Study of Dynamic Aperture for PETRA III Ring

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DESY

Overview

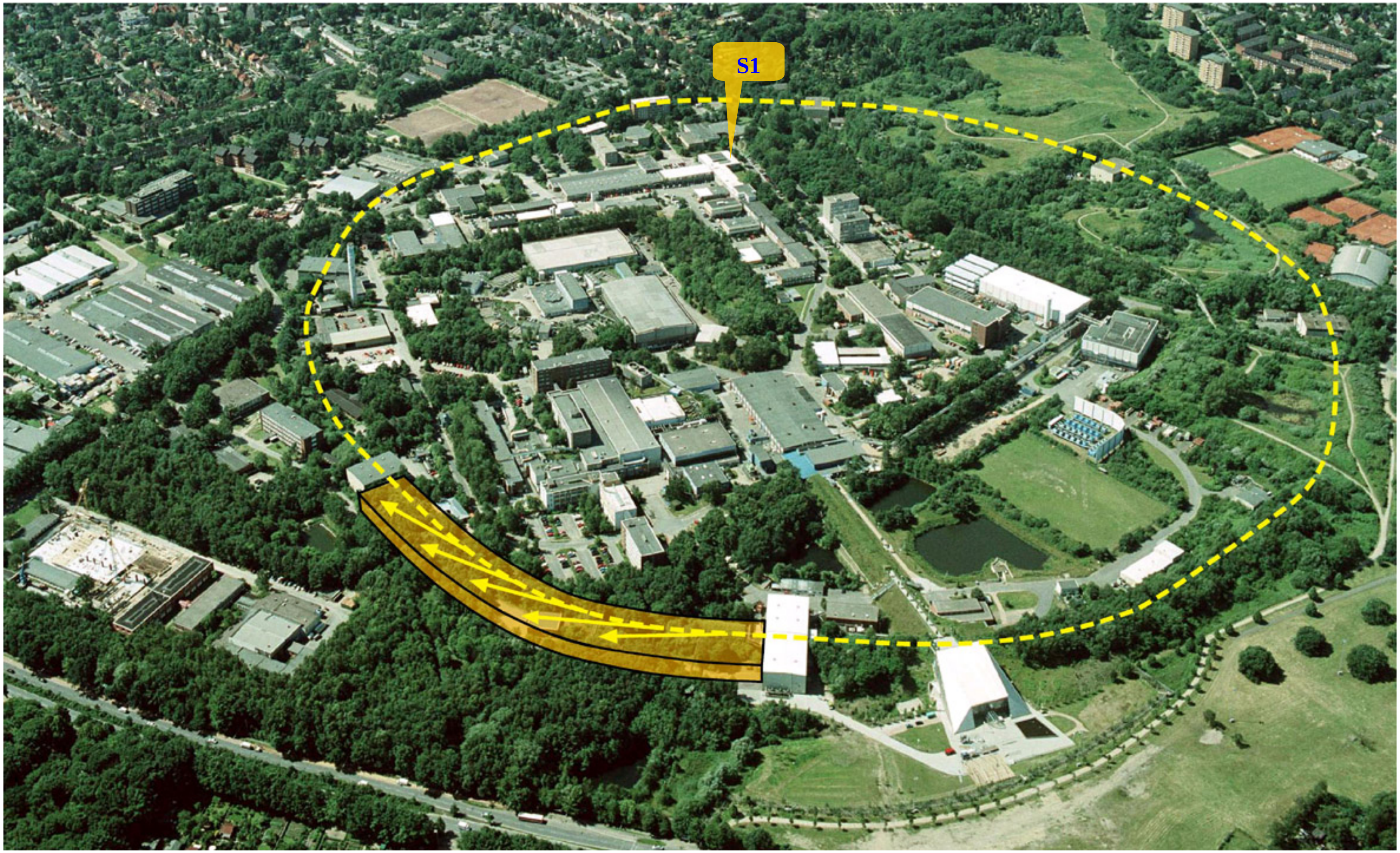
- Introduction
- Dynamics of damping wigglers
- Choice of machine tunes, and optimization of sextupole configuration
- Simulation results

