An analysis of nonlinear harmonic generation in high gain free electron laser

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Content

Basic Equations

NHG in SASE

NHG in HGHG

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Basic Equations

$$\frac{d}{dz}\tilde{a}_{sn} \simeq \frac{r_e n_e a_u \delta_{pn} \lambda_{sn}}{\gamma}\left\langle e^{-in\phi}\right\rangle$$

SVEA for all harmonic

$$\frac{d^2\phi}{dt^2} = -\frac{c^2}{\gamma^2} 2a_u k_u \operatorname{Re}\sum_j \delta_{pj} k_{sj} \tilde{a}_{sj} e^{ij\phi}$$

$\delta_p = [J,J]_n$, n=1,3,5,.....

$$\phi = (k_{s1} + k_u)z - \omega_1 t = \phi_0 + \phi_0{}'z + \Delta\phi$$

$$\Delta\phi = \sum_j \Delta\phi_j$$

phase change due to interaction with *j*th harmoic optical field:

$$\Delta\phi_j = \operatorname{Re} A_j e^{ij\phi_0} = \left|A_j\right| \cos(j\phi_0 + \psi_j),$$

$$A_j = \left|A_j\right| e^{i\psi_j} = -\frac{2a_u k_u \delta_{pj} k_{sj}}{\gamma^2} e^{ij\phi_0{}'z} i\int \frac{\partial}{\partial(j\phi_0')}\tilde{a}_{sj} e^{-ij\phi_0{}'(z-z')} dz'$$

Using

$$e^{-in\Delta\phi_j} = e^{-in|A_j|\cos(j\phi_0+\psi_j)} = \sum_{m=-\infty}^{\infty} (-i)^m J_m(n|A_j|)e^{im(j\phi_0+\psi_j)}$$

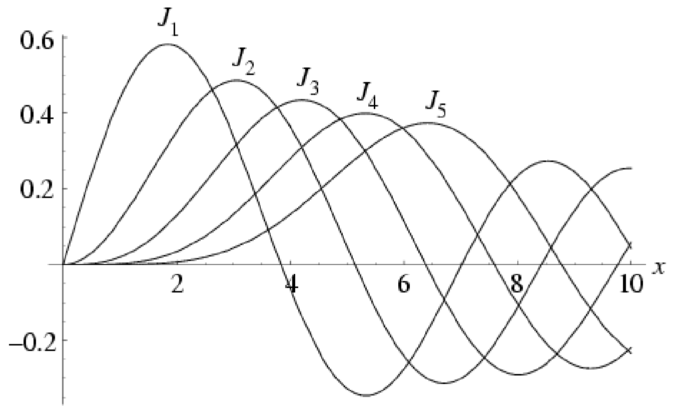
After average over uniform ϕ_0

$$\frac{d}{dz}\tilde{a}_{sn} \simeq \frac{r_e n_e a_u \delta_{pn} \lambda_{sn}}{\gamma} \left\langle e^{-in\phi_0' z}(-i)^n \sum_{\Sigma m_j j=n} \prod_j J_{m_j}(n|A_j|)e^{im_j\psi_j} \right\rangle_{\phi_0'}$$

$$\sum_j jm_j = n$$

all harmonic interaction contribute

but many of them can be ignored



$$\frac{d}{dz}\tilde{a}_{sn} \sim \sum_{\Sigma m_j j=n} \prod_j J_{m_j}(n|A_j|)e^{im_j\psi_j}$$

discussion

- Small A_j

$$A_j = -\frac{2a_u k_u \delta_{pj} k_{sj}}{\gamma^2} e^{ij\phi_0' z} i \int \frac{\partial}{\partial(j\phi_0')} \tilde{a}_{sj} e^{-ij\phi_0'(z-z')} dz'$$

$$\frac{d}{dz}\tilde{a}_{sn} \sim \sum_{\Sigma jm_j=n} \prod_j A_j^{m_j}$$

$$\sum_j jm_j = n$$

$$J_{m_j}(n|A_j|) = \left(\frac{-inA_j e^{-ij\phi_0' z}}{2}\right)^{m_j} \sum_{j=0}^{\infty} \frac{(-1)^j}{j!(j+m_j)!}\left(\frac{n|A_j|}{2}\right)^{2j}$$

$$\frac{d}{dz}\tilde{a}_{sn} \sim \sum_{\Sigma m_j j=n} \prod_j J_{m_j}\left(n\left|A_j\right|\right)e^{im_j\psi_j} \qquad \sum_j jm_j = n$$

