Quantum Regime of SASE-FEL

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Outline

1. Classical FEL, Superradiance and SASE

2. Quantum FEL model

3. Gain and spectrum in quantum FEL

4. Spectral “purification” in Quantum SASE

5. Fluctuations and energy spread effect

6. Towards a Quantum x-ray SASE-FEL

CLASSICAL MODEL

$$\frac{\partial^2 \theta_j}{\partial \bar{z}^2} = -(Ae^{i\theta_j} + c.c.)$$

$$\frac{\partial A}{\partial \bar{z}} + \frac{\partial A}{\partial z_1} = \frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_j}$$

R.Bonifacio, C.Pellegrini, L.Narducci, Opt. Comm. (1984)

$$\bar{z} = z / L_g$$

$$L_g = \lambda_w / 4\pi\rho$$ gain length

$$z_1 = (z - v_r t) / L_c$$

$$L_c = \lambda / 4\pi\rho$$ cooperation length

$$\theta = (k + k_w)z - \omega t;$$

$$\rho |A|^2 = \frac{P_{e.m.}}{P_{beam}}$$

$$\rho \propto \frac{1}{\gamma_r}\left(\frac{a_w \lambda_w}{\sigma}\right)^{2/3} I^{1/3}$$ FEL parameter

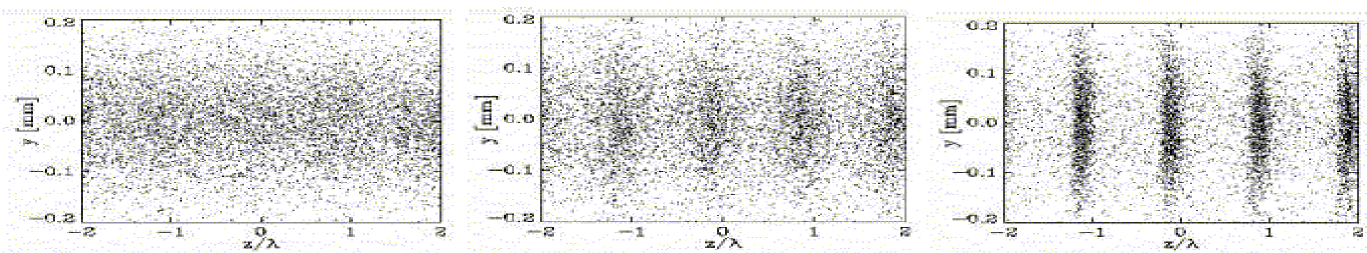
HIGH-GAIN REGIME

- exponential growth of intensity and bunching
- start up from noise
- saturation at A~1 (P_{rad} ~ρ P_{beam}) after several L_g

b~0 → b~0.8

bunching:

$$b = \frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_j}$$

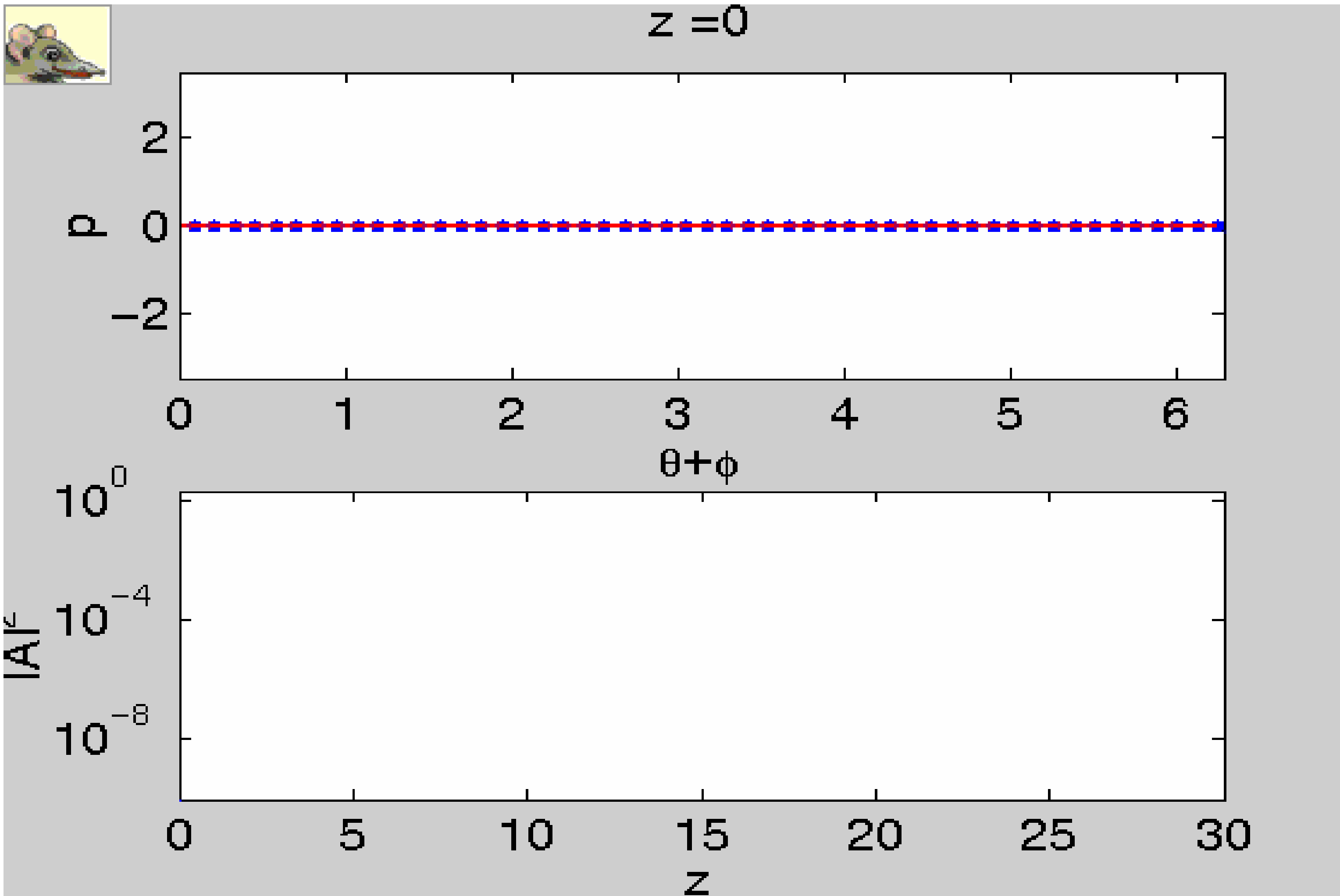


wiggler length (several L_g)

neglecting SLIPPAGE: $\frac{\partial A}{\partial z_1} = 0$ (STEADY-STATE regime)

electrons behave as coupled pendula in a self-consistent potential:

$$V(\theta + \varphi) = 2|A|\cos(\theta + \varphi)$$



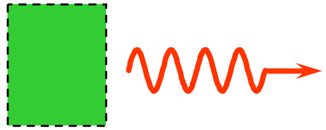
$$p = \frac{\gamma - \gamma_r}{\rho \gamma_r}$$

SLIPPAGE EFFECT: SUPERRADIANCE

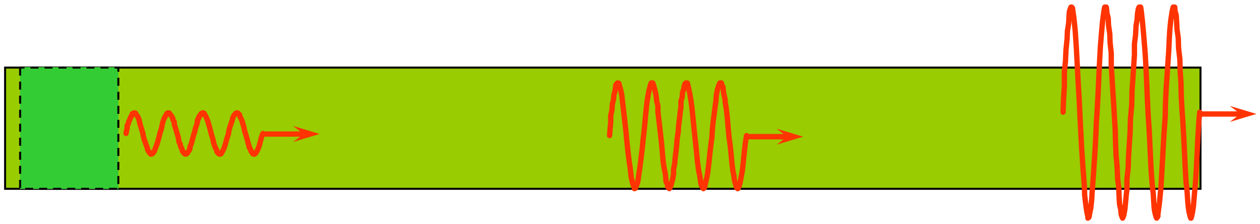
R. Bonifacio, B.W. McNeil, P. Pierini PRA (1989)

Particles at the trailing edge of the beam never receive radiation from particles behind them: they radiate in a SUPERRADIANT PULSE or SPIKE which propagates forward.

if $L_b << L_c$ the SR pulse remains small (weak SR).



if $L_b >> L_c$ the weak SR pulse gets amplified (strong SR) as it propagates forward through beam with no saturation.

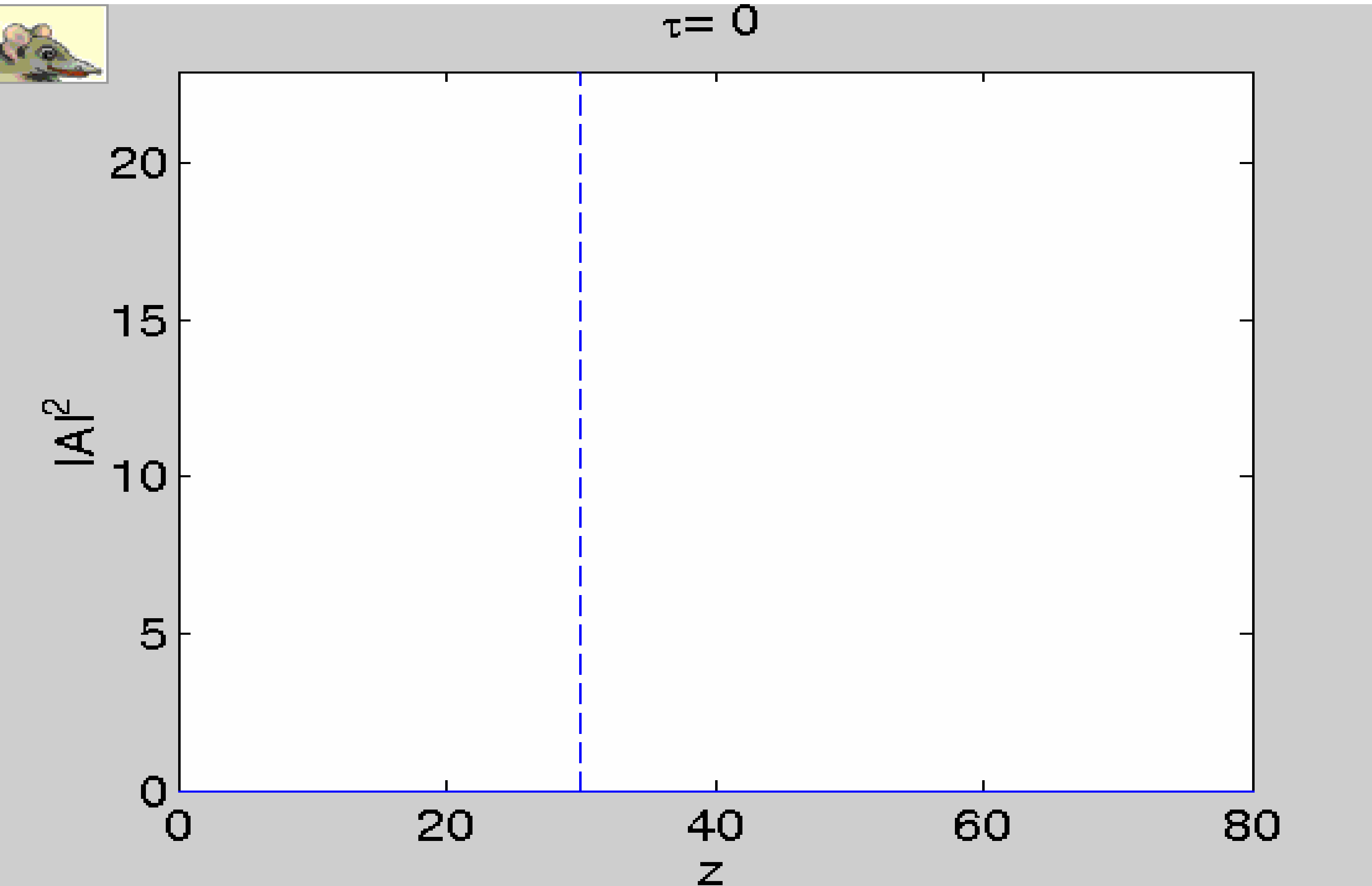


The SR pulse is a self-similar solution of the propagation equation.