Introduction

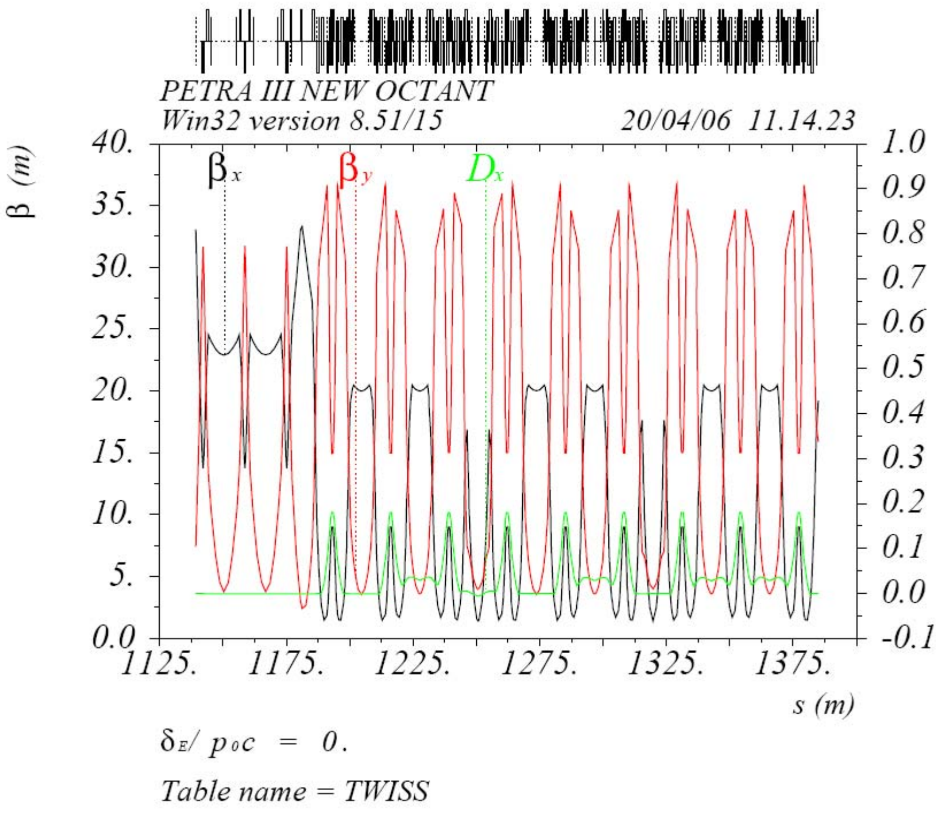
- Main parameters of PETRA III

parameters	value
Energy (GeV)	6
Number of insertion devices	13
Max. current (mA)	>100
Horizontal emittance (nm rad)	1.0
Vertical emittance (nm rad)	0.01

