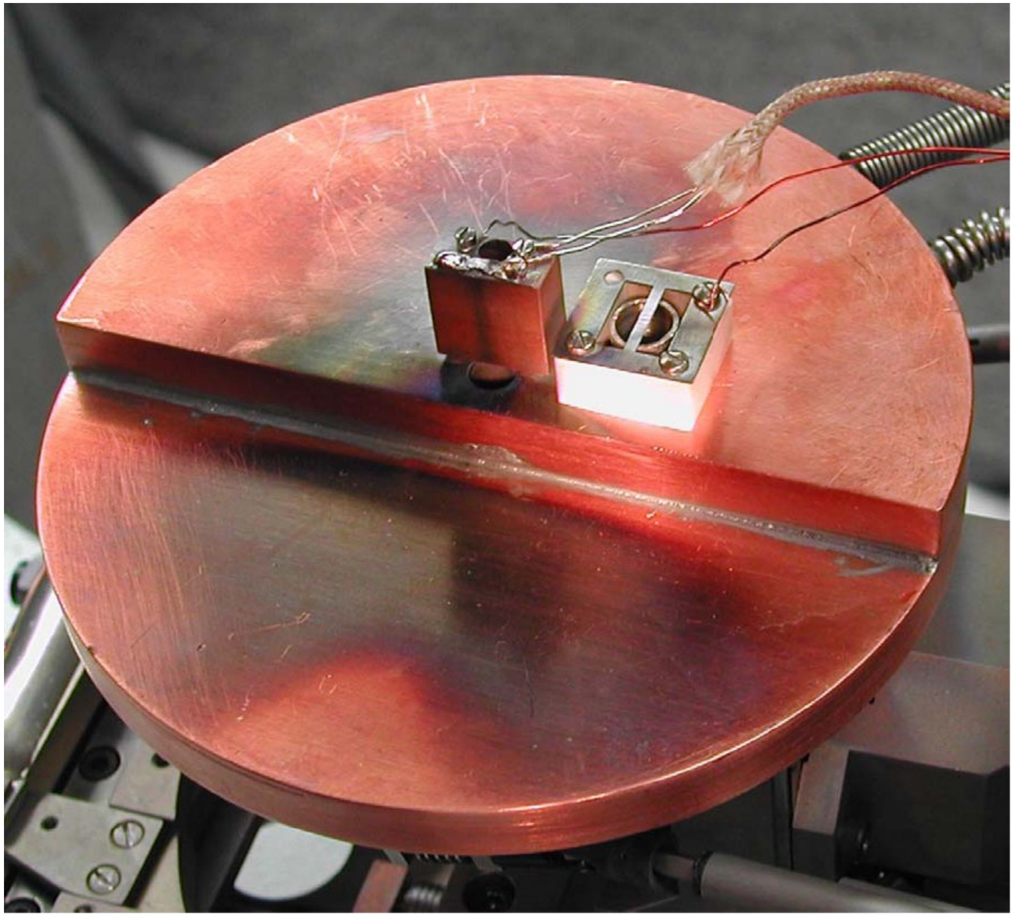
$I \leq 1$ mA

λg = 173 μm, d = 100 mm, w = 62 μm,

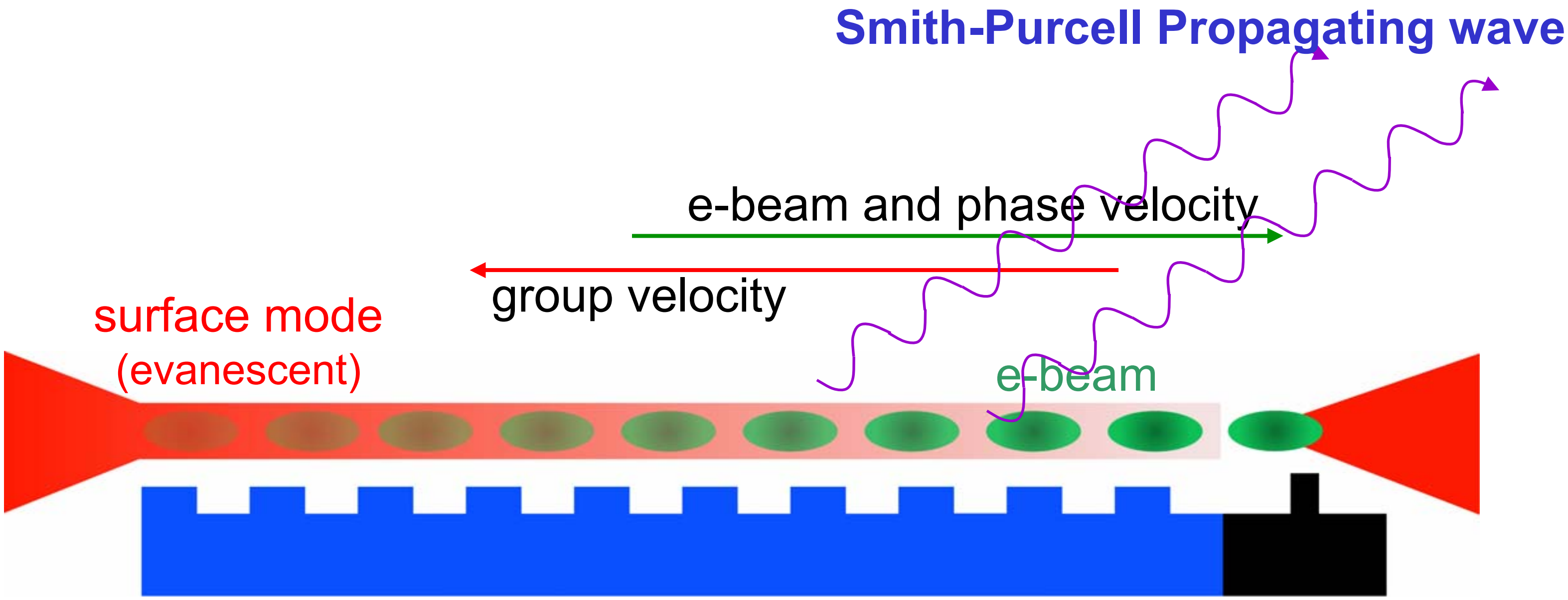
b = 10 μm, L = 12.7 mm

SEM-Based Smith-Purcell Radiator at the U of C, After the Dartmouth Set-Up (O. Kapp, A. Crewe, KJK)