STRONG SUPERRADIANCE
from a coherent seed

$$\frac{\partial A}{\partial \overline{z}} + \frac{\partial A}{\partial z_1} = \left\langle e^{-i\theta} \right\rangle \qquad \left(z_1 = \frac{z - vt}{L_c} \right)$$

L_b=30 L_c, 0<z<50 L_g resonant seed (δ=0)



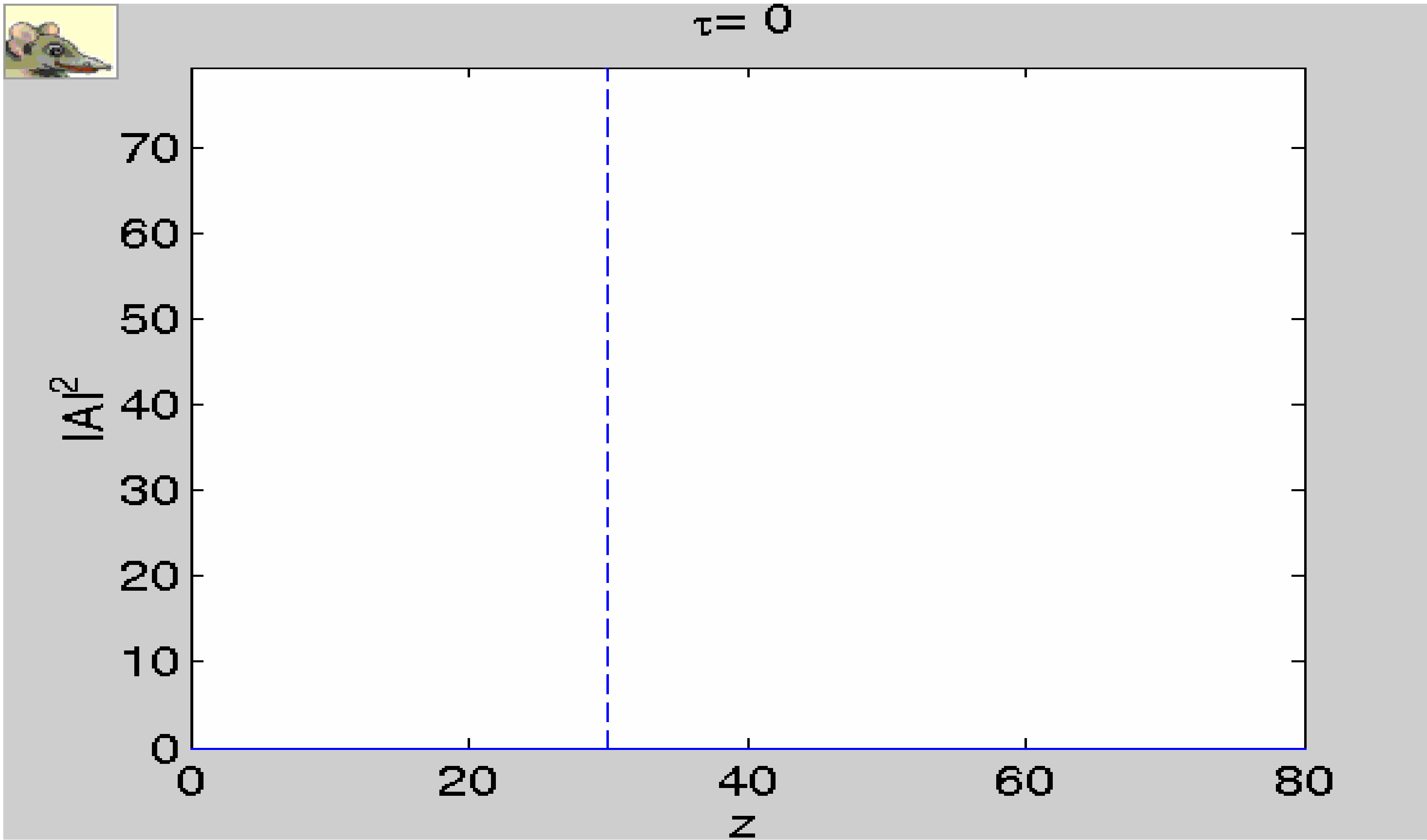
$\tau \rightarrow z/L_g$

$z \rightarrow (z\text{-}vt)/L_c$

STRONG SUPERRADIANCE
from a detuned coherent seed

$$\frac{\partial A}{\partial \overline{z}} + \frac{\partial A}{\partial z_1} = \left\langle e^{-i\theta} \right\rangle \qquad \left(z_1 = \frac{z - vt}{L_c} \right)$$

L_b=30 L_c, 0<z<50 L_g , detuned seed (δ=2)



SASE mode for FELs

Ingredients of Self Amplified Spontaneous Emission (SASE)

i) Start up from noise
ii) Propagation effects (slippage)
iii) Superradiance instability

⇩

R.Bonifacio, L. De Salvo, P.Pierini, N.Piovella, C. Pellegrini, PRL (1994)

each cooperation length in the e-beam radiates a SR spike which is amplified when it propagates forward on the beam

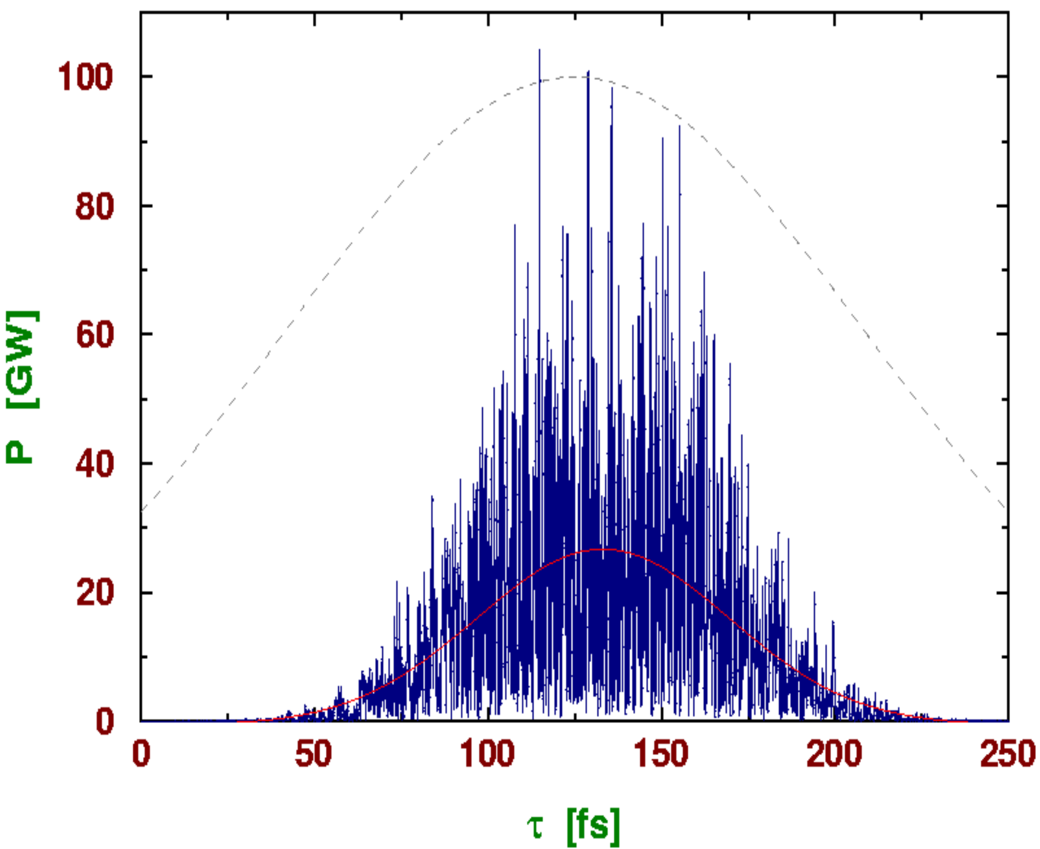
SASE mode is proposed as a method for producing 'coherent' X-ray radiation (LCLS, Desy,..)

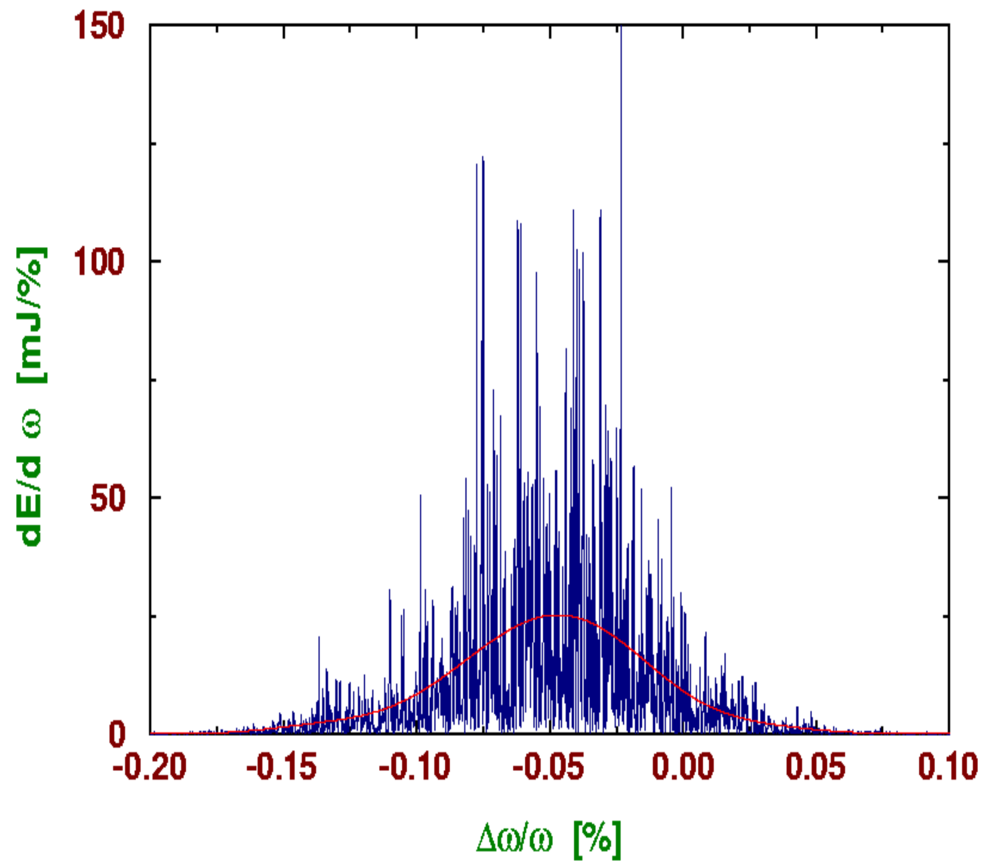
DRAWBACKS OF 'CLASSICAL' SASE

Time profile has many random spikes (n= L_b/L_c)

Broad and noisy spectrum at short wavelengths (x-ray FELs)

simulations from DESY for the SASE experiment





NEW QUANTUM-SASE REGIME

- In a QUANTUM REGIME an FEL behaves as a TWO-LEVEL system
- electrons emit coherent photons as in a LASER
- in the SASE mode the spectrum is intrinsically narrow (‘quantum purification’)
- the transition between the classical and the quantum regimes depends on a single parameter:

$$\overline{\rho} = \left(\frac{mc\ \gamma_r}{\hbar k} \right) \rho$$

QUANTUM EFFECTS IN FELs

CLASSICAL LIMIT OF FEL:

momentum spread: $mc\,(\delta\gamma) >> \hbar k$: photon recoil

Since in the classical regime $(\delta\gamma / \gamma_r) \approx \rho$ ⇨ $\bar{\rho} >> 1$ $\bar{\rho} = \left(\frac{mc\ \gamma_r}{\hbar k}\right)\rho$

many recoils implies many photons, hence..
classically, each electron emit many photons

$$\frac{< N >_{photon}}{N} = \bar{\rho}\,|A|^2 >> 1$$

(since A~1)

the QUANTUM REGIME of an FEL occurs when:

$$mc\,(\delta\gamma) \le \hbar k \quad \Rightarrow \quad \bar{\rho} < 1$$

each electron emits only a single photon! $(\bar{\rho}\,|A|^2 = 1)$

⇓

COHERENCE

Quantum FEL behaves like a two-level system
(i.e. a 'laser')