- Small signal

$$\frac{d}{dz}\tilde{a}_{sn} \sim J_1\left(n\left|A_n\right|\right)e^{i\psi_n}$$

no coupling

First term of power series

Linear case

Small signal gain

$$g = -n\left(\frac{\delta_{pn}}{\delta_{p1}}\right)^2 (4\pi N\rho)^3 \frac{\partial}{\partial x}\sin c^2\frac{x}{2}, \quad x=n\phi_0'L$$

n=1: fundamental case

- High gain: NHG, fundamental dominant

$$\frac{d}{dz}\tilde{a}_{sn} \approx \frac{r_e n_e a_u \delta_{pn}\lambda_{sn}}{\gamma}\left\langle J_n\left(n\left|A_1\right|\right)e^{in(\psi_1-\phi_0'z)}\right\rangle_{\phi_0'}(-i)^n$$

n=1: result of:

J. C.Gallardo *et.al*. Phys Rev.A36(1987)3222

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NHG in SASE

$$\frac{d}{dz}\tilde{a}_{sn} \approx \frac{r_e n_e a_u \delta_{pn} \lambda_{sn}}{\gamma} \left\langle J_n(n|A_1|) e^{in(\psi_1 - \phi_0' z)} \right\rangle_{\phi_0'} (-i)^n$$

$$= \frac{r_e n_e a_u \delta_{pn} \lambda_{sn}}{\gamma} \left\langle \left(\frac{-inA_1 e^{-i\phi_0' z}}{2} \right)^n \sum_{m=0}^{\infty} \frac{(-1)^m}{m!(m+n)!} \left(\frac{n|A_1|}{2} \right)^{2m} \right\rangle_{\phi_0'}$$

Consider exponential gain, mono-energetic resonant e-:

($z > 3\ L_g$, $\phi_0' = 0$)

$$\tilde{a}_{s1} \approx \frac{a_{ef}}{3} e^{\mu_1 z}, \quad \mu_1 = k_u \rho (i + \sqrt{3})$$

$$e^{-i\phi_0' z} A_1 = -\frac{2a_u \delta_{p1} k_u k_{s1}}{\gamma^2} i \int \frac{\partial}{\partial \phi_0'} \tilde{a}_{s1} e^{-i\phi_0'(z-z')} dz'$$

$$\approx -\frac{2a_u \delta_{p1} k_u k_{s1}}{\gamma^2} i \frac{a_{s0}}{3} \frac{\partial}{\partial \phi_0'} \frac{e^{\mu_1 z}}{\mu_1 + i\phi_0'} \Bigg|_{\phi_0' = 0} = -\frac{2a_u \delta_{p1} k_u k_{s1}}{\gamma^2} \frac{\tilde{a}_{s1}}{\mu_1^2}$$

$$|A_1|^2 \approx \left(\frac{2a_u \delta_{p1} k_u k_{s1}}{\gamma^2} \right)^2 \left| \frac{\tilde{a}_{s1}}{\mu_1^2} \right|^2 = \frac{4P_1}{\rho P_e}$$

$$\psi_1 = k_u \rho z - \frac{\pi}{3}$$

$\because$ $P_1 < P_s \sim \rho P_e$

$\therefore$ $A_j <\sim 1$

$\tilde{a}_{ef}$: effective input power of SASE, see my next report

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Further approximate, before saturation