Layout of storage ring



New octant DBA optics

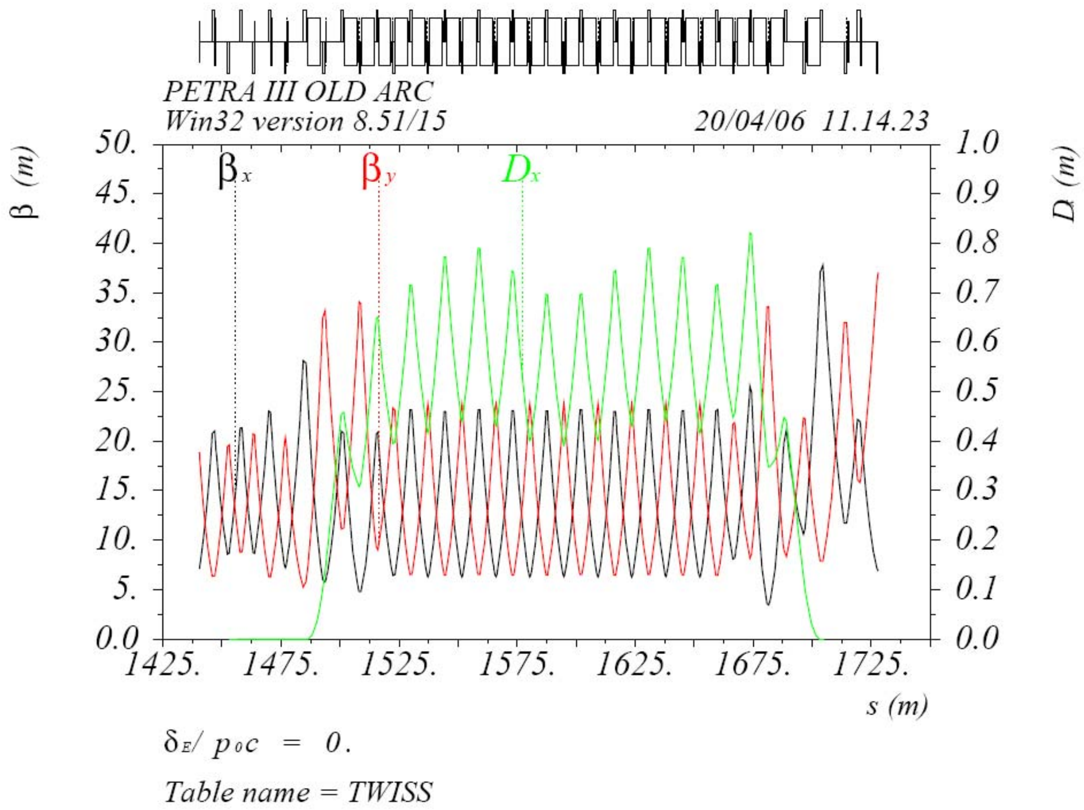


Design requirements (users):
- 5 & 2m long insertion devices
- low and high b_x

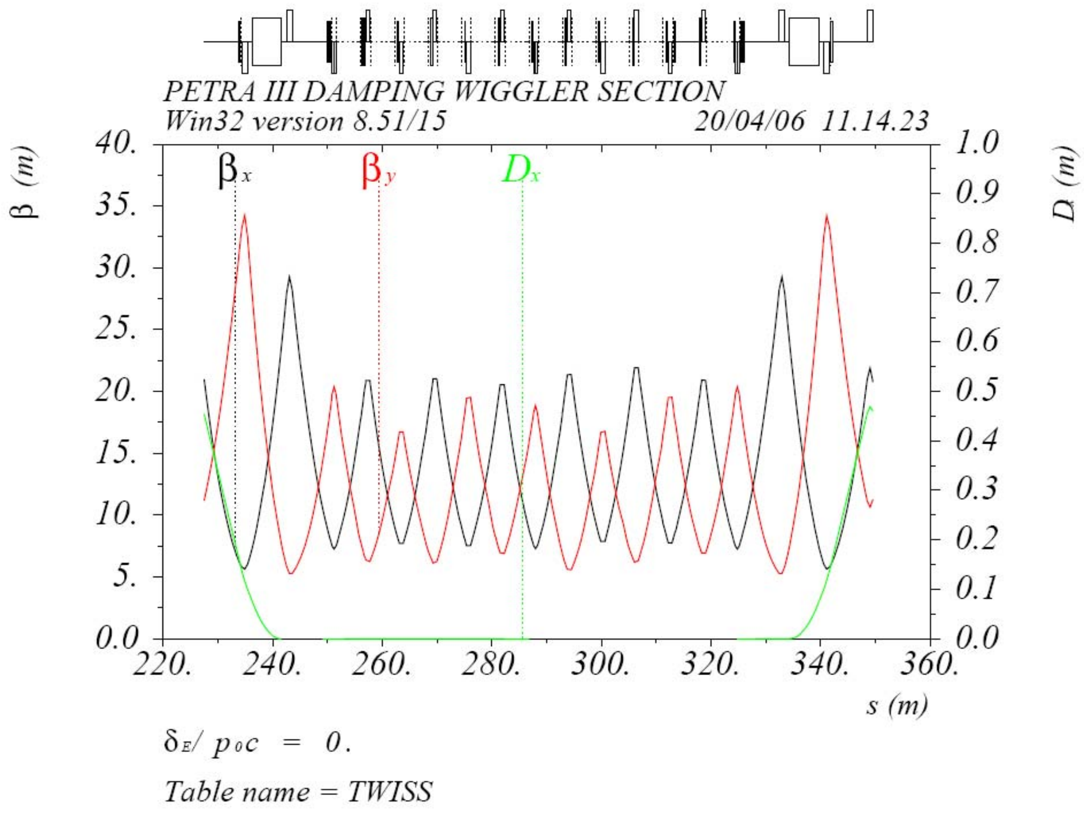
Design constrains (machine):
- low ξ, low ε