Heated Specimen Stage and Possible Black Body Radiation Background



Smith-Purcell FEL is a Backward Wave Oscillator



2D Smith-Purcell BWO Theory

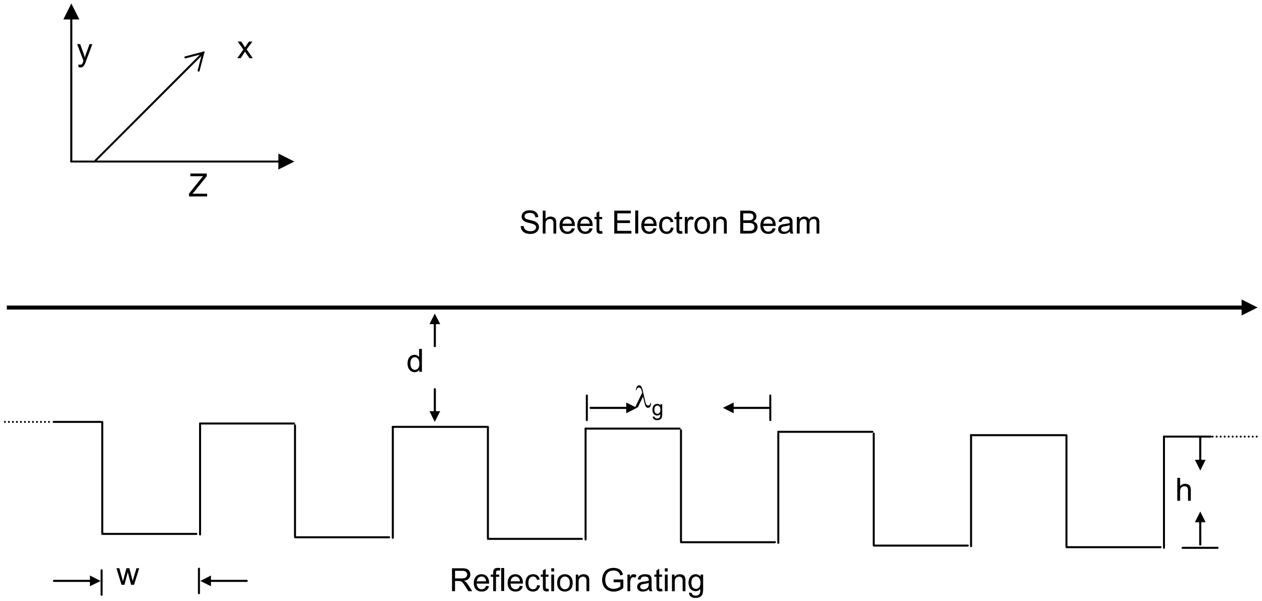
- Electron beam filling uniformly above grating
  - H.L. Andrews and C.A. Brau, Phys. Rev. ST Accel. Beams **7**, 070701 (2004).
  - H. L. Andrews, C.H. Boulware, C.A. Brau, and J. D. Jarvis, Phys. Rev. ST Accel. Beams **8**, 050703 (2005).

- Sheet beam at distance *d* above grating
  - PIC simulation:

    J.T. Donahue and J. Gardelle, Phys. Rev. ST Accel. Beams **8**, 060702 (2005)
  - Analytical theory and Maxwell-Lorentz simulations:

    Vinit Kumar and Kwang-Je Kim, Physical Review E **73**, 026501 (2006).

Sheet Beam Smith-Purcell BWO



E- Field, Energy Modulation, and Bunching; Three-Fold Way for FELs

- E_z-Field gives rise to energy modulation

$$\frac{d\eta}{dz} = \frac{q}{\gamma mc^2} E_z(z,t) \qquad \eta = \frac{\gamma - \gamma_0}{\gamma_0}$$

- Energy modulation gives rise to bunching

$$\frac{d\xi}{dz} = -\frac{\eta}{c\beta^3\gamma^2}$$

- Bunching gives rise to surface mode

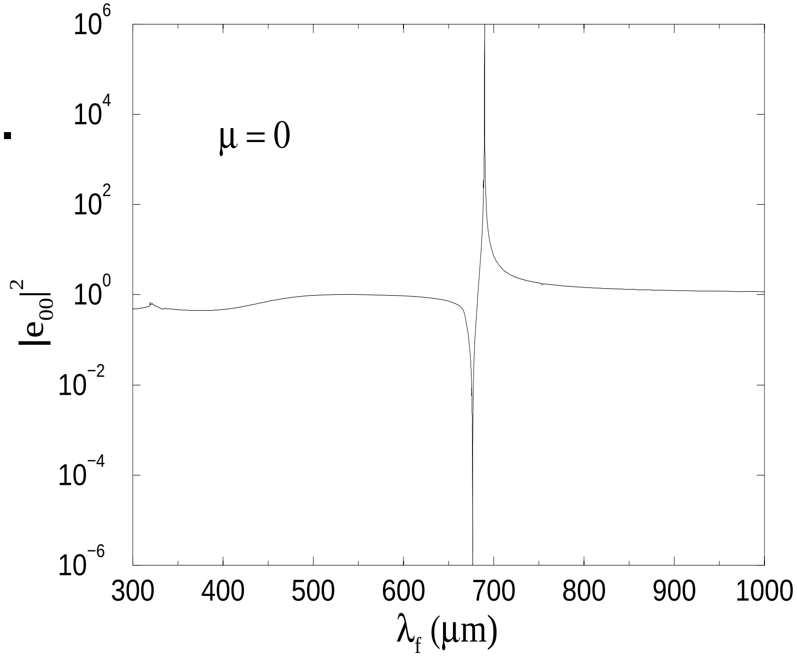
$$E_z = \frac{i\Gamma_0}{2\varepsilon_0\omega}\left(e_{00}\, e^{-2\Gamma_0 b} - 1\right) K_0(\omega)\, e^{i\alpha_0 z}$$