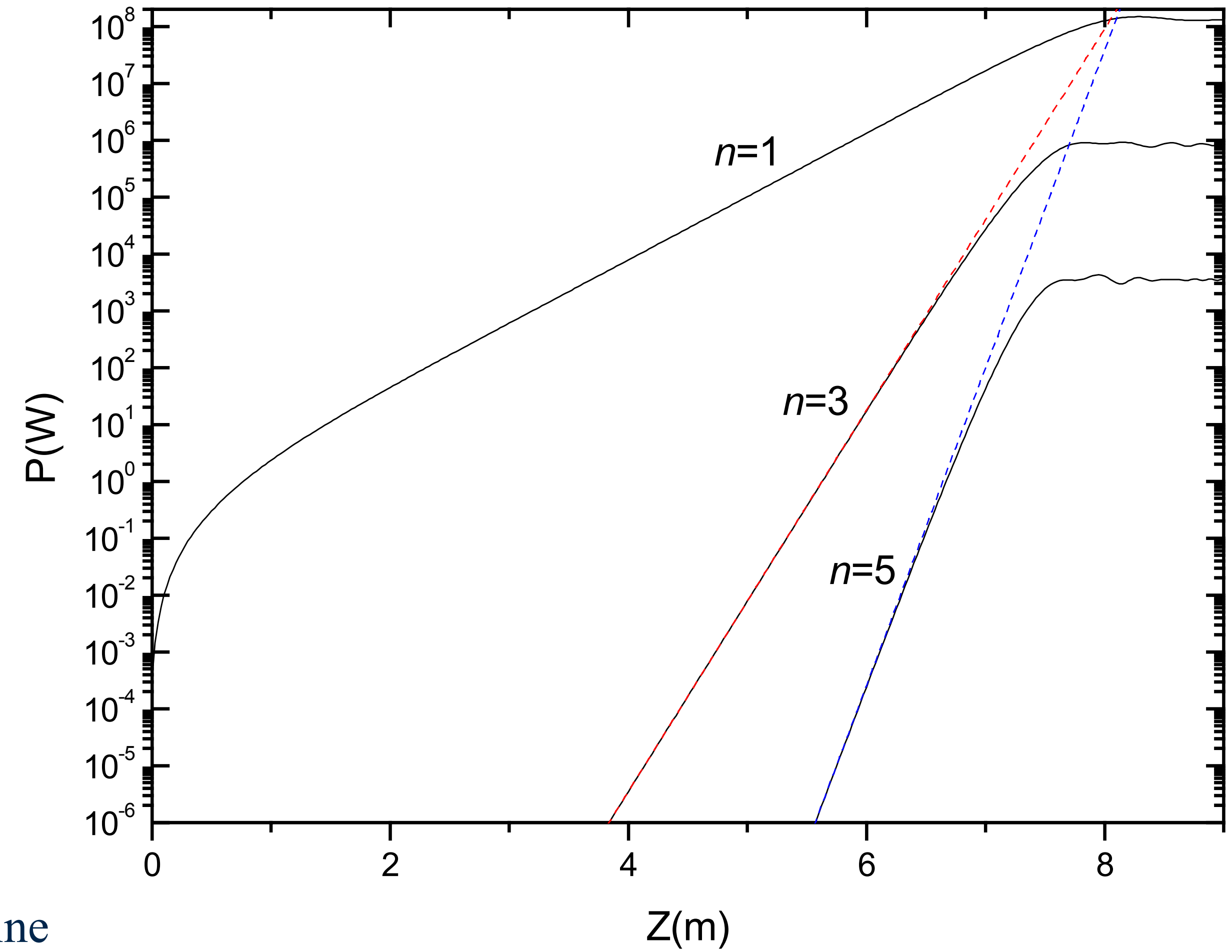
$$\frac{d}{dz}\tilde{a}_{sn} \approx \frac{r_e n_e a_u \delta_{pn} \lambda_{sn}}{\gamma n!}\left(\frac{-inA_1}{2}\right)^n$$

$$\frac{P_n}{\rho P_e} \approx \left(\frac{n^{n-1}\delta_{pn}}{n!\delta_{p1}}\right)^2 \left(\frac{P_1}{\rho P_e}\right)^n = \left(\frac{n^{n-1}\delta_{pn}}{n!\delta_{p1}}\right)^2 \left(\frac{P_{10}}{9\rho P_e}\right)^n e^{n\frac{z}{L_g}}$$

$\delta_p = [J,J]_n$, n=1,3,5,....