Design consequences:
- D_x small => no sextupoles

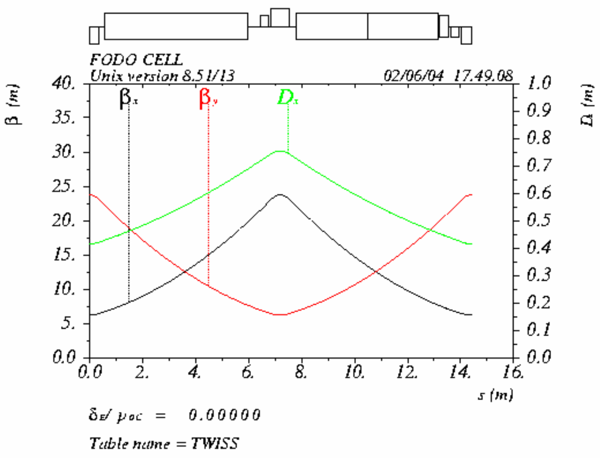
Old FODO octant

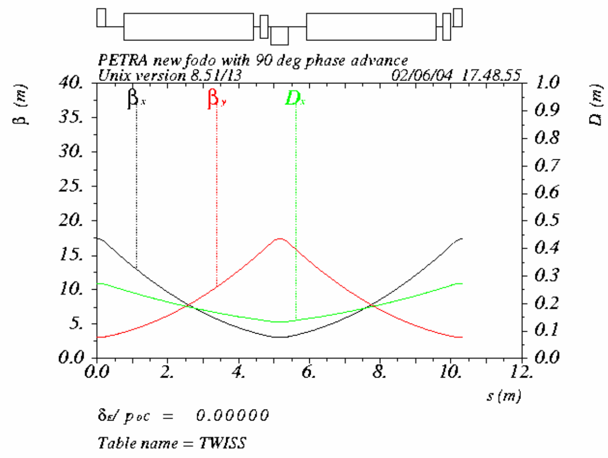


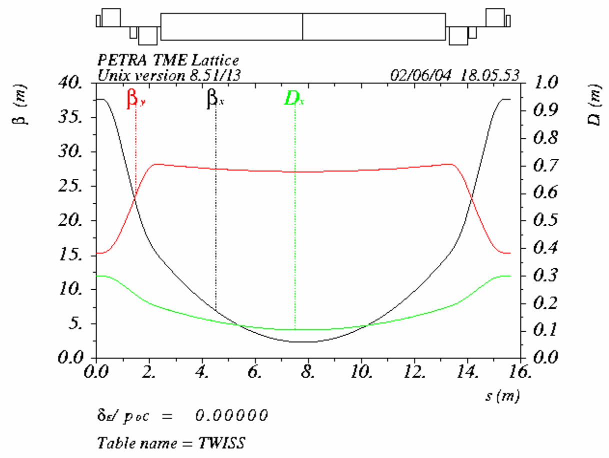
Damping wiggler section



Old Octant Cell Options





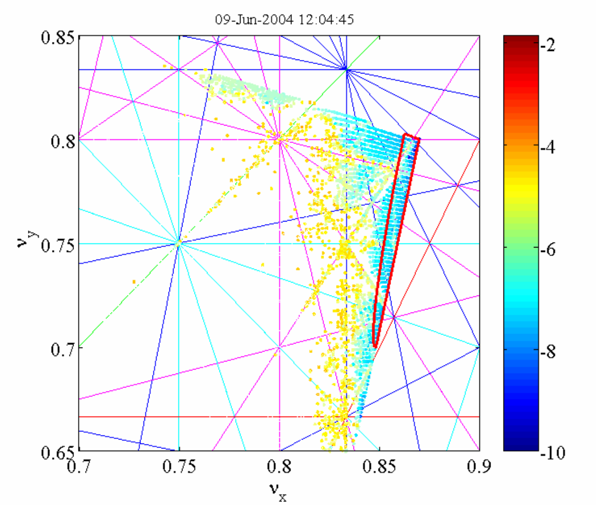


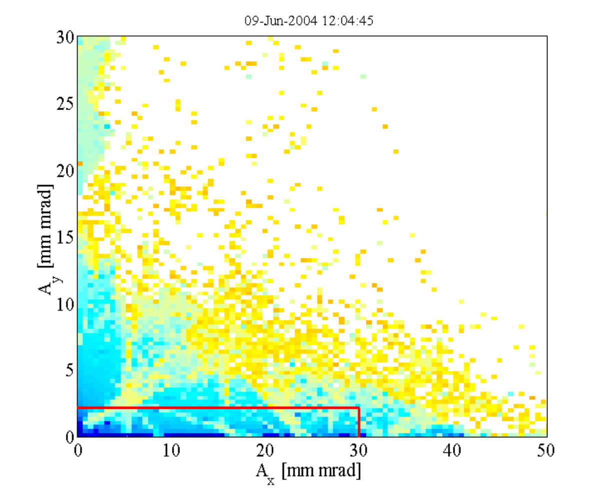
- 72° FODO cell
- Lattice unchanged
- Damping wiggler needed to reach 1nmrad

- 90° short FODO cell
- Lattice and hardware new
- Small dispersion, strong focusing leads to strong chromatic sextupoles

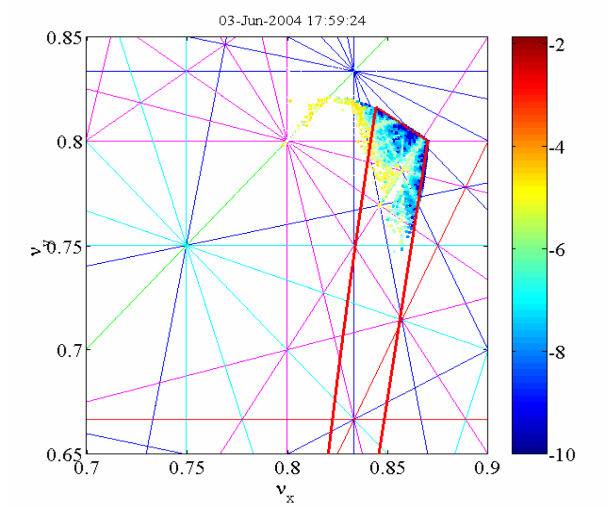
- 'TME' cell
- Lattice new
- Hardware reused
- Small dispersion, strong focusing leads to strong chromatic sextupoles

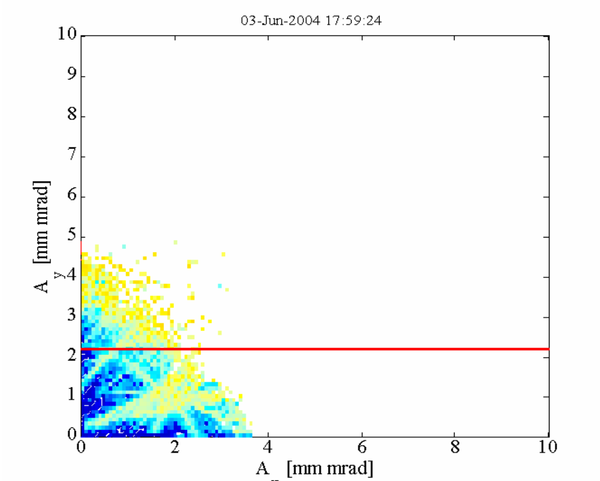
Comparison of lattices (1)