- *Quadratic equation for growth rate if e_{00} is a smooth function**
- *However, e_{00} is singular !*

*K.-J. Kim and S. B. Song, Nucl. Instrum. Methods Phys. Res. A 475, 158 (2001).

Singularity in e_{00} and Freely Propagating Surface Mode

- **The reflection coefficient e_{00} diverges at λ=690 m.**
- **Freely propagating surface mode at this λ.**
- **For a non-zero growth rate (μ) it has a simple pole**

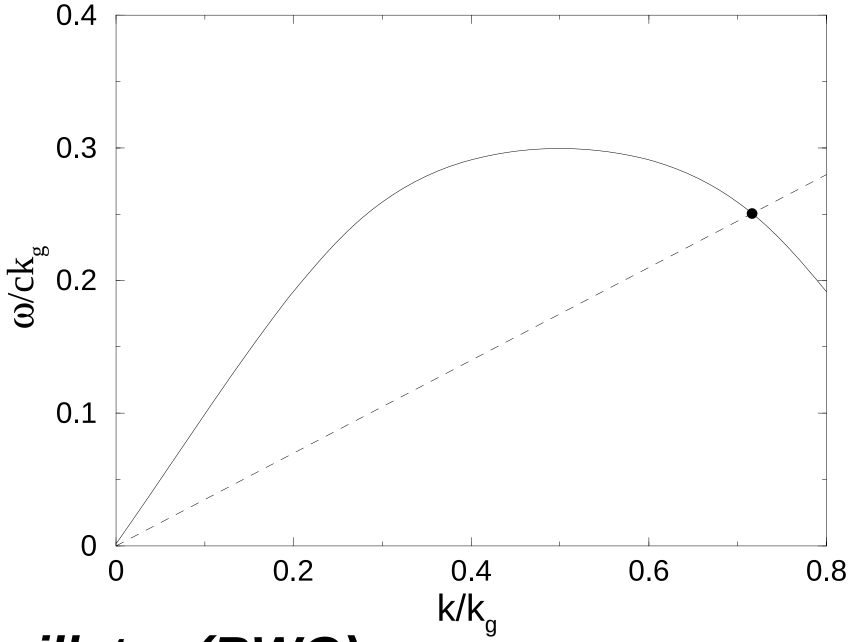


$$\boxed{e_{00}(\mu) = \frac{-i\chi}{\mu} + \chi_1}$$

$$\boxed{\mu E^{sur} = \frac{dE^{sur}}{dz} = \frac{IZ_0\chi}{2\beta\gamma\Delta y} e^{-2\Gamma_0 b} \left\langle e^{-i\psi} \right\rangle}$$

Thus we recover cubic equation !

*Surface Mode Has Negative Group Velocity**



- **Phase velocity = $\omega/k_z = \beta c$**
- **$d\omega/dk_z < 0$**
- ***Thus SP-FEL is a Backward Wave Oscillator (BWO)***
- **Optical energy accumulates exponentially to saturation *without feedback mirrors***

*H.L. Andrews et al., Phys. Rev. ST Accel. Beams. 8, 050703 (2005)

Including Time Dependence via

Time-dependent Maxwell equation:

$$\frac{\partial E}{\partial t} - v_g \frac{\partial E}{\partial z} = -\frac{IZ_0 \chi v_g}{2\beta\gamma\Delta y} e^{-2\Gamma_0 b}\left\langle e^{-i\psi}\right\rangle$$

$$E^{sc} = \frac{-iIZ_0}{2\beta\gamma\Delta y}\left(1-\chi_1 e^{-2\Gamma_0 b}\right)\left\langle e^{-i\psi}\right\rangle$$