QUANTUM FEL MODEL

Procedure :

Describe N particle system as a Quantum Mechanical ensemble

↓

Write a Schrödinger equation for macroscopic wavefunction

$$\Psi(\theta, \bar{z})$$

Include slippage (i.e. propagation) using a multiple-scaling approach → $\Psi(\theta, \bar{z}, z_1)$

Canonical Quantization

$$\dot{\theta} = \frac{p}{\bar{\rho}} = \frac{\partial H}{\partial p}$$

$$\dot{p} = -\bar{\rho}\left(Ae^{i\theta} + c.c\right) = -\frac{\partial H}{\partial \theta}$$

$$H = \frac{p^2}{2\bar{\rho}} - i\bar{\rho}\left(Ae^{i\theta} - c.c.\right)$$

$$\left[p = \frac{mc(\gamma - \gamma_0)}{\hbar k}\right]$$

$$[\hat{\theta}, \hat{p}] = i \quad \Rightarrow \quad p \to \hat{p} = -i\frac{\partial}{\partial \theta} \quad \Rightarrow \quad i\frac{\partial \Psi}{\partial \bar{z}} = \hat{H}\Psi$$

$$i\frac{\partial \Psi}{\partial \bar{z}} = -\frac{1}{2\bar{\rho}}\frac{\partial^2 \Psi}{\partial \theta^2} - i\bar{\rho}\left[A(\bar{z})e^{i\theta} - c.c.\right]\Psi$$

$$\frac{dA}{d\bar{z}} = \int_0^{2\pi} d\theta \left|\Psi(\theta, \bar{z})\right|^2 e^{-i\theta} + i\delta A$$

G. Preparata, PRA (1988)

QUANTUM FEL PROPAGATION MODEL

θ describes spatial evolution of Ψ on scale of λ
z_1 describes spatial evolution of A and Ψ on scale of $L_c >> \lambda$.

$(z_1 = (z - vt)/L_c)$

Using the multiple-scale method we have derive the model:

$$i\frac{\partial \Psi}{\partial \bar{z}} = -\frac{1}{2\bar{\rho}}\frac{\partial^2 \Psi}{\partial \theta^2} - i\bar{\rho}\left[A(\bar{z}, z_1)e^{i\theta} - c.c.\right]\Psi$$

$$\frac{\partial A}{\partial \bar{z}} + \frac{\partial A}{\partial z_1} = \int_0^{2\pi} d\theta \left|\Psi(\theta, \bar{z}, z_1)\right|^2 e^{-i\theta} + i\delta A$$

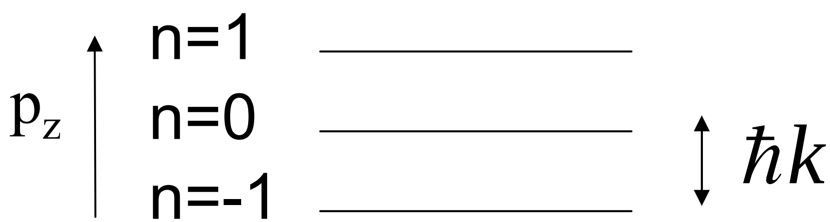
R. Bonifacio, N. Piovella, G.R.M. Robb, M. Cola, Optics Comm. (2005)

Momentum representation:

$$\Psi(\theta,\bar{z},z_1)=\sum_{n=-\infty}^{\infty}c_n(\bar{z},z_1)e^{in\theta}$$

$e^{in\theta}$ is the momentum eigenstate corresponding to eigenvalue $n(\hbar k)$

discrete changes of momentum : $p_z \sim mc(\gamma-\gamma_r)= n\,(\hbar k)$, $n=0,\pm1,..$



$$\frac{\partial c_n}{\partial \bar{z}}=-\frac{in^2}{2\bar{\rho}}c_n-\bar{\rho}\left(Ac_{n-1}-A^*c_{n+1}\right)$$

$$\frac{\partial A}{\partial \bar{z}}+\frac{\partial A}{\partial z_1}=\sum_{n=-\infty}^{\infty}c_n c_{n-1}^*+i\delta A$$

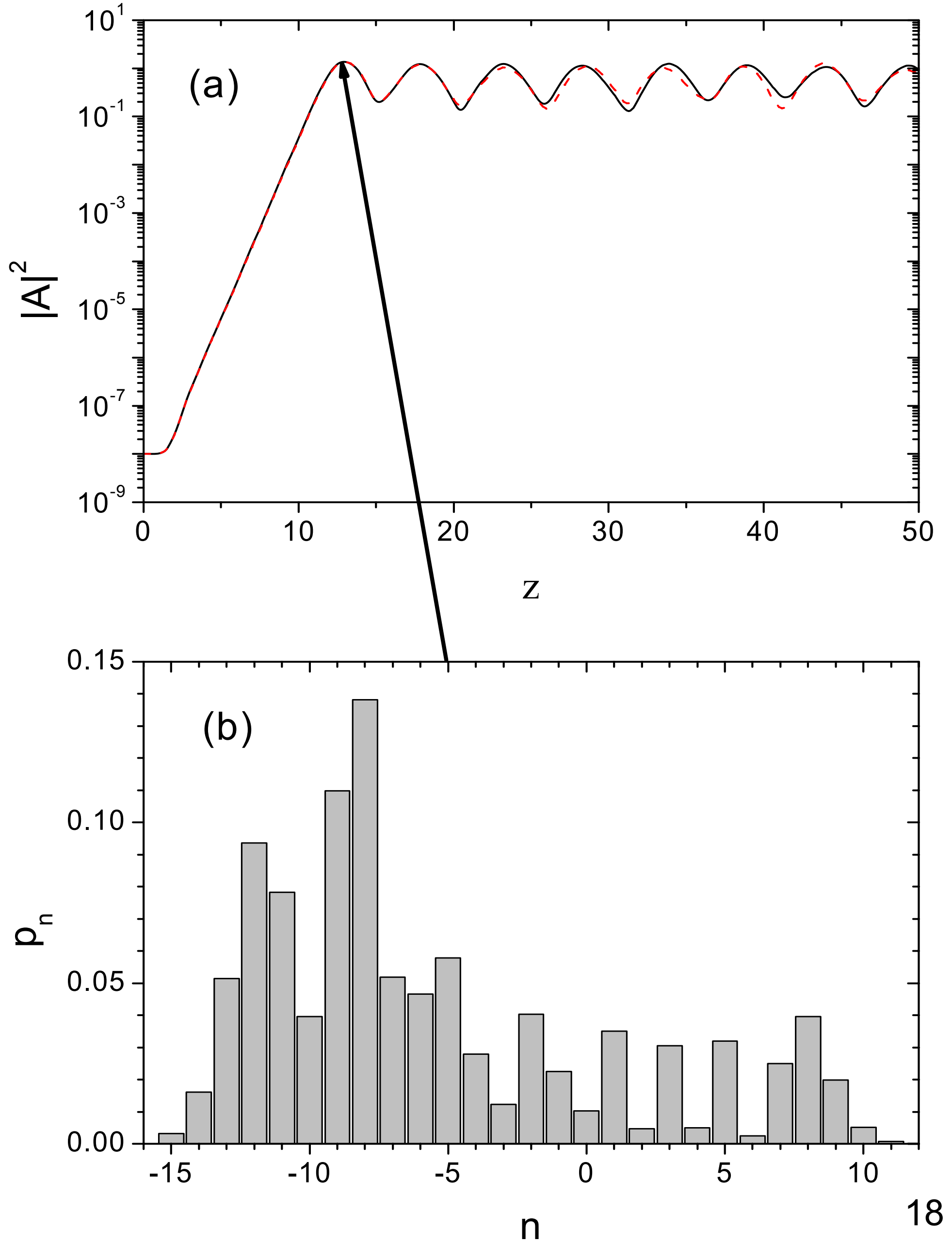
$$\left(\frac{\partial A}{\partial z_1} = 0 \right)$$

classical limit is recovered for

$$\bar{\rho} >> 1$$

many momentum states occupied, both with n>0 **and** n<0

$\bar{\rho}$=10, δ=0, no propagation

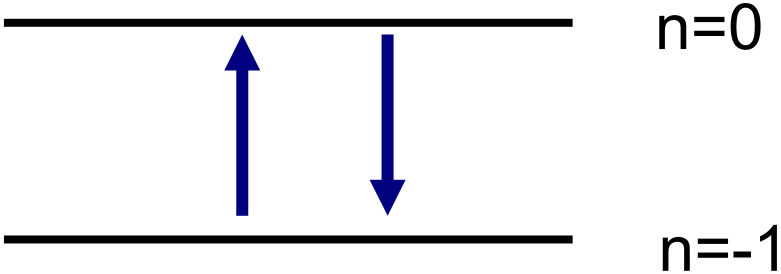


Quantum limit for $\bar{\rho} \leq 1$

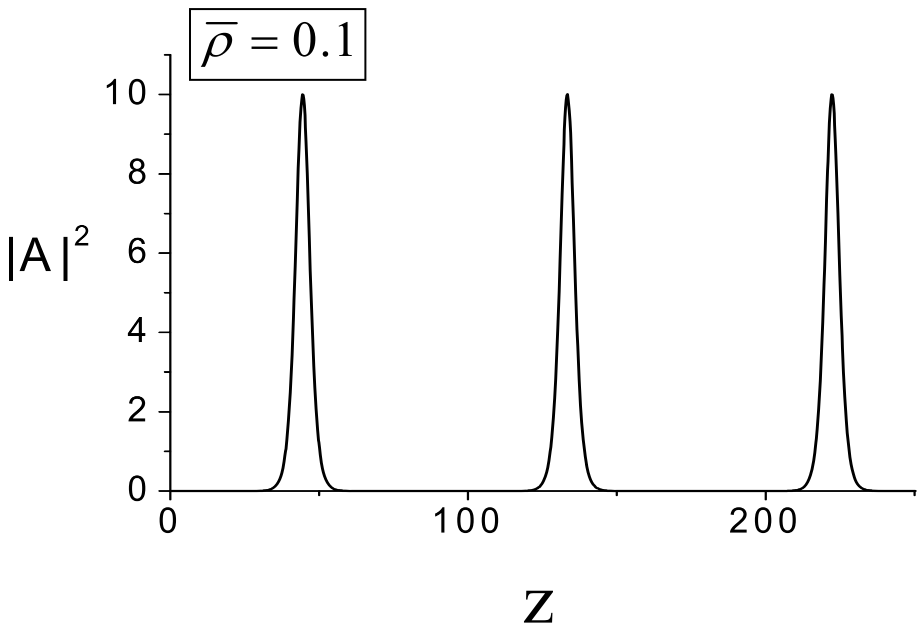
$$\left(\frac{\partial A}{\partial z_1} = 0 \right)$$