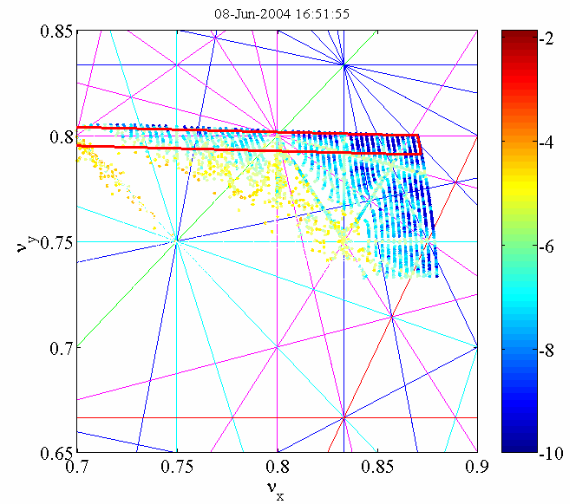


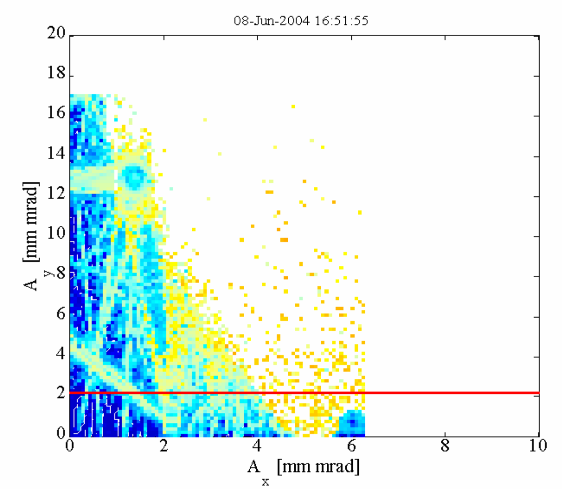
FODO 72





Short FODO 90

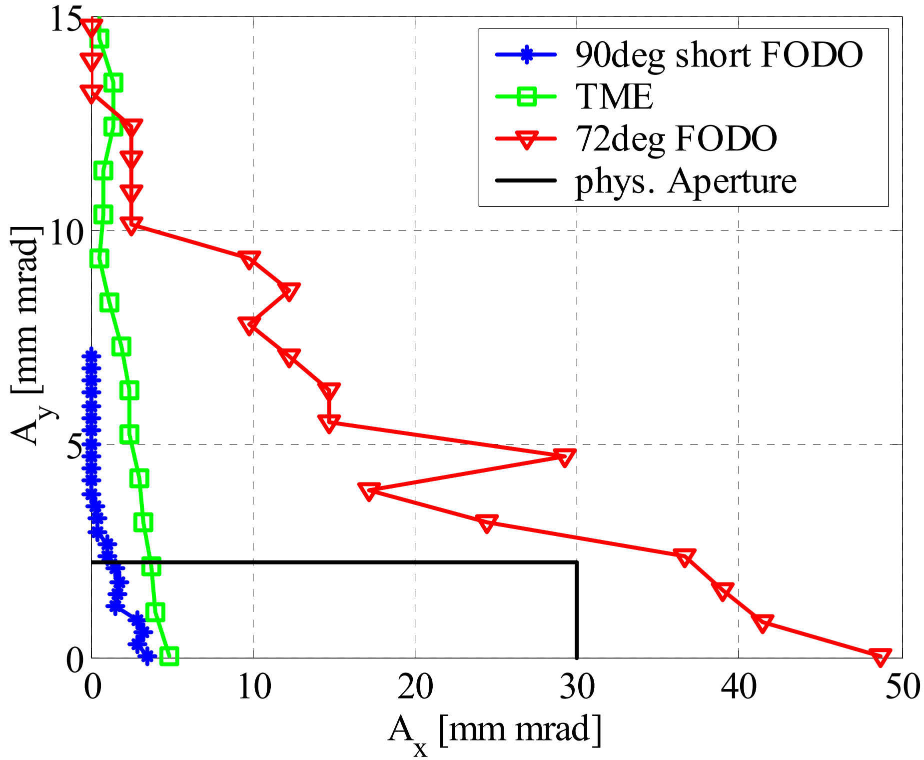




TME

Comparison of lattices (2)

Dynamic Aperture for injection:



Dynamics of damping wigglers

- Fitting wiggler field map into analytical formulae;
- Constructing transformation matrix for linear optics match;
- Constructing nonlinear map for simulation (dynamic aperture calculation).

Wiggler Field Calculations

$$B_x = \frac{k_x}{k_y} B_0 \sinh(k_x x)\sinh(k_y y)\cos(kz)$$

$$B_y = B_0 \cosh(k_x x)\cosh(k_y y)\cos(kz)$$

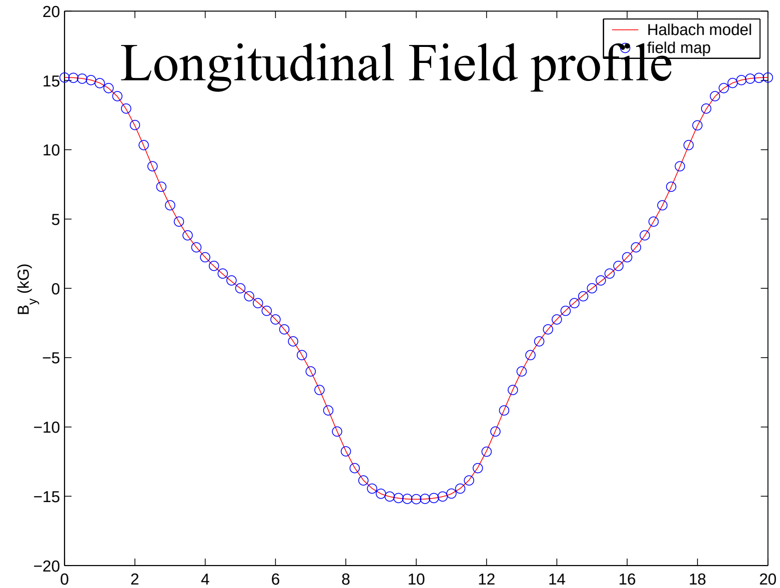
$$B_z = -\frac{k}{k_y} B_0 \cosh(k_x x)\sinh(k_y y)\sin(kz)$$

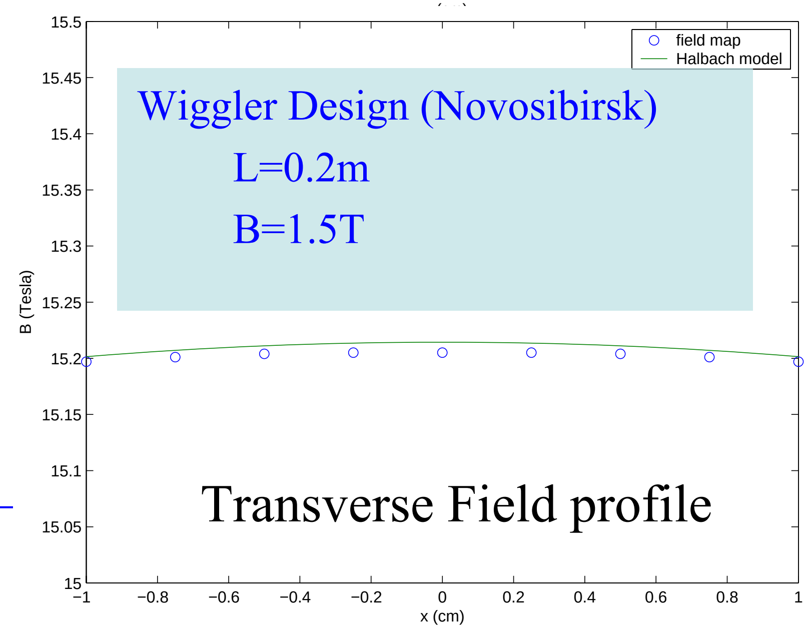
$$k_x^2 + k_y^2 = k^2 = \left(\frac{2\pi}{\lambda}\right)^2$$