$\mu \Rightarrow \partial / \partial t \pm v \partial / \partial$

± **According to forward and backward**

Lorentz equation:

$$\frac{\partial \gamma_i}{\partial t} + v\frac{\partial \gamma_i}{\partial z} = \frac{ev}{mc^2}\left(E+E^{sc}\right)e^{i\psi_i} \quad + c.c.$$

$$\frac{\partial \psi_i}{\partial t} + v\frac{\partial \psi_i}{\partial z} = \frac{\omega_s}{\beta^2\gamma^2}\frac{\left(\gamma_i-\gamma_p\right)}{\gamma_p}$$

[*] First obtained for microwave circuit by N. S. Ginzburg et al., Sov. Radiophys. Electron., 21, 728 (1979), See also B. Levush et al., IEEE Trans. Plasma Sci., 20, 263 (1992).

Boundary Conditions for a BWO

- No bunching at the entrance of the grating:

$$\psi_i(\varsigma = 0, \tau) = 0$$

- No energy modulation at the entrance

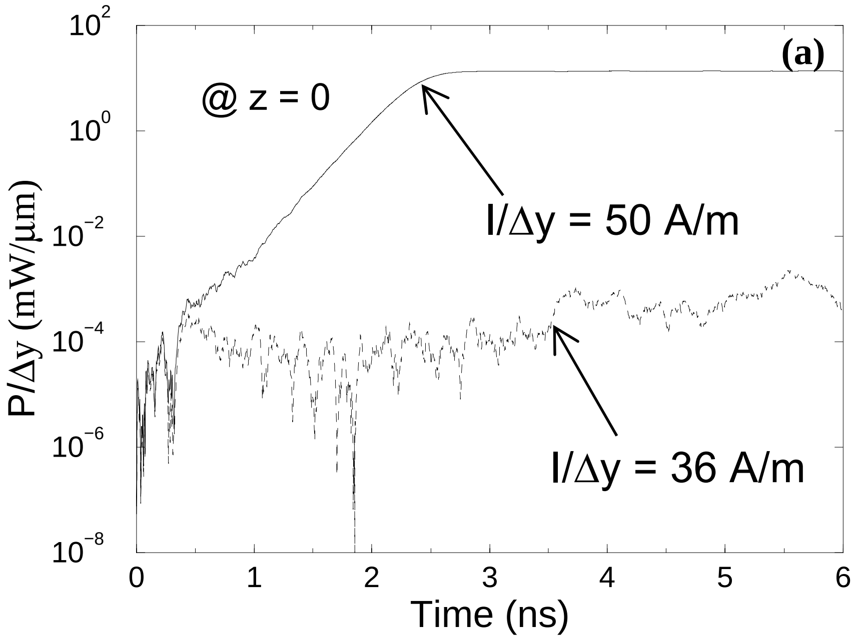
$$\eta_i(\varsigma = 0, \tau) = 0$$

- Optical energy propagates towards the entrance:

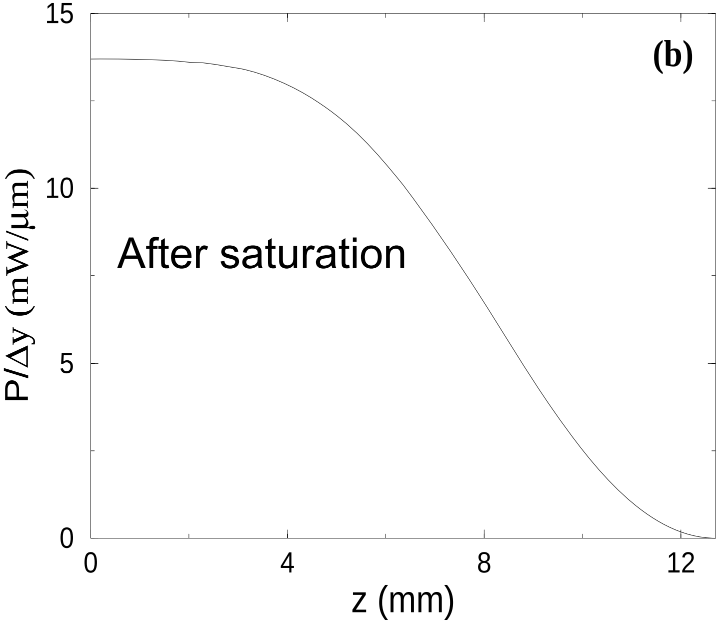
$$\varepsilon(\varsigma = 1, \tau) = 0$$