Numerical result

(for LEUTL FEL)



Solid line

$$\frac{d}{dz}\tilde{a}_{sn} \approx \frac{r_e n_e a_u \delta_{pn} \lambda_{sn}}{\gamma} \left\langle J_n(n|A_1|) e^{in(\psi_1 - \varphi_0' z)} \right\rangle_{\varphi_0'} (-i)^n$$

Dashed line

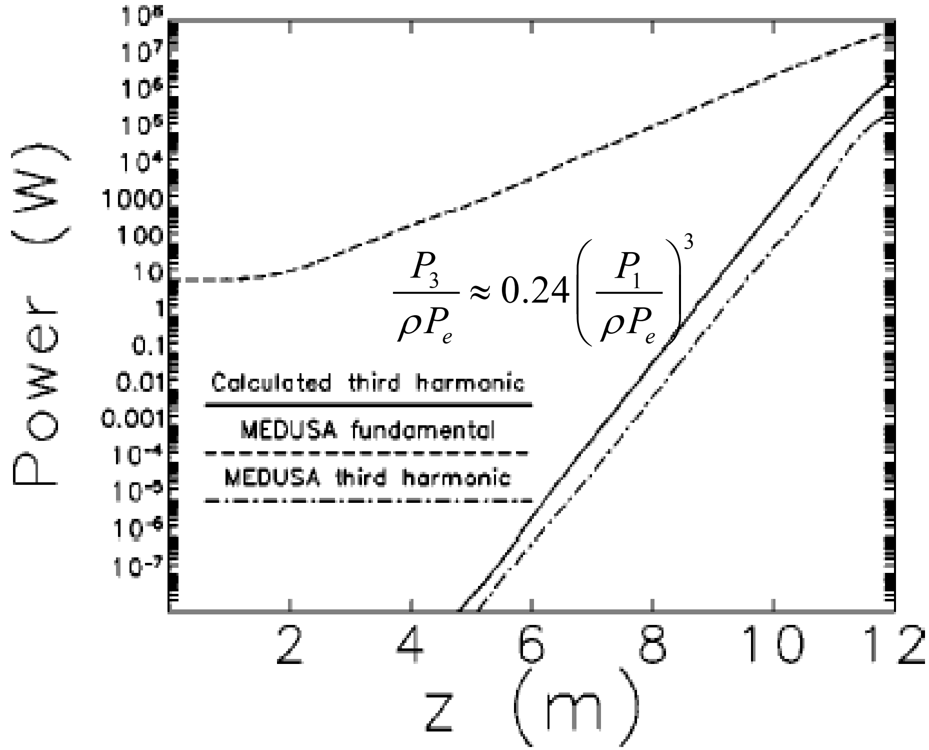
$$\frac{P_n}{\rho P_e} \approx \left(\frac{n^{n-1}\delta_{pn}}{n!\,\delta_{p1}} \right)^2 \left(\frac{P_1}{\rho P_e} \right)^n$$

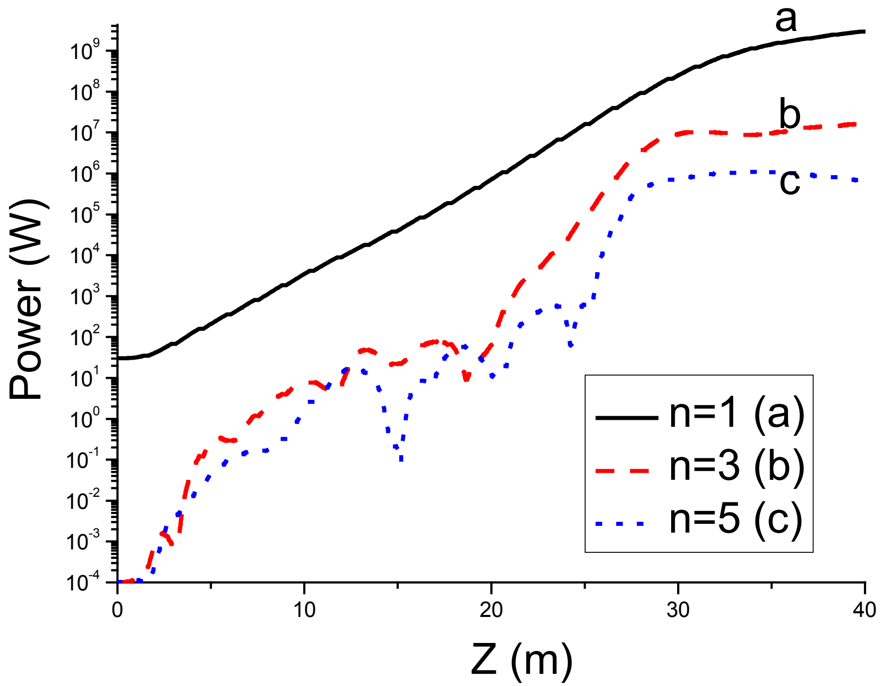
LEUTL FEL:

	here	simulation (MEDUSA)*
Saturation	P_{3s}=0.9MW	2.67MW
	P_{1s}=148MW	70MW

$$\frac{P_3}{\rho P_e} \approx 0.13\left(\frac{P_1}{\rho P_e}\right)^3$$

* Z.Huang，K.-J.Kim，Phys.Rev.E 62(5), (2000)7295;





Primary numerical result (for HTF)
from basic Eq.s (program TDH1D)

NHG in HGHG

In the gain section of HGHG*:

$$\frac{d}{dz}\tilde{a}_{sn_2} \simeq \frac{r_e n_e a_{u2} \delta_{pn_2} \lambda_{sn_2}}{\gamma} \left\langle e^{-in_2\phi_2} \right\rangle$$

$\delta_{pn2}(a_{u2})$

$$\phi_2 = \underline{\phi}_{20} + \phi_{02}' z_2 + \Delta\phi_{20} + \sum_j \Delta\phi_j$$

$n_1=\lambda_{r1}/\lambda_{r2}$, $n_2=\lambda_{r2}/\lambda_s$

$$\underline{\phi}_{20} = n_1\phi_{10} + n_1\phi_{10}'(l_1 + N_d\lambda_{u1}) \qquad \phi_{02}' = \phi_{10}'\lambda_{u1}/\lambda_{u2}$$

Bunching due to seeding laser $(\lambda_{s0}=\lambda_{r1})$

$$\Delta\phi_{20} \simeq -n_1\Delta\xi\cos(\phi_{10} + \frac{\phi_{10}' l_1}{2})$$

$$\Delta\xi = \frac{2k_{u1}k_{s1}a_{s1}a_{u1}\delta_{p1}}{\gamma^2} l_1 (N_d\lambda_{u1} + \frac{l_1}{2} + z_2\frac{\lambda_{u1}}{n_1\lambda_{u2}})$$

$\delta_{p1}(a_{u1})$

*JiaQika,; Nucl.Instr. & Meth. A**519**, (2004) 489

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Bunching due to harmonic

$$\Delta\phi_j \simeq -\frac{2a_{u2}k_{u2}\delta_{pj}k_{sj}}{\gamma^2}\,\mathrm{Re}\,e^{ij(\underline{\phi}_{20}+\Delta\phi_{20}+\phi_{02}'z)}\,i\int\frac{\partial}{\partial(j\phi_{02}')}\tilde{a}_{sj}e^{-ij\phi_{02}'(z-z')}dz'$$

$$= \mathrm{Re}\,A_j e^{ij(\underline{\phi}_{20}+\Delta\phi_{20})} = \left|A_j\right|\cos(j(\underline{\phi}_{20}+\Delta\phi_{20})+\psi_j)$$

$$\Delta\phi_{20} \simeq -n_1\Delta\xi\cos(\phi_{10}+\frac{\phi_{10}'l_1}{2})$$

Using

$$e^{-in_2\Delta\varphi_j} = \sum_{m=-\infty}^{\infty}(-i)^m J_m(n_2\left|A_j\right|)e^{im(j\underline{\phi}_{20}+j\Delta\phi_{20}+\psi_j)}$$

$$\underline{\phi}_{20} = n_1\phi_{10} + n_1\phi_{10}'(l_1+N_d\lambda_{u1})$$

We obtain

$$\frac{d}{dz}\tilde{a}_{sn_2} = \frac{r_e n_e a_{u2}\delta_{pn_2}\lambda_{sn_2}}{\gamma} *$$

$$\left\langle e^{-in_2\phi_{02}'z}\sum_{\sum_j(jm_jn_1+h_j)=n_1n_2}\prod_{j=1}(-i)^{m_j+h_j}J_{m_j}(n_2\left|A_j\right|)e^{im_j\psi_j}J_{h_j}(n_1(jm_j-n_2\delta_{1j})\Delta\xi)e^{-i(N_d\lambda_u+\frac{l_1}{2})h_j\phi_{10}'}\right\rangle_{\phi_{10}'}$$