$$\vec{B} = \sum_{n=1,3,\ldots,N} B_n \vec{f}_n\left(k_{x,n}, k_{y,n}, nk, x, y, z\right)$$

$$k_{x,n}^2 + k_{y,n}^2 = (nk)^2$$

Halbach Formulae





Particle Motion in Wiggler Field

4th order Hamiltonian of planar Wiggler

$$\tilde{H}(z) = -\frac{eB_0}{p_0 k}\sin kz \cdot x' \quad \cdots dipole$$

$$+\frac{1}{2}x'^2 + \frac{1}{2}y'^2 \quad \cdots drift \quad space$$

$$+\frac{e^2B_0^2}{2p_0^2k^2}\sin^2 kz \cdot k^2y^2 \quad \cdots quadrupole$$

$$-\frac{eB_0}{2p_0k}\sin kz \cdot k^2y^2 \cdot x' \quad \cdots sextupole$$

$$+\frac{e^2B_0^2}{6p_0^2k^2}\sin^2 kz \cdot k^4y^4 \quad \cdots octupole$$

$$+\cdots(higher \quad order)$$

Don't cancel after one period (drift space, quadrupole, octupole)

Cancel after one period (dipole, sextupole)

Linear map of wiggler

$$M(z, z+\Delta z) = e^{:-\Delta z \widetilde{H}(z):}$$

$$\begin{cases} Mx = x + \Delta z \cdot x' - \Delta z \dfrac{eB_0}{(1+\delta) p_0 k} \sin(kz + \phi_0) + \cdots \\ Mx' = x' + \Delta z \dfrac{e^2 B_0^2 k_x^2}{(1+\delta)^2 p_0^2 k^2} x_0 \sin^2(kz + \phi_0) + \cdots \\ My = y + \Delta z \cdot y' + \cdots \\ My' = y' - \Delta z \dfrac{e^2 B_0^2 k_y^2}{(1+\delta)^2 p_0^2 k^2} y_0 \sin^2(kz + \phi_0) + \cdots \end{cases}$$

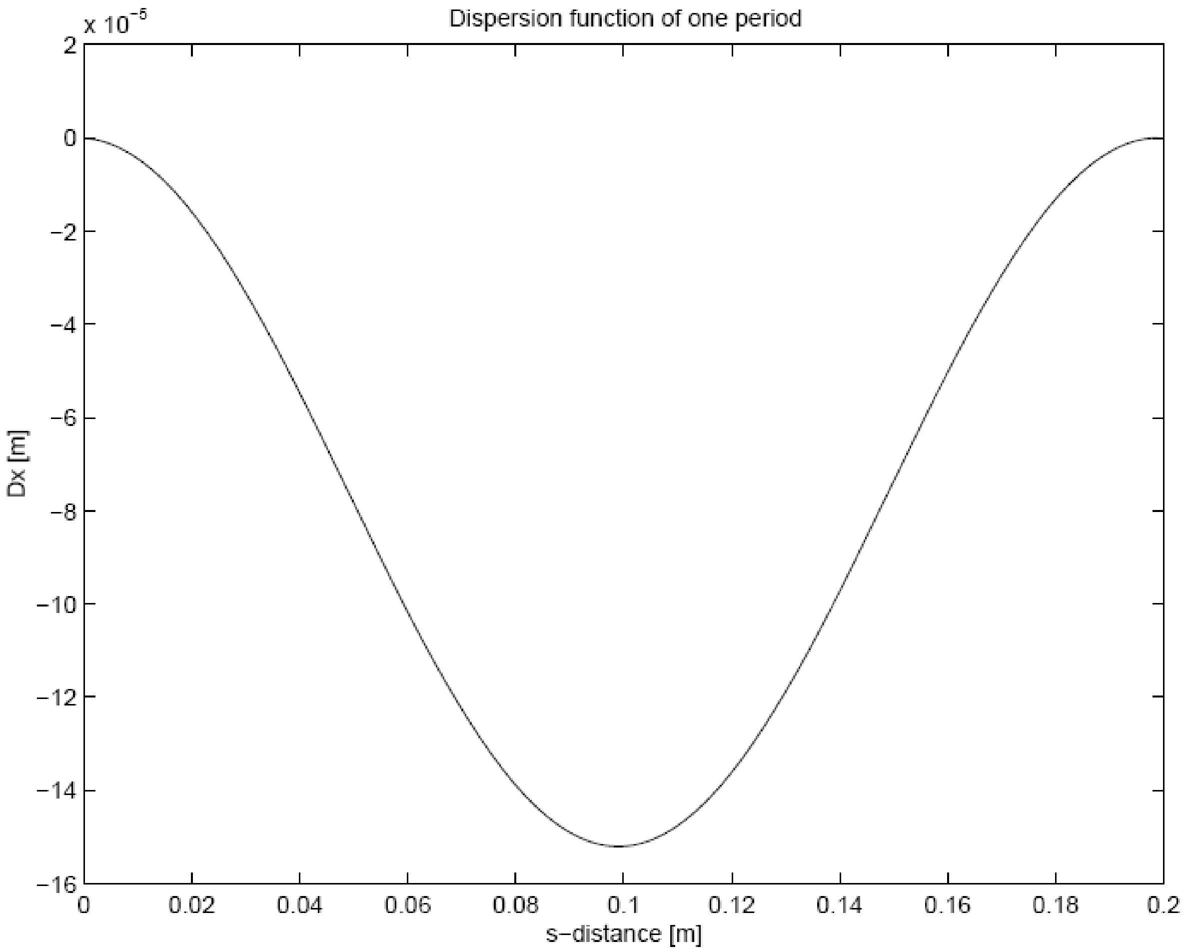
Linear transformation matrix of wiggler

$$\vec{x} = (x, x', y, y', z, \delta, 1)^T$$

$$\begin{bmatrix} 1 & \Delta z & 0 & 0 & 0 & \Delta z \frac{eB_0}{p_0 k}\sin(kz+\phi_0) & m_{17} \\ K_x \Delta z & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta z & 0 & 0 & 0 \\ 0 & 0 & -K_y \Delta z & 1 & 0 & 0 & 0 \\ 0 & -\Delta z \frac{eB_0}{p_0 k}\sin(kz+\phi_0) & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