Distributed feedback→Oscillation if curent is higher than a threthold value

Simulation Results: Start Current and Saturation



Energy conversion efficiency = 0.8%



For I/Δy = 50 A/m, at saturation, P/Δy = 13.7 mW/μm

Power e-folding time = 0.2 ns (simulation)

0.17 ns (analytic formula)

Lasing wavelength = 694.5 μm (simulation)

694 μm (analytic formula)

Analytic Solution in the Linear Regime (cont'd)

- Nontrivial solution if

$$\left(\kappa_1^2 - Q\right)\left(\kappa_2 - \kappa_3\right)e^{\kappa_1} + \left(\kappa_2^2 - Q\right)\left(\kappa_3 - \kappa_1\right)e^{\kappa_2} + \left(\kappa_3^2 - Q\right)\left(\kappa_1 - \kappa_2\right)e^{\kappa_3} = 0$$

- This is a transcendental equation on ν. Find that there is a threshold value of J above which ν has a positive real part.

$\Rightarrow$ Start current condition

$$\frac{I_s}{\Delta y} = 7.685\, I_A \frac{\beta^4 \gamma^4 \lambda}{2\pi^2 \chi L^3} e^{2\Gamma_0 b}$$

Start-Current for 2D Sheet Beam

$$\frac{dI}{dx} > \frac{dI_S}{dx} = J_S(\eta)\frac{I_A}{2\pi\chi}\frac{\beta^3\gamma^4}{kL^3}e^{2\Gamma_0 d}$$

d: distance of the sheet beam from the grating surface

Γ_o : $\sqrt{k_z^2 - k^2}$

k: $2\pi/\lambda$

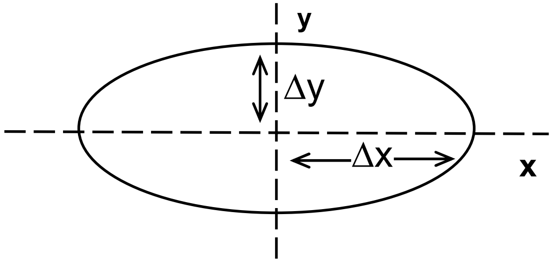
(k_z, α): (Real, Imaginary) part of the propagation constant

χ : Residue of the singularity

$J_s(\eta)$: dimensionless start current ($\eta = \alpha L$)

Beam Design for SP-FELs

- For clarity, assume KV distribution



$\sigma_x = \Delta x/2$

$\sigma_y = \Delta y/2$

- Choose β* ≥ L at the grating center (beam size variation is small)

- For a good overlap of evanescent wave with e-beam

$$d \le 1/2\Gamma_o,\ \Delta y = 1/2\Gamma_o, B_y{}^* = L,\ \varepsilon_y \le \frac{\beta\gamma}{(4\Gamma_0)}\frac{1}{L}.$$