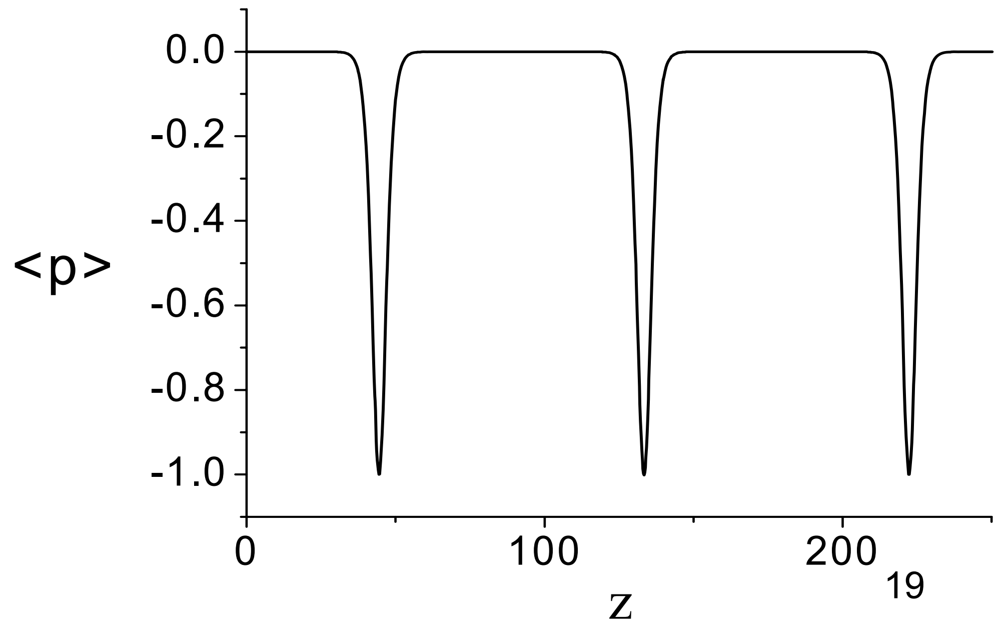
Only TWO momentum states : p=0 and p= - ħk

$$\Psi(\theta, \bar{z}) \propto c_0(\bar{z}) + c_{-1}(\bar{z}) e^{-i\theta}$$



Dynamics are those of a 2-level system coupled to an optical field, as in a LASER