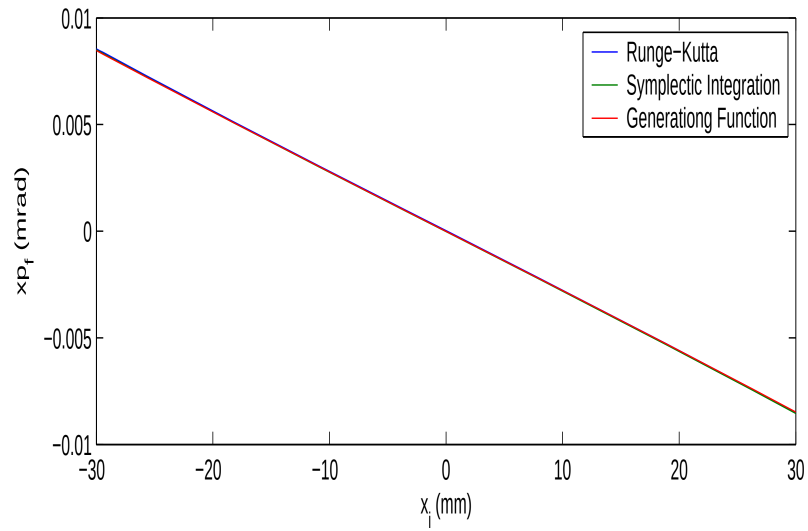
$$m_{17} = -\frac{eB_0 \Delta z}{p_0 k}\sin(kz+\phi_0) \quad K_{x,y} = \frac{e^2 B_0^2 k_{x,y}^2}{(1+\delta)^2 p_0^2 k^2}\sin^2(kz+\phi_0)$$

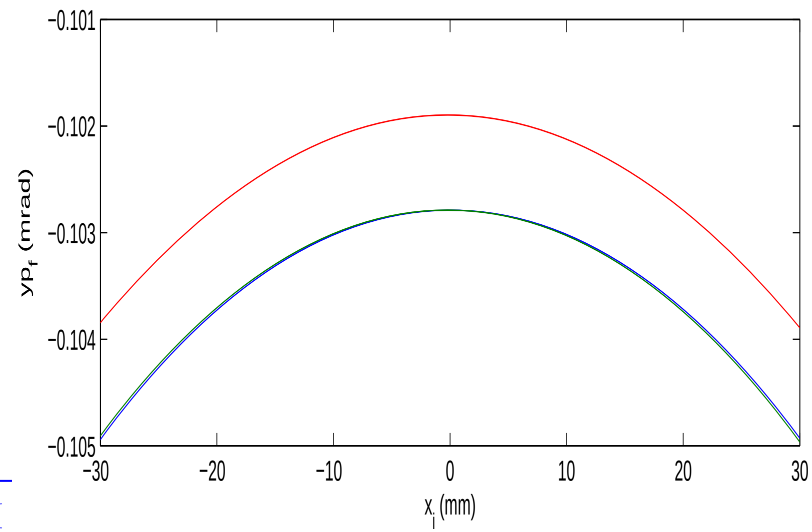
Dispersion of one period



Nonlinear Map: Particle Tracking

- Runge-Kutta Integration through fitted field maps (non-symplectic)
- Symplectic Integration through fitted field maps (Y. Wu)
- Numerical Generating Function from numerical R-K integration (M. Scheer)
  $F(qx_i, px_f, qy_i, py_f)$
  $= a_{klmn} qx_i^k px_f^l qy_i^m py_f^n$





The comparison of different tracking methods

Tracking method	symplecticity	speed	accuracy
Runge-Kutta method	no	slow	high
Symplectic integration	yes	slow	high
Generating function	yes	fast	high

Normal Form to optimize sextupole configuration

- One-turn-map;
- Concatenation of one-turn-map;
- Norm form technique to construct 1st order nonlinear coefficients;
- Minimization of those coefficients and tunes dependence of amplitudes.

1^{st} order nonlinear coefficients of sextupole

$$C_{00003} = \sum \lambda_i D_{xi}{}^3$$

$$C_{10002} = \frac{3i}{2} \sum \lambda_i \sqrt{2\beta_x} D_{xi}{}^2 e^{i\phi_x}$$

$$C_{01002} = -\frac{3i}{2} \sum \lambda_i \sqrt{2\beta_x} D_{xi}{}^2 e^{-i\phi_x}$$

$$C_{20001} = -\frac{3}{2} \sum \lambda_i \beta_x D_{xi} e^{2i\phi_x}$$

$$C_{02001} = -\frac{3}{2} \sum \lambda_i \beta_x D_{xi} e^{-2i\phi_x}$$

$$C_{11001} = 3 \sum \lambda_i \beta_x D_{xi}$$

$$C_{00201} = \frac{3}{2} \sum \lambda_i \beta_y D_{xi} e^{2i\phi_y}$$

$$C_{00021} = \frac{3}{2} \sum \lambda_i \beta_y D_{xi} e^{-2i\phi_y}$$

$$C_{00111} = -3 \sum \lambda_i \beta_y D_{xi}$$

$$C_{30000} = i \sum \lambda_i \left(\frac{\beta_x}{2}\right)^{\frac{3}{2}} e^{3i\phi_x}$$

$$C_{03000} = -i \sum \lambda_i \left(\frac{\beta_x}{2}\right)^{\frac{3}{2}} e^{-3i\phi_x}$$

$$C_{21000} = 3i \sum \lambda_i \left(\frac{\beta_x}{2}\right)^{\frac{3}{2}} e^{i\phi_x}$$

$$C_{12000} = -3i \sum \lambda_i \left(\frac{\beta_x}{2}\right)^{\frac{3}{2}} e^{-i\phi_x}$$

$$C_{10200} = -\frac{3i}{4} \sum \lambda_i \sqrt{2\beta_x} \beta_y e^{i(\phi_x+2\phi_y)}$$

$$C_{01020} = \frac{3i}{4} \sum \lambda_i \sqrt{2\beta_x} \beta_y e^{-i(\phi_x+2\phi_y)}$$

$$C_{10020} = -\frac{3i}{4} \sum \lambda_i \sqrt{2\beta_x} \beta_y e^{i(\phi_x-2\phi_y)}$$

$$C_{01200} = \frac{3i}{4} \sum \lambda_i \sqrt{2\beta_x} \beta_y e^{i(-\phi_x+2\phi_y)}$$

$$C_{10110} = \frac{3i}{2} \sum \lambda_i \sqrt{2\beta_x} \beta_y e^{i\phi_x}$$

$$C_{01110} = -\frac{3i}{2} \sum \lambda_i \sqrt{2\beta_x} \beta_y e^{-i\phi_x}$$

Optimization results

- Four families of sextupole are used for chromaticity correction;

```
The K2L value of sextupoles:
S1(K2L) =-0.61940484
S2(K2L) = 0.75713146
S3(K2L) =-0.65309156
S4(K2L) = 0.78349853
```

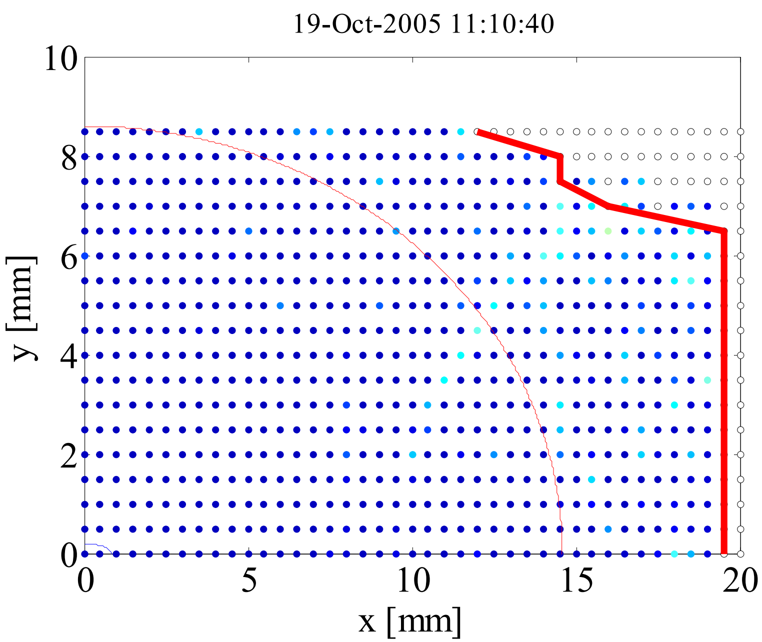
- Harmonic sextupole at dispersion-free section does NOT help improve DA.

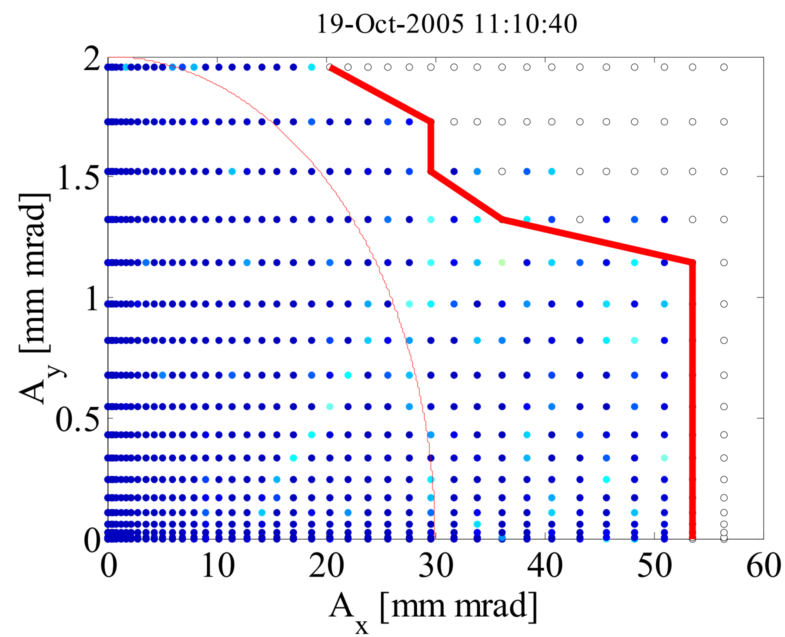
```
The K2L value of sextupoles:
S1(K2L) =-0.62774759
S2(K2L) = 0.75538557
S3(K2L) =-0.64480679
S4(K2L) = 0.78533184
S5(K2L) =-0.01954510
S6(K2L) = 0.01404320
S7(K2L) = 0.01168470
```

Simulation