- ***Mode behavior in (x,z) similar to free space wave*** λ_z

$$\varepsilon_x \le \beta\gamma\frac{\lambda_z}{4\pi}$$

- ***Electron beam x-size the same as the optical beam***

$$\Delta x = 2\sqrt{\frac{\lambda_z L}{4\pi}}$$

Operation of 3D Smith-Purcell BWO

- Start current condition

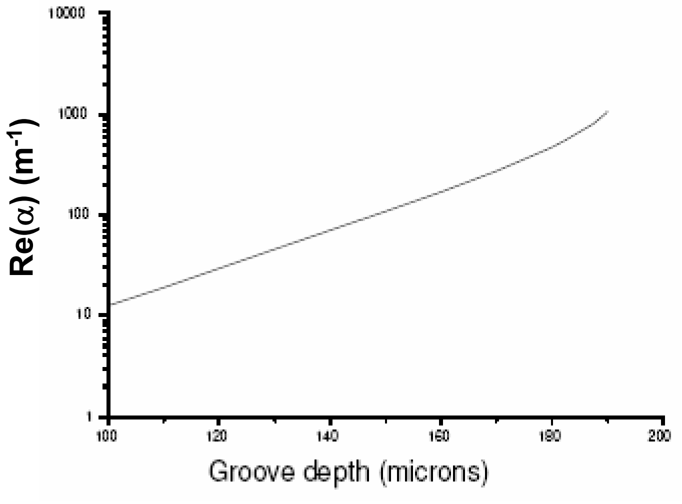
$$I > I_S = \frac{\pi}{2}\frac{dI_S}{dx}\Delta x$$

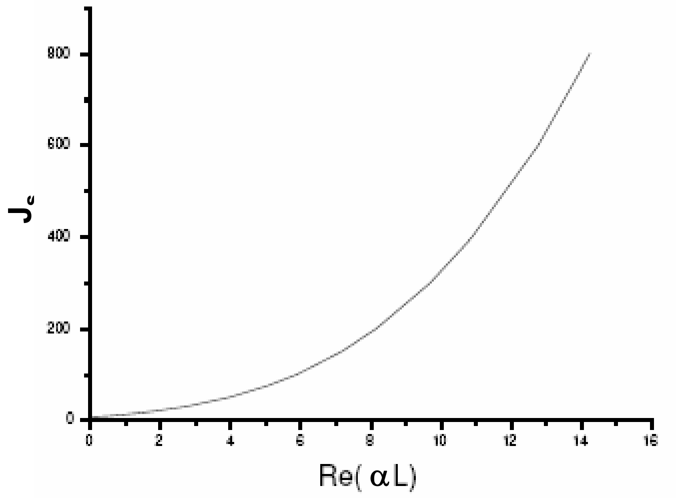
- Efficiency

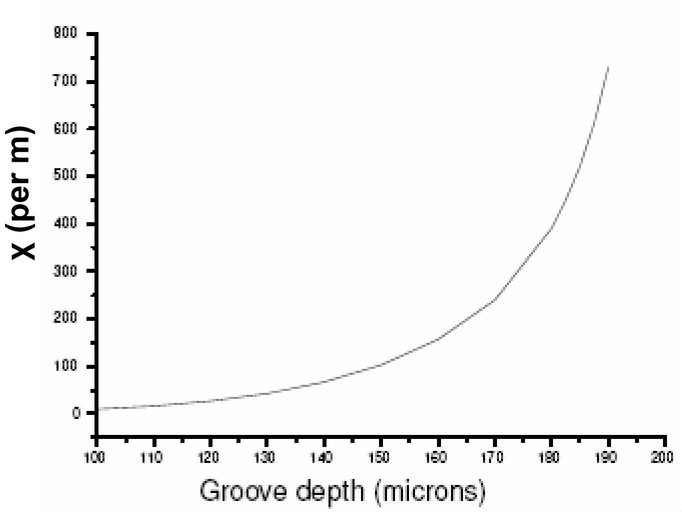
$$P^{opt} \geq P_s^{opt} = \eta_{eff} V I_S \qquad \eta_{eff} \approx \frac{\lambda_z}{2L}$$