$$jm_jn_1 + h_j = n_1n_2$$

$$\delta_{1j} = \begin{cases}1, & (j=1)\\ 0, & (j\neq 1)\end{cases}$$

Dominant: seeding, and fundamental of gain section

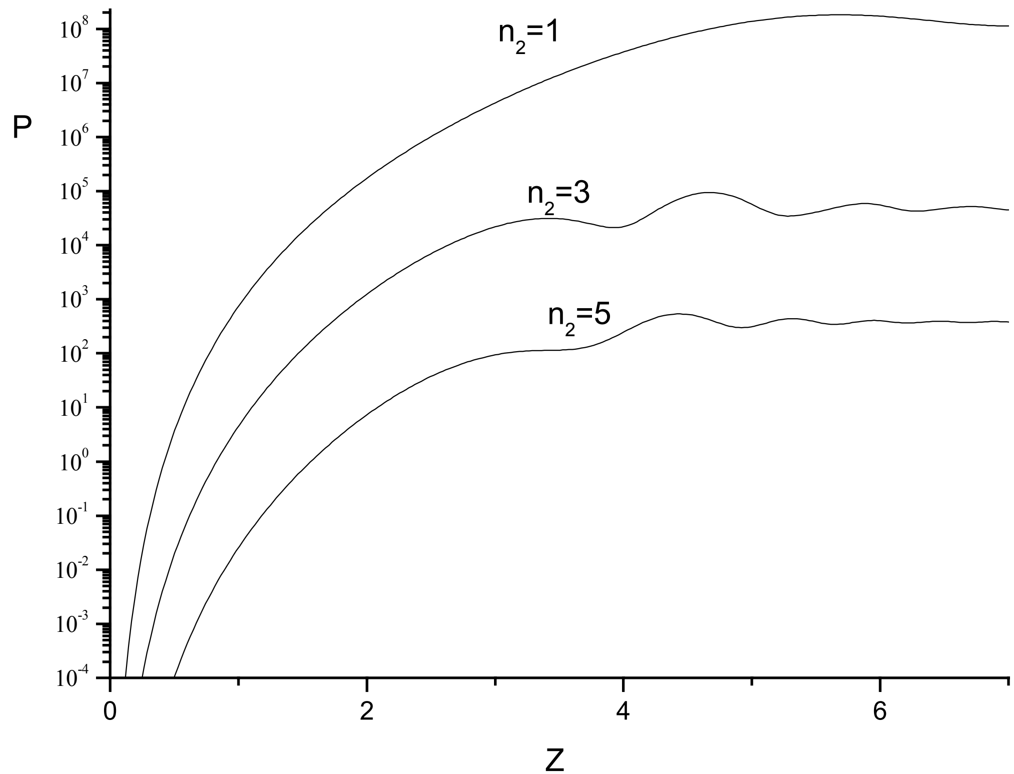
$$\frac{d}{dz}\tilde{a}_{sn_2} \simeq \frac{r_e n_e a_{u2}\delta_{pn_2}\lambda_{s2}}{\gamma}\left\langle e^{-in_2\phi_{02}'z}\sum_{(mn_1+h)=n_1n_2}(-i)^m i^h J_m(n_2|A_1|)J_h(h\Delta\xi)e^{im\psi_1}e^{-ih(N_d\lambda_u+\frac{l_1}{2})\phi_{10}'}\right\rangle_{\phi_{10}'}$$

$$mn_1 + h = n_1 n_2$$

$n_2=1$ $n_1=n$: HGHG

$$\frac{d}{dz}\tilde{a}_{s2} \simeq \frac{r_e n_e a_{u2}\delta_{p2}\lambda_{s2}}{\gamma}\left\langle e^{-i\phi_{02}'z}[i^{n_1}J_0(|A_1|)J_{n_1}(n_1\Delta\xi)e^{-i(N_d\lambda_u+\frac{l_1}{2})n_1\phi_{10}'}-iJ_1(|A_1|)e^{i\psi_1}]\right\rangle$$

$$\frac{d}{dz}\tilde{a}_{sn_2} \simeq \frac{r_e n_e a_{u2} \delta_{pn_2} \lambda_{s2}}{\gamma} \left\langle e^{-in_2\phi_{02}'z} \sum_{(mn_1+h)=n_1 n_2} (-i)^m i^h J_m(n_2|A_1|) J_h(h\Delta\xi) e^{im\psi_1} e^{-ih(N_d\lambda_u+\frac{l_1}{2})\phi_{10}'} \right\rangle_{\phi_{10}'}$$



Primary numerical result for NHG in HGHG

More detailed calculation are on going…

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Thank you