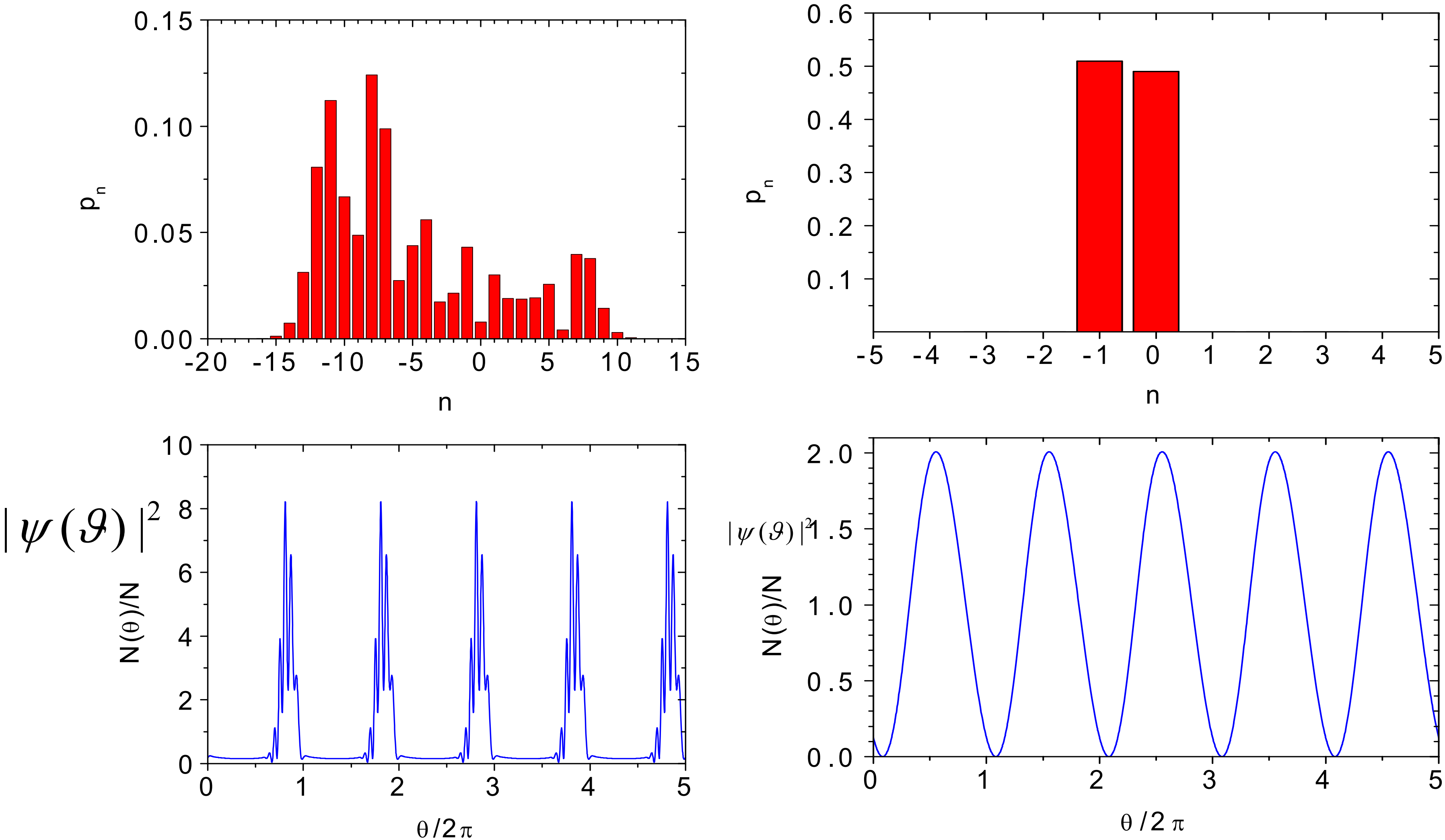




Bunching and density grating

CLASSICAL REGIME $\overline{\rho}>>1$

QUANTUM REGIME $\overline{\rho}<1$



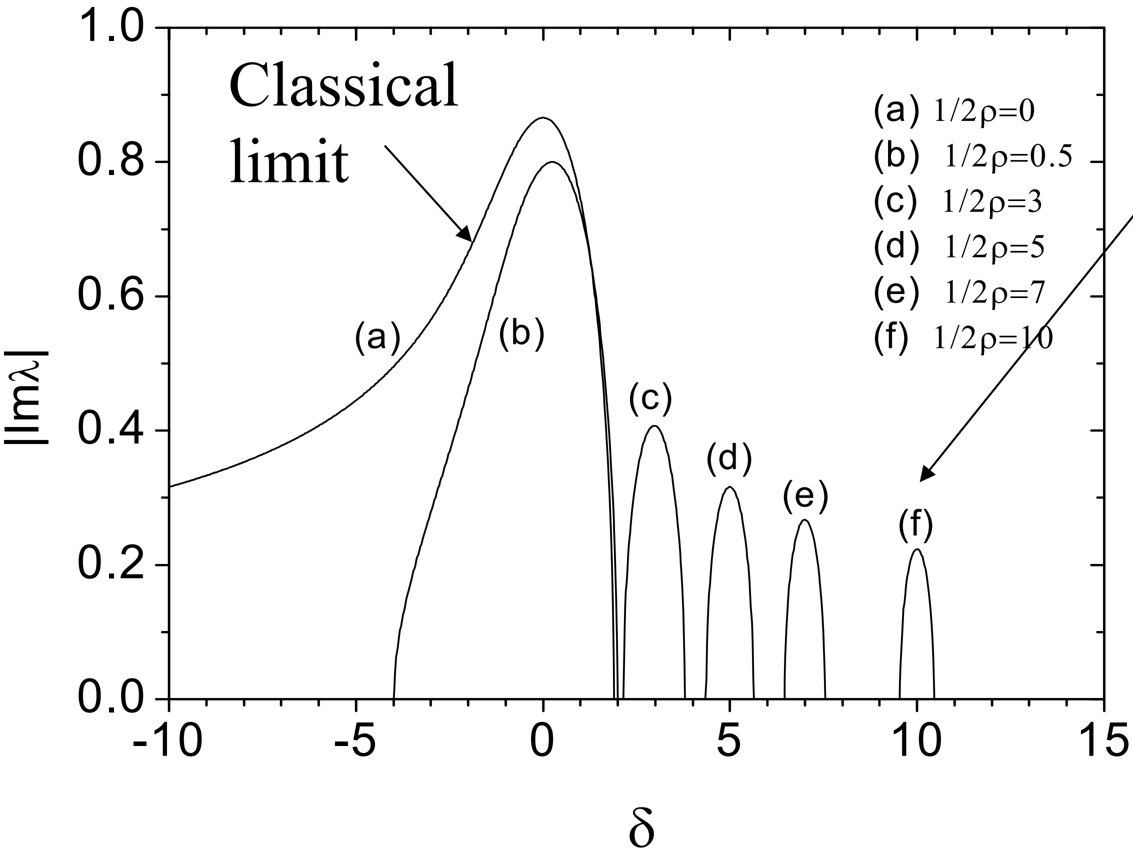
Quantum Linear Theory

$$\left(A \propto e^{i\lambda\bar{z}}\right)$$

$$(\lambda - \Delta)\left(\lambda^2 - \frac{1}{4\bar{\rho}^2}\right) + 1 = 0$$

$$\Delta = \delta + \frac{n}{\bar{\rho}} \quad ; \text{n=0, -1, ...}$$

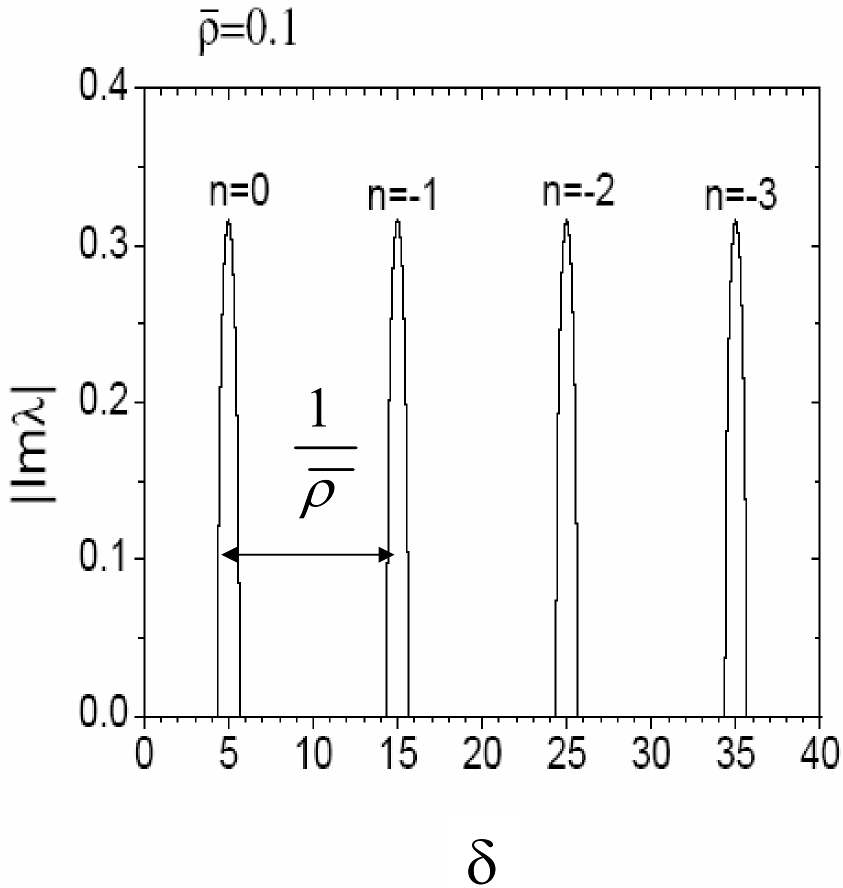
Quantum regime for $\bar{\rho}<1$

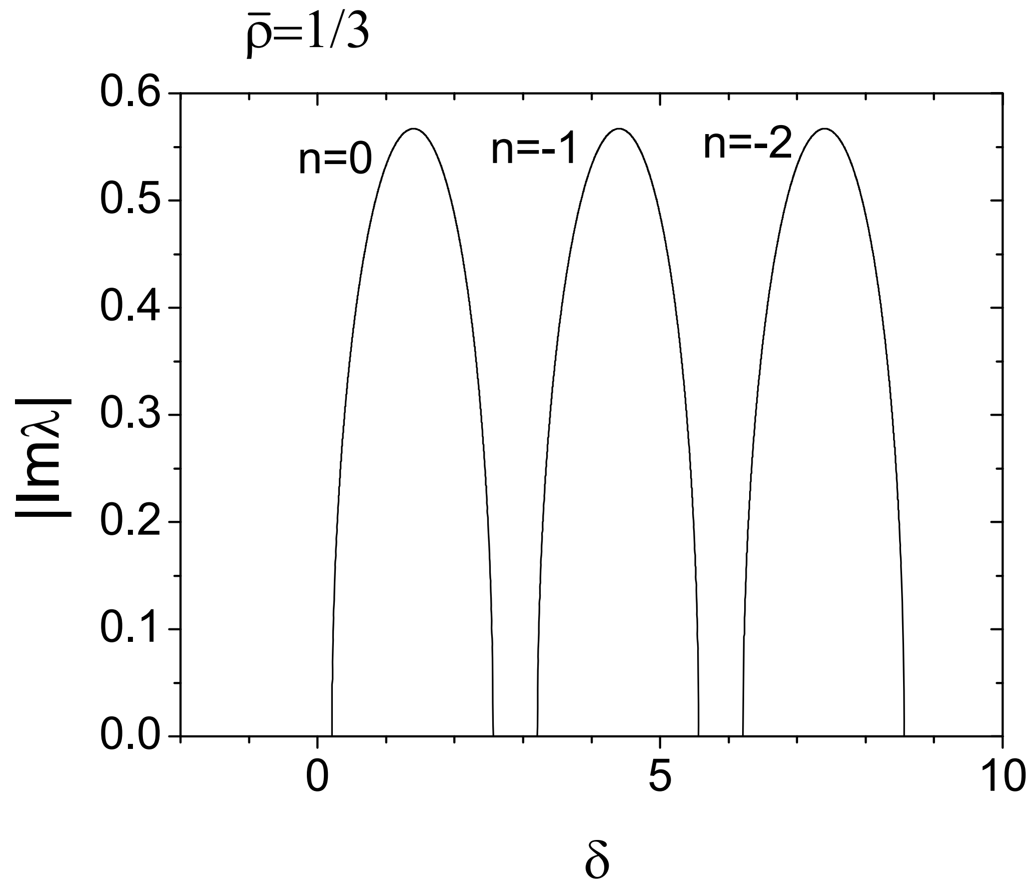


Resonance: $\delta = \dfrac{1}{2\bar{\rho}}$

$$mc(\gamma_0 - \gamma_r) = \hbar k / 2$$

width $\propto \sqrt{\bar{\rho}}$





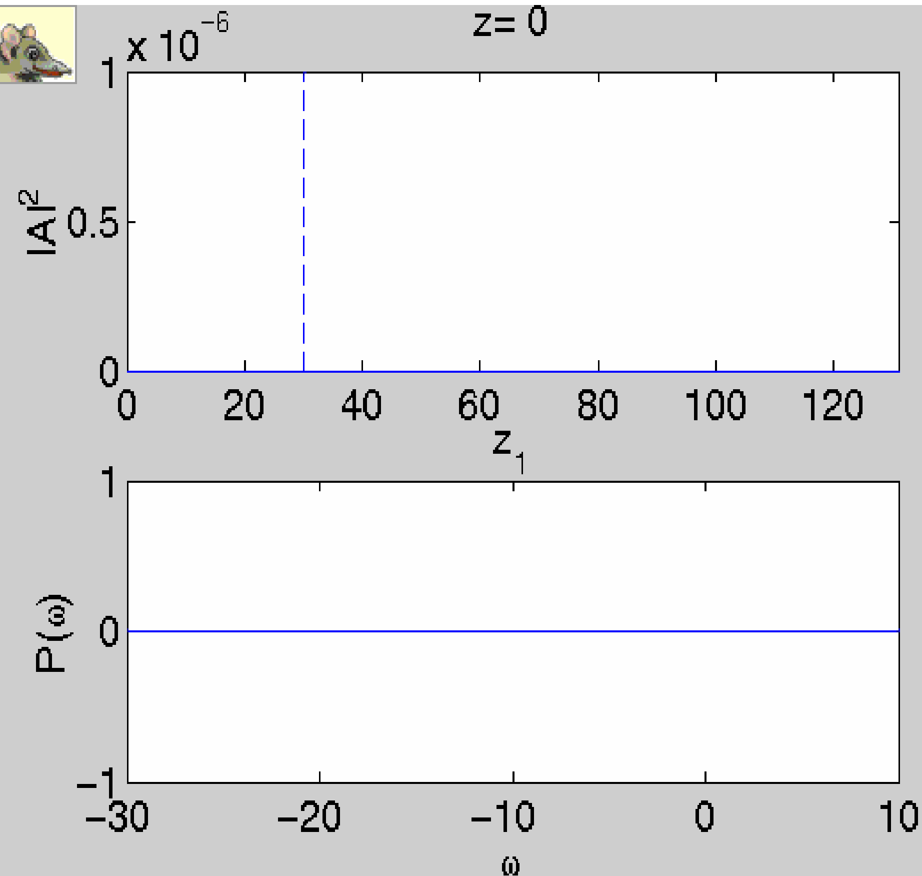
for $\bar{\rho} > 0.4$ lines overlap: - transition to classical regime.

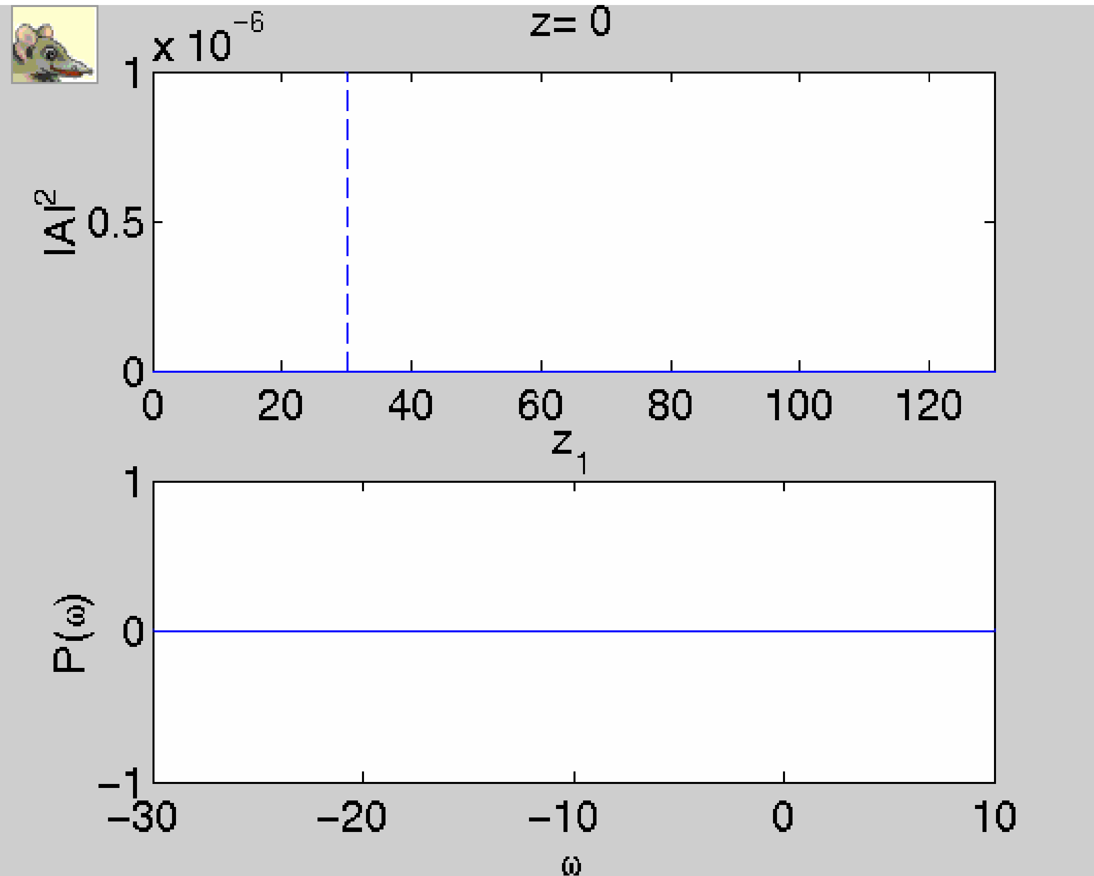
SASE : NUMERICAL SIMULATIONS using the quantum model

$$L_b = 30L_c$$

CLASSICAL REGIME: $\bar{\rho} = 5$

QUANTUM REGIME: $\bar{\rho} = 0.1$

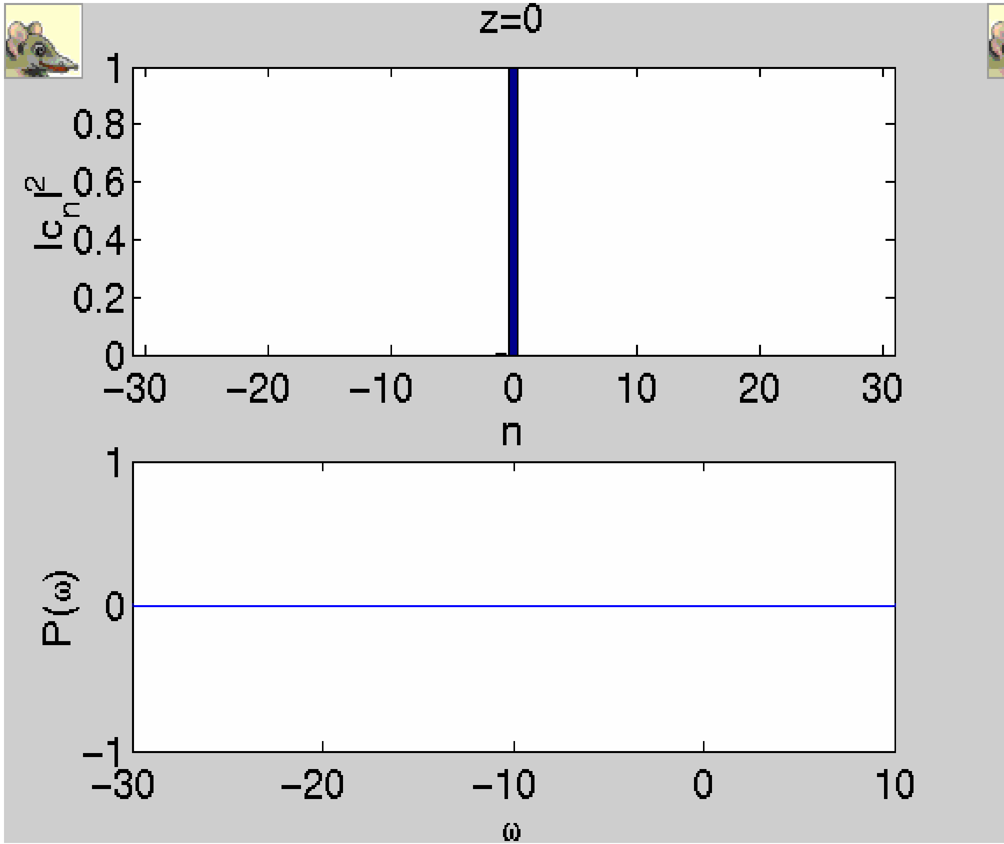


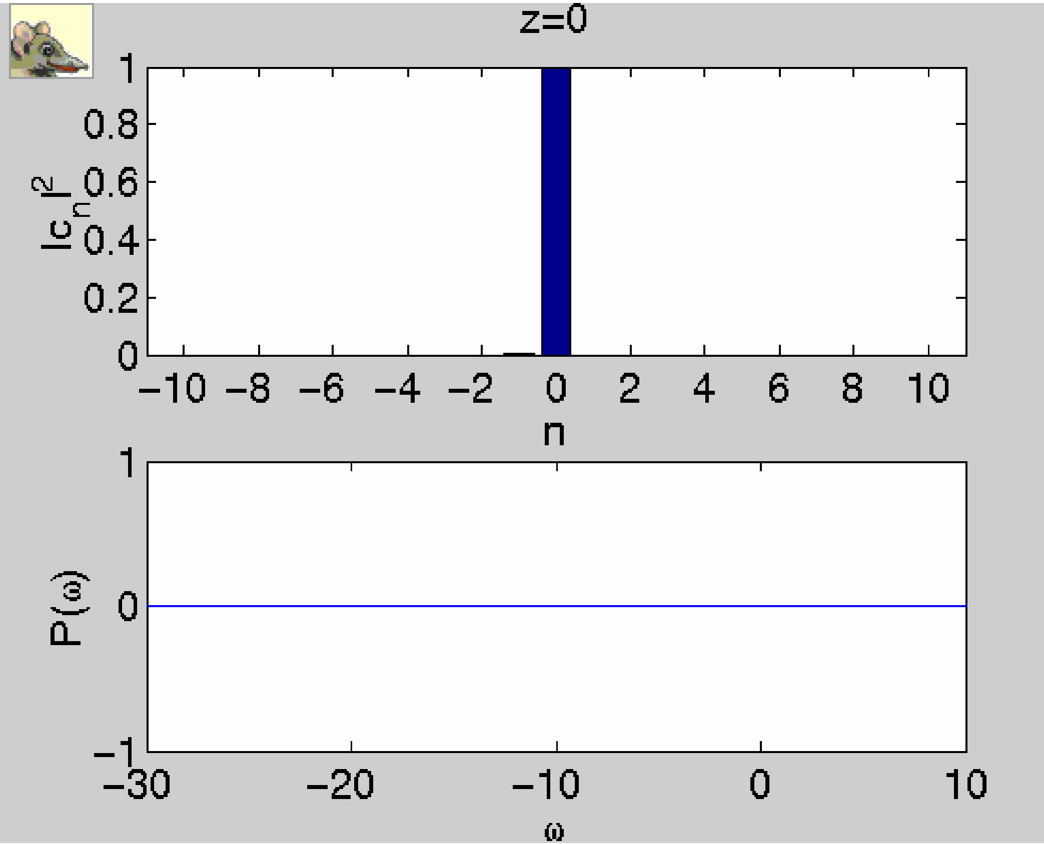


momentum distribution for SASE

CLASSICAL REGIME: $\bar{\rho} = 5$

QUANTUM REGIME: $\bar{\rho} = 0.1$

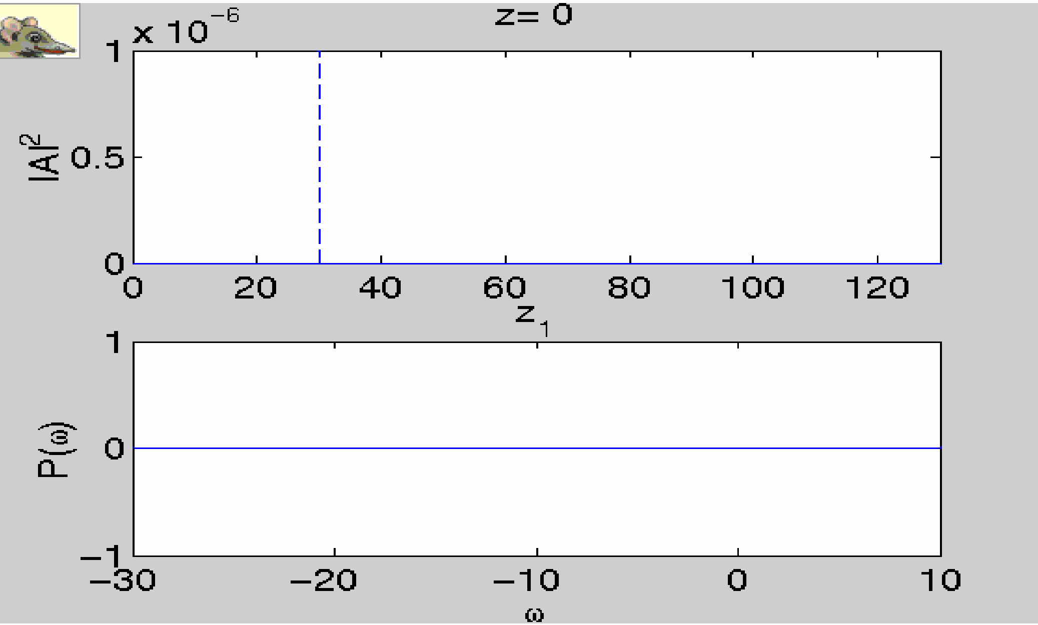




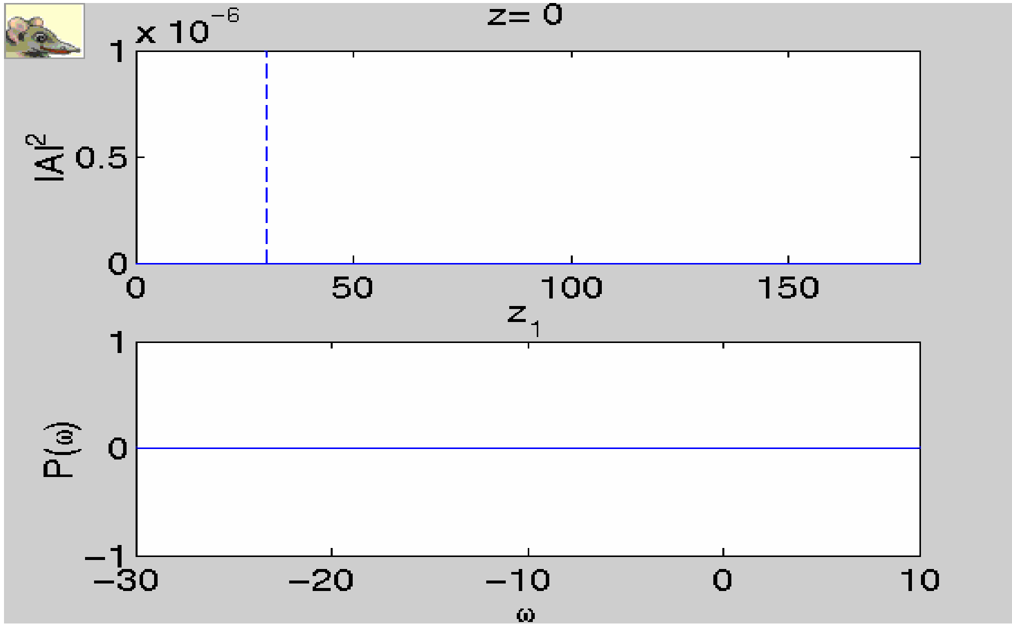
Classical behaviour :
both n<0 and n>0 occupied

Quantum behaviour :
sequential SR decay, only n<0

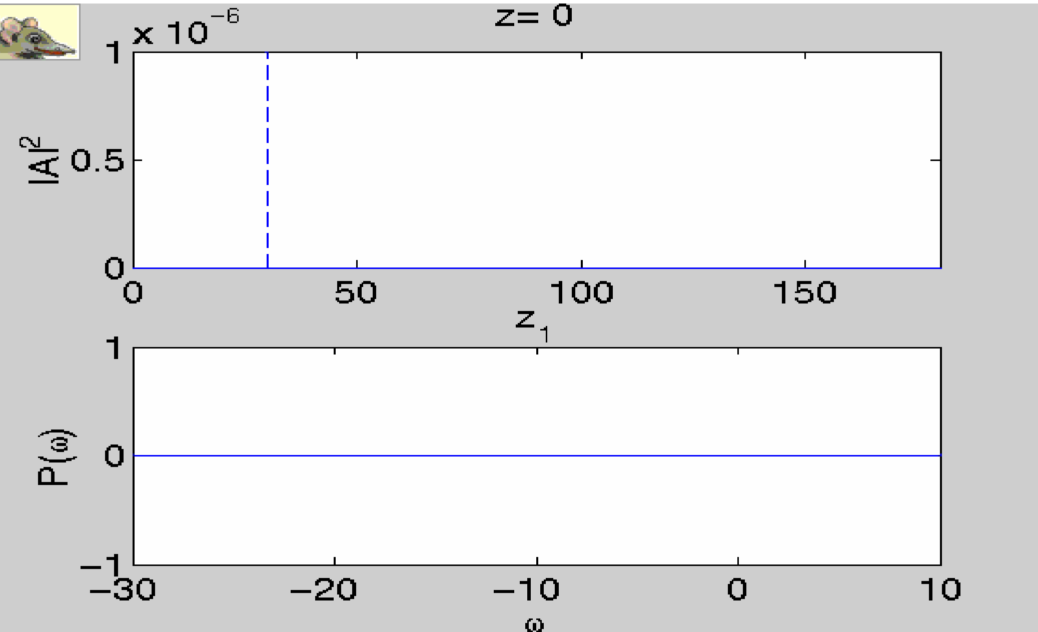
$\bar{\rho} = 0.1 \quad 1/\bar{\rho} = 10$



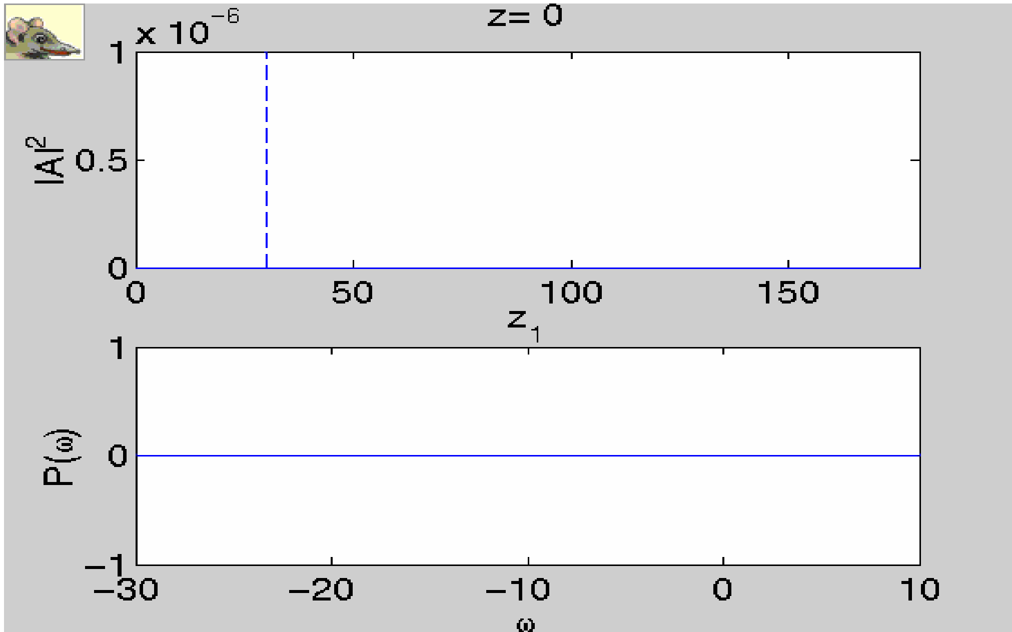
$\bar{\rho} = 0.2 \quad 1/\bar{\rho} = 5$



$\bar{\rho} = 0.3 \quad 1/\bar{\rho} = 3.3$

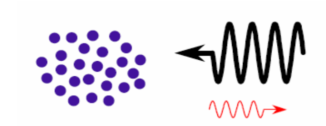


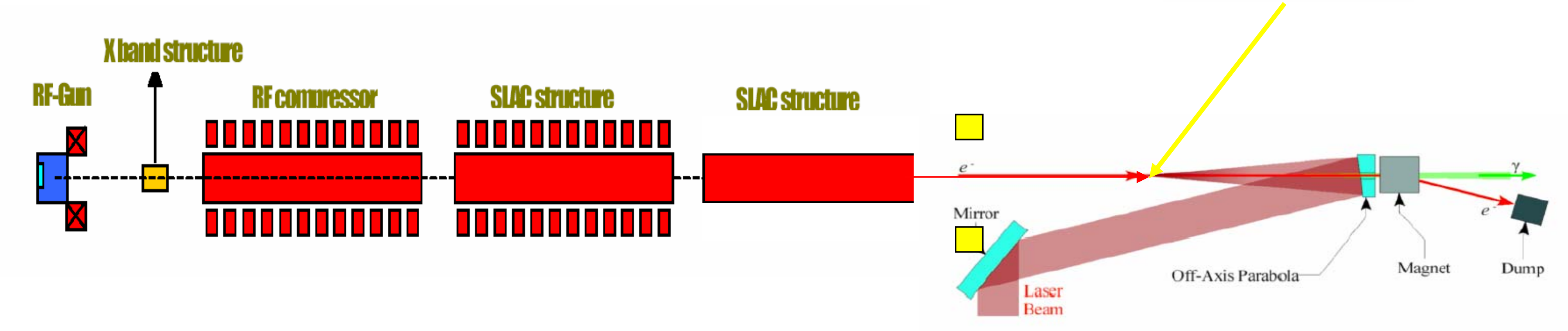
$\bar{\rho} = 0.4 \quad 1/\bar{\rho} = 2.5$



QFEL

INFN project for a feasibility study of a quantum x-ray FEL (~1 Å) in the SASE mode with a laser wiggler (0.8 μm, 10-100 TW)



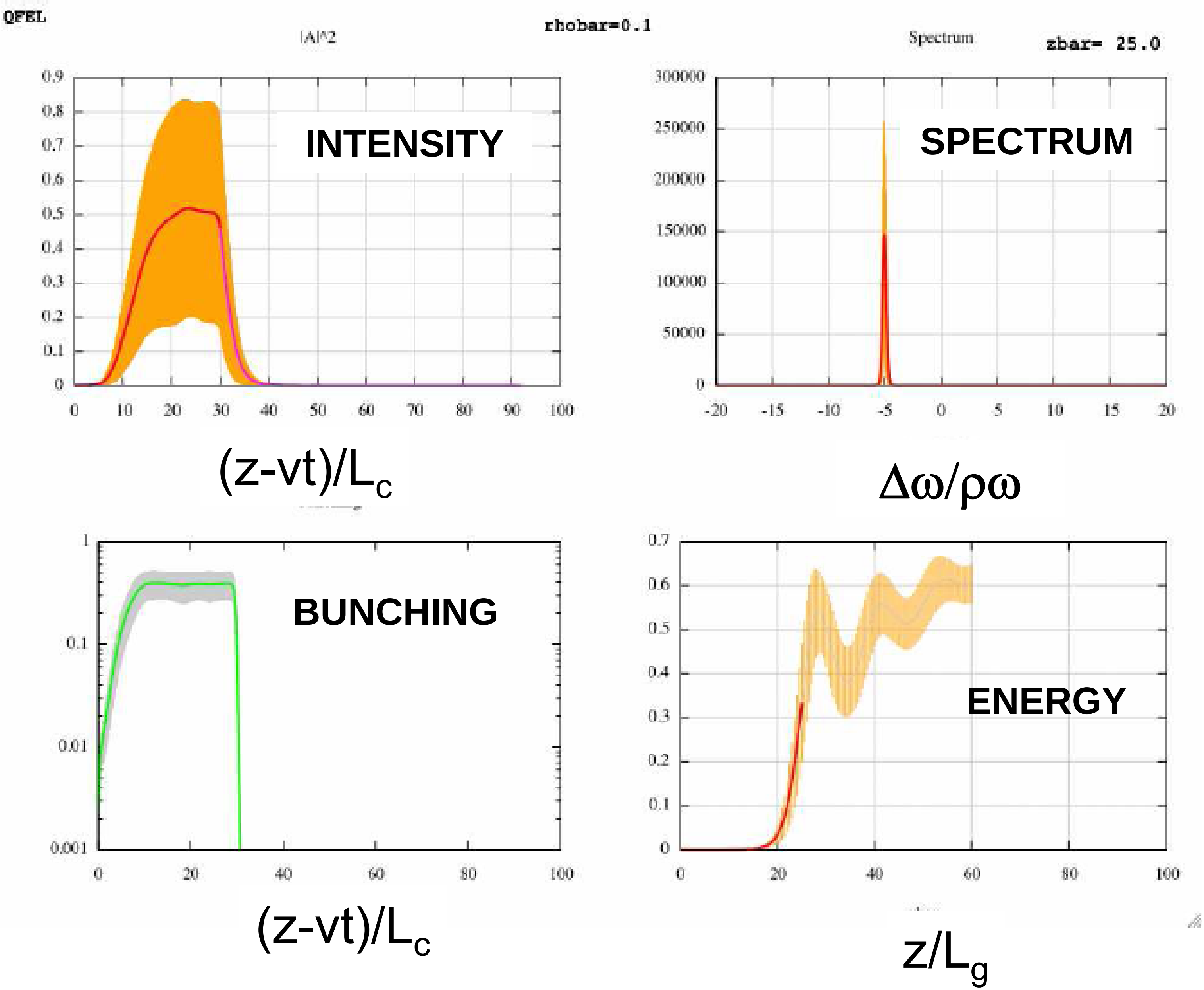


under development at LNF (Frascati) for SPARC/PLASMON-X

main goals:

1. quantum 3D model
2. development of a 3D numerical code
3. definition of the experimental constraints
4. demonstration of the feasibility of a Quantum-SASE FEL experiment

shot-to-shot fluctuations in the QUANTUM SASE



$$\bar{\rho} = 0.1$$
$$b_0 = 0.01$$
$$L_b = 30L_c$$

INITIAL ENERGY SPREAD EFFECTS:

$$\frac{\partial c_n}{\partial \bar{z}} = -in\left(\frac{n}{2\bar{\rho}} + \delta\right)c_n - \bar{\rho}\left(Ac_{n-1} - A^* c_{n+1}\right)$$

$$\frac{\partial A}{\partial \bar{z}} + \frac{\partial A}{\partial z_1} = \sum_{n=-\infty}^{\infty} \int d\delta\, G(\delta) c_n c_{n-1}^*$$

G(δ) ; initial distribution of width σ_δ

linear theory → GENERALIZED DISPERSION relation: $\left(A \propto e^{i\lambda\bar{z}}\right)$

$$\lambda - \Delta + \bar{\rho}\int_{-\infty}^{\infty}\frac{d\delta}{\lambda+\delta}\left\{G\left(\delta+\frac{1}{2\bar{\rho}}\right) - G\left(\delta-\frac{1}{2\bar{\rho}}\right)\right\} = 0$$

$$\left(\Delta = \bar{\delta} + \frac{n}{\bar{\rho}} - \frac{\Delta\omega}{\rho\omega}\right)$$

N. Piovella, R. Bonifacio, NIMA (2006)

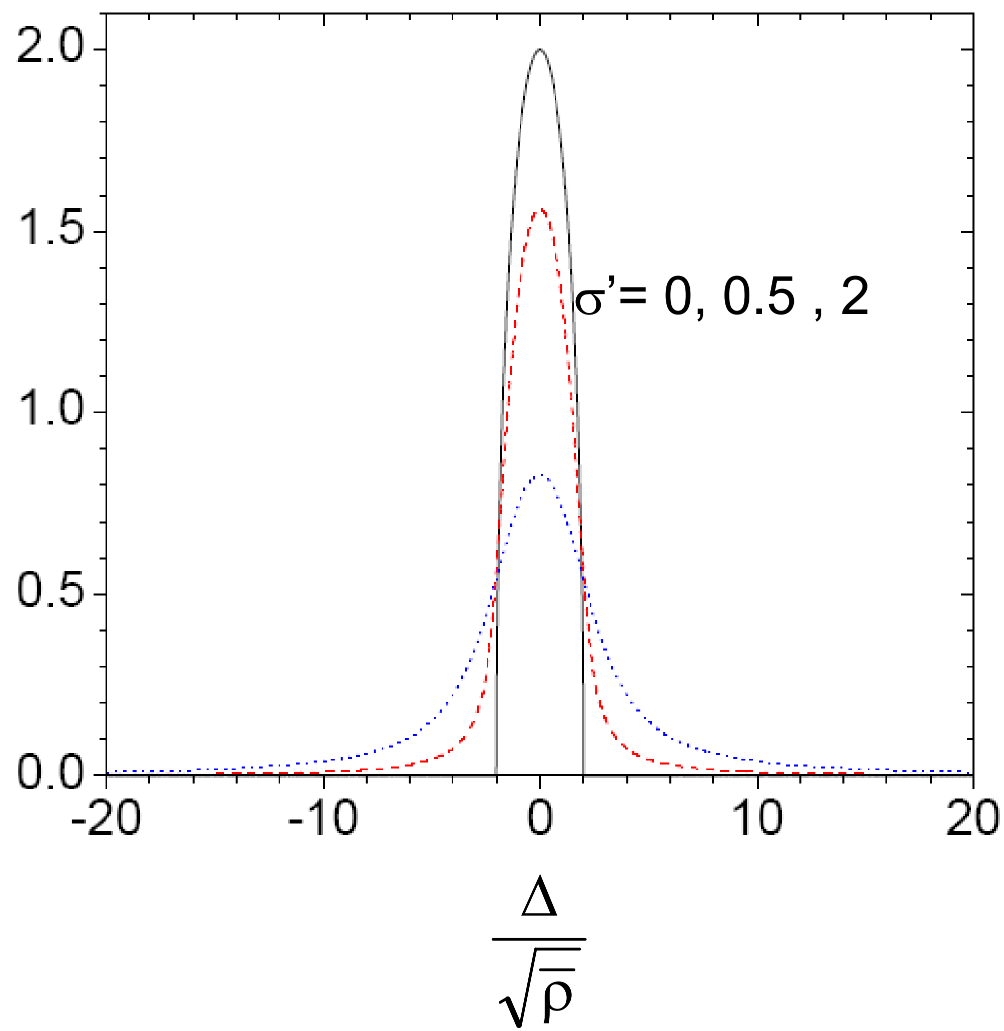
when G(δ) is Lorentzian →

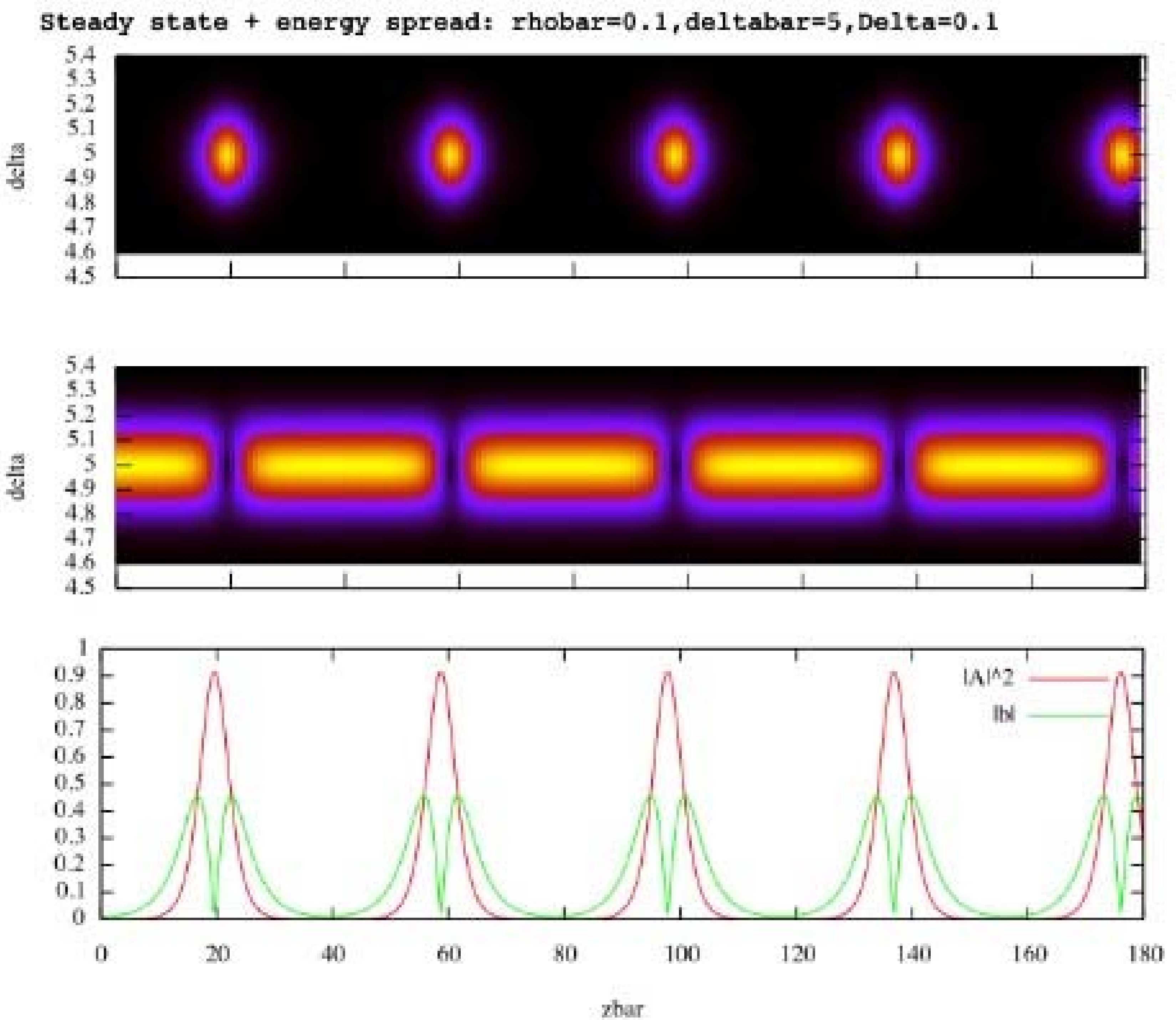
$$(\lambda - \Delta)\left[(\lambda - i\sigma)^2 - \frac{1}{4\bar{\rho}^2}\right] + 1 = 0$$

in the quantum limit $\bar{\rho} < 1$

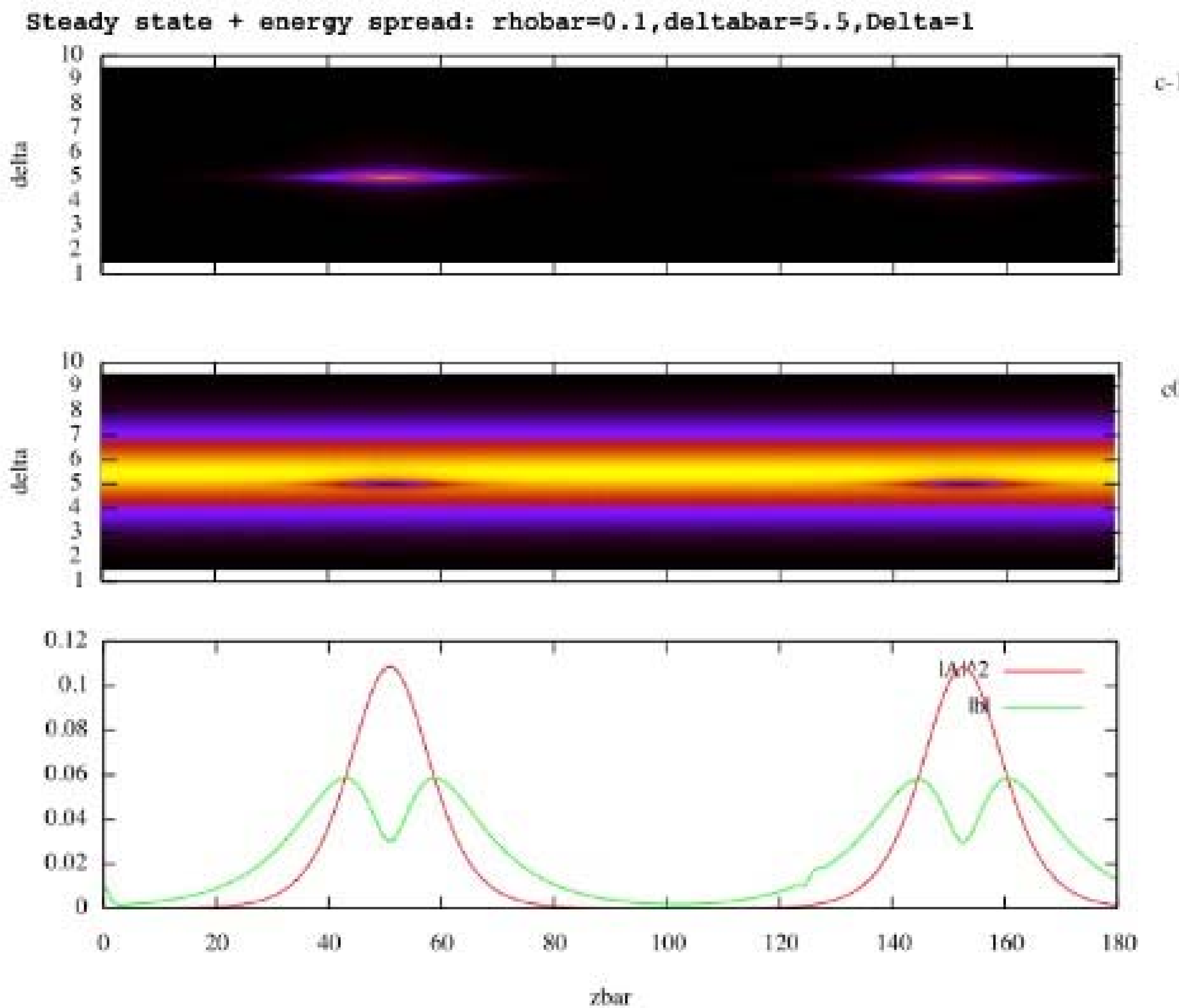
$$\text{max gain} = \frac{\sqrt{\bar{\rho}}}{L_g}\left[\sqrt{4 + \sigma'^2} - \sigma'\right]$$

$$\sigma' = \frac{\sigma}{\sqrt{\bar{\rho}}} = \frac{1}{\sqrt{\bar{\rho}}}\left(\frac{\delta\gamma_0}{\rho\gamma_0}\right)$$





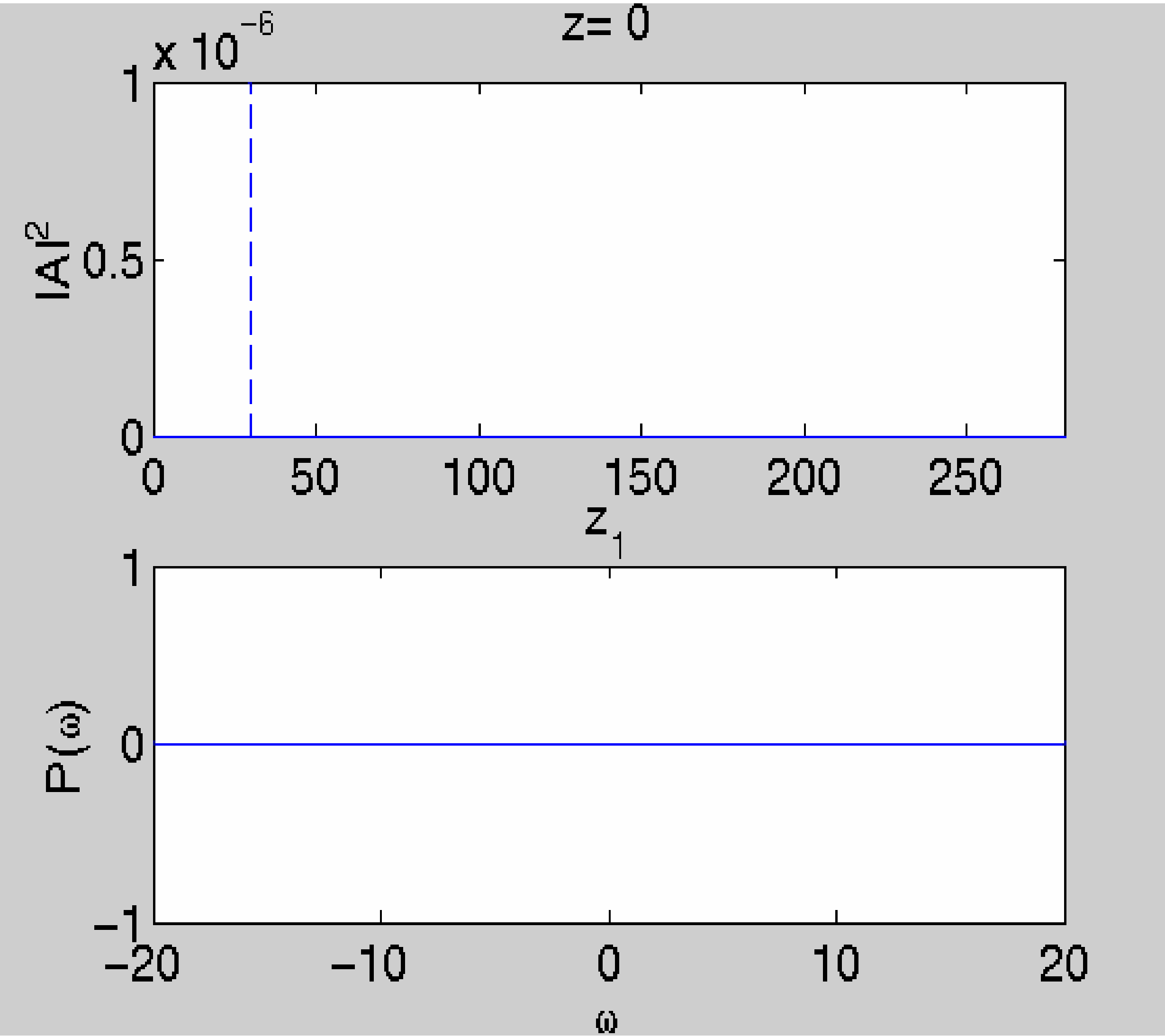
Steady state + energy spread: rhobar=0.1,deltabar=5,Delta=0.1
delta
c-1
c0
|A|^2
|b|
zbar



Steady state + energy spread: rhobar=0.1,deltabar=5.5,Delta=1
c-1
c0
delta
|A|^2
|b|
zbar

QUANTUM SASE with spread ($\bar{\rho}$=0.1 and σ=1)

$$\left(\frac{\delta\gamma_0}{\gamma_0} = \rho\right)$$



coherence is preserved if:

$$mc\delta\gamma_0 < \hbar k$$

$$\left(\sigma << \frac{1}{\bar{\rho}}\right)$$

but gain is reduced if

$$\frac{\delta\gamma_0}{\gamma_0} \geq \rho\sqrt{\bar{\rho}}$$

CONCLUSIONS

- The new QUANTUM regime of SASE-FEL could generate x-ray radiation with a much narrower spectrum than classical SASE

- Our quantum propagation model describes both classical ($\bar{\rho}$>>1) and quantum ($\bar{\rho}$<1) SASE mode operation.

- In the quantum regime FELs behaves as two-level system (Laser), with multiple lines in the spectrum.

- We are working to extend the 1D Quantum FEL model into a full 3D model, with an e.m. wiggler (QFEL project).