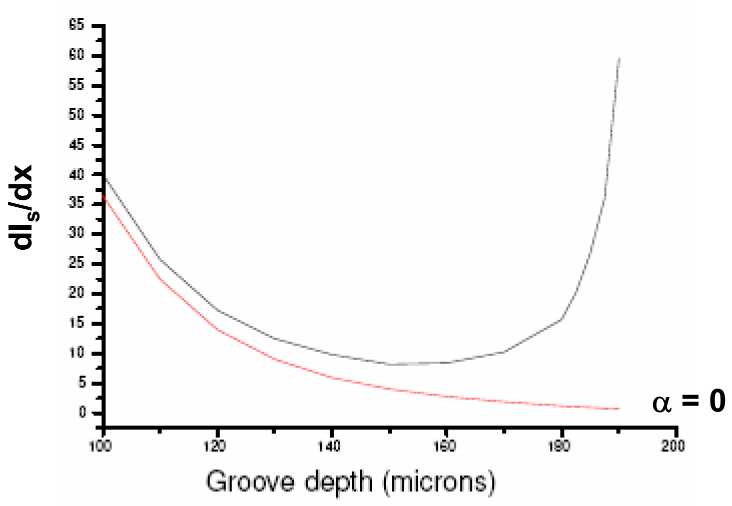
- Outcoupling via
  - Mode conversion at entrance
  - Bunched beam radiation at exit
  - Propagating wave at harmonic frequencies

Grating Optimization (qV = 35 keV)









Example Parameters

Grating

$\lambda g = 173\ \mu\text{m}$

$h = 150\ \mu\text{m}$

$L = 1.27\ \text{cm}$

Electron Beam

$qV = 35\ \text{keV}$

$\beta = 0.352$

$\Delta y = 23.7\ \text{mm}$

$\varepsilon_y \leq 4.15\times10^{-9}\ \text{m-r}$

$\varepsilon_x \leq 8.37\times10^{-6}\ \text{m-r}$

$\Delta x = 1.1\ \text{mm}$

$I_s = 13.4\ \text{mA}$

$P_s^{ebeam} = 470\ \text{W}$

BWO

$\lambda = 790\ \mu$

$\lambda_z = 278\ \mu\text{m}$

$\upsilon_g = -0.1\ c$

$\eta_{eff} \sim 1\%$

$P_S^{opt} \approx 5\text{W}$

A Thermionic Line Source

- Thermionic cathode (planar, elliptical)

$$\left(\varepsilon_{cx},\varepsilon_{cy}\right)=0.5\left(\Delta x_c,\Delta y_c\right)\sqrt{k_B T/mc^2}$$

- T = 2500 K $\sqrt{k_B T/mc^2}=0.65\times10^{-3}$

- $\Delta x_c \leq 26$ mm, $\Delta y_c \leq 13$ μm

- $J_c=\dfrac{I_s}{\pi(\Delta x_c)(\Delta y_c)}\geq 1.3\,A/cm^2$

- Tungsten: J_c = 1-3 A/cm^2

Flat Beam Technique

- Start with angular momentum-dominated round source

$$\frac{\varepsilon_x}{\varepsilon_y} = \left(\frac{qB\, r_c^{\,2}}{mc\, 4\varepsilon_l} \right)^2$$

- $r_c = 2\varepsilon_l \Big/ \sqrt{k_B T / mc^2}, \quad B = k_b T \big/ \left(q \varepsilon_y c \right)$

- Choose: $\varepsilon_y = 4.15 \times 10^{-9}$ m-r
- T = 2500 K ⇒ B = 1.73 kG
- Choose $\varepsilon_x/\varepsilon_y$ = 600 (A_o experiment: 100)

$$\Rightarrow \varepsilon_x \big/ \varepsilon_y = 600 \quad (A_o \text{ experiment} : 100)$$

- $\varepsilon_l = 1 \times 10^{-7}$ m-r, r_c = 314 μm
- $J_c = 4.3$ A/cm^2

Conclusions

- We have developed a theory of SP-FELs driven by sheet beams operating as a BWO, using Maxwell-Lorentz equations.
- Simple formula for start current is derived from linear analysis .
- Results from a simulation code based on Maxwell-Lorentz equations agree with linear theory where applicable and give saturation behavior.
- The sheet beam theory can be used for designing a **portable** SP FEL for THz radiation.