Longitudinal Coherence Preservation & Chirp Evolution in a High Gain Laser Seeded Free Electron Laser Amplifier

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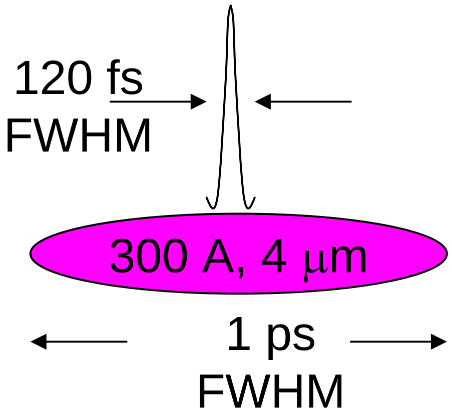
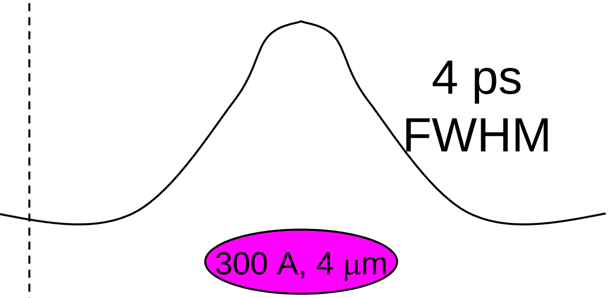
Motivation: Explain Experimental Data

Superradiance:



Chirped Seed:



10 m NISUS Undulator

Spectrometer

FROG Spectrogram

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Wigner Fcn for a Chirped Gaussian Seed Pulse

The electric field of the chirped seed laser is assumed to be,

$$E_s(t, z) = E_0 e^{i(k_0 z - \omega_0 t)} e^{-(\alpha + i\beta)(t - z/c)^2}$$

Chirp

The Wigner function is defined as,

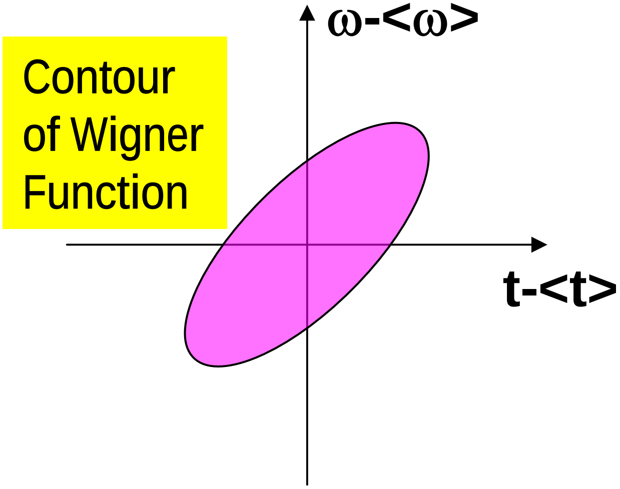
$$W(t, \omega, z) = \int E(t - \tau/2, z) E^*(t + \tau/2, z) e^{-i\omega\tau} d\tau$$

The Wigner function for the seed laser is a Gaussian:

$$W_s(t, \omega, z) = |E_0|^2 \sqrt{\frac{2\pi}{\alpha}} \exp\left\{ - \left[4\left(t - \frac{z}{c}\right)^2 (\alpha^2 + \beta^2) - 4\beta\left(t - \frac{z}{c}\right)(\omega - \omega_0) + (\omega - \omega_0)^2 \right] / (2\alpha) \right\}$$

Chirped Seed Pulse: Longitudinal Coherence

The moments of the Gaussian seed are as follows:



$$
\begin{cases}
\langle t \rangle = \frac{z}{c} = \frac{z}{v_g} \\
\langle \omega \rangle = \omega_0 \\
\sqrt{\langle (t - \langle t \rangle)^2 \rangle} = \sigma_{t,\mathrm{seed}} = \frac{1}{2\sqrt{\alpha}} \\
\sqrt{\langle (\omega - \langle \omega \rangle)^2 \rangle} = \sigma_{\omega,\mathrm{seed}} = \sqrt{\frac{\alpha^2+\beta^2}{\alpha}} \\
\langle (t - \langle t \rangle)(\omega - \langle \omega \rangle) \rangle = \frac{\beta}{2\alpha}
\end{cases}
$$

The longitudinal emittance of the Gaussian seed is defined as:

$$
\varepsilon_{\mathrm{Light}} \equiv \sqrt{\langle (t - \langle t \rangle)^2 \rangle \langle (\omega - \langle \omega \rangle)^2 \rangle - \langle (t - \langle t \rangle)(\omega - \langle \omega \rangle) \rangle^2}
$$

= ½

Coasting Beam High Gain FEL Amplification

Using an FEL Green function approach pioneered by J.M. Wang to solve the self consistent Vlasov-Maxwell eqns, the electric field is given by $E(t,z) = A(\theta,Z)e^{i(kz-\omega_0 t)}$ where,

Chirped Seed / **Green Fcn**

$$A(\theta,Z)\cong e^{\rho(\sqrt{3}+i)Z}\int_0^\infty d\xi\, e^{\overbrace{-\frac{(\theta-\xi)^2}{\omega_0^2}(\alpha+i\beta)}^{\text{Chirped Seed}}\overbrace{-\rho(\sqrt{3}+i)\left[9(\xi-Z/3)^2/(4Z)\right]}^{\text{Green Fcn}}}$$

$$\mathrm{v_g} \equiv \frac{\omega_0}{k_0 + {}^2\!/\!_3\, k_w}$$

Evaluating the integral yields,

$$E_{\mathrm{FEL}}(t,z) = E_{0,\mathrm{FEL}}\, e^{\rho(\sqrt{3}+i)k_w z} \times e^{i(k_0 z-\omega_0 t)} e^{-[\alpha(z)+i\beta(z)](t-z/v_g)^2}$$

E-field is of the same form as seed but with z-dependent moments!

Wigner Fcn & Coherence After FEL Interaction

WF is of the same form as seed but with z-dependent moments:

$$\sigma_t(z) = \sqrt{\frac{4\sigma^2_{\omega,\mathrm{seed}} + \left(6 + 2\sqrt{3}\frac{\beta}{\alpha}\right)\sigma^2_{\omega,\mathrm{GF}}(z) + \frac{3}{\alpha}\sigma^4_{\omega,\mathrm{GF}}(z)}{12\sigma^2_{\omega,\mathrm{GF}}(z)\left[\sigma^2_{\omega,\mathrm{seed}} + \sigma^2_{\omega,\mathrm{GF}}(z)\right]}}$$

$$\sigma_\omega(z) = \sqrt{\frac{1}{\frac{1}{\sigma^2_{\omega,\mathrm{seed}}} + \frac{1}{\sigma^2_{\omega,\mathrm{GF}}(z)}}}$$

Coherence Length ~ $1/\sigma_\omega(z)$

$$\langle (t - \langle t \rangle)(\omega - \langle \omega \rangle) \rangle \equiv \frac{\beta(z)}{2\alpha(z)} = \frac{1}{2\sqrt{3}} \frac{\sigma^2_{\omega,\mathrm{seed}} + \sqrt{3}\frac{\beta}{\alpha}\sigma^2_{\omega,\mathrm{GF}}(z)}{\sigma^2_{\omega,\mathrm{seed}} + \sigma^2_{\omega,\mathrm{GF}}(z)}$$

$$\sigma_{\omega,GF}(z) \equiv \sqrt{\frac{3\sqrt{3}\rho\omega_0^2}{k_w z}}$$

Compute $\varepsilon(z) = ½ \Rightarrow$ Coherence is Preserved!

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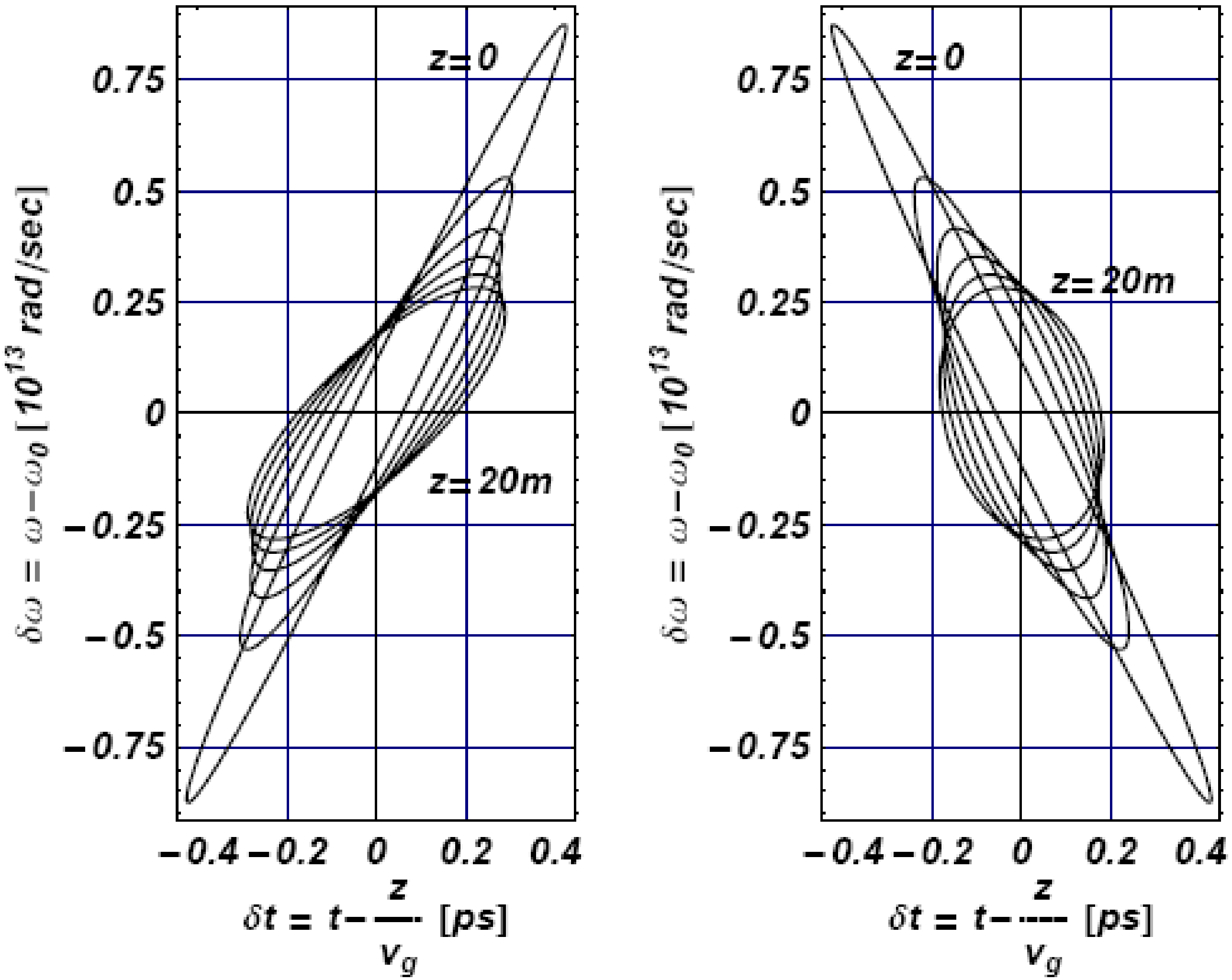
Phase Space Evolution: Numerical Example



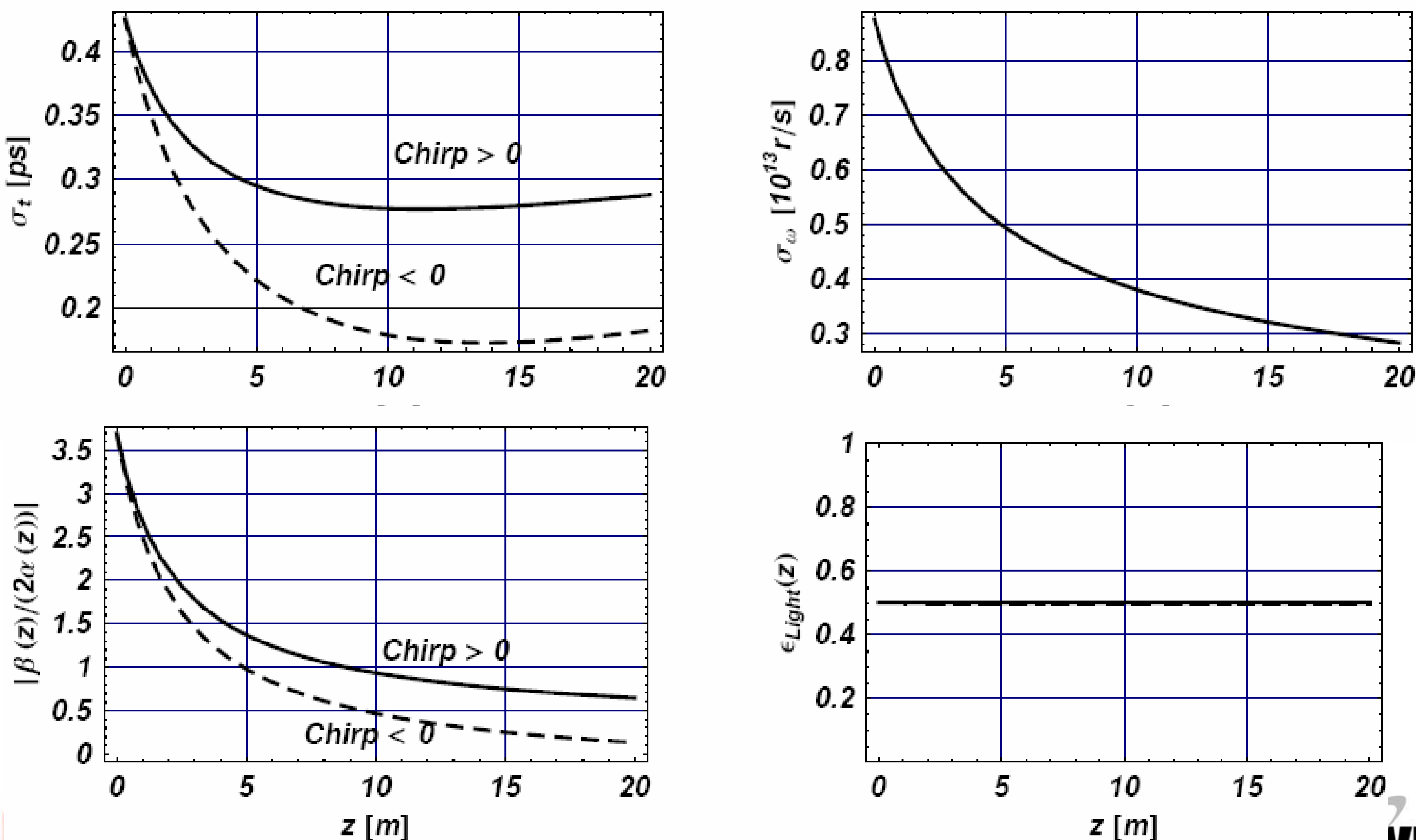
$\rho = 10^{-3}$

$\lambda = 800$ nm

$T_{FWHM} = 1$ ps

$\omega_{FWHM} = 7$ nm

Evolution of the Moments: Numerical Example



Canonical Transformation & ABCD Matrix

The convolution integral is of the general form of an integral representation of an *ABCD* canonical transform, as such the phase space area & longitudinal coherence is preserved:

$$E(\xi) \sim \frac{1}{\sqrt{2\pi i B}} \int Exp\left[\frac{i}{2B}\left(A\xi^2 - \xi\xi' + D\xi'^2\right)\right] E(\xi') d\xi'$$

Think of Diffraction & Lenses for Transverse Plane

Associated with the canonical transformation is a symplectic *ABCD* matrix with GVD (ReB) & gain (ImB):

$$M_{ABCD} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & -\frac{2ik_w z}{9\left(i+\sqrt{3}\right)\rho\omega_0} \\ 0 & 1 \end{pmatrix}$$

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ABCD Matrix & Complex Pulse Parameter

As in diffraction problems, the *ABCD* matrix can be used to transform the complex Gaussian pulse parameter, $p(z)$,

$$\frac{1}{p(z)} \equiv -\frac{2\beta(z)}{\omega_0} + i\frac{2\alpha(z)}{\omega_0}$$

as follows,

$$p(z) = \frac{A\,p(0) + B}{C\,p(0) + D}$$

to obtain the new chirp and pulse duration after the FEL interaction confirming the results computed as moments of the Wigner function.

Concluding Remarks

- FEL Coasting Beam High Gain Green Function can be Characterized by an ABCD Canonical Transformation

- FEL Process = GVD & Gain Modifies the Seed Chirp, Pulse Length & Bandwidth

- Longitudinal Coherence of the Seed is Preserved in the High Gain Exponential Regime

Acknowledgements & References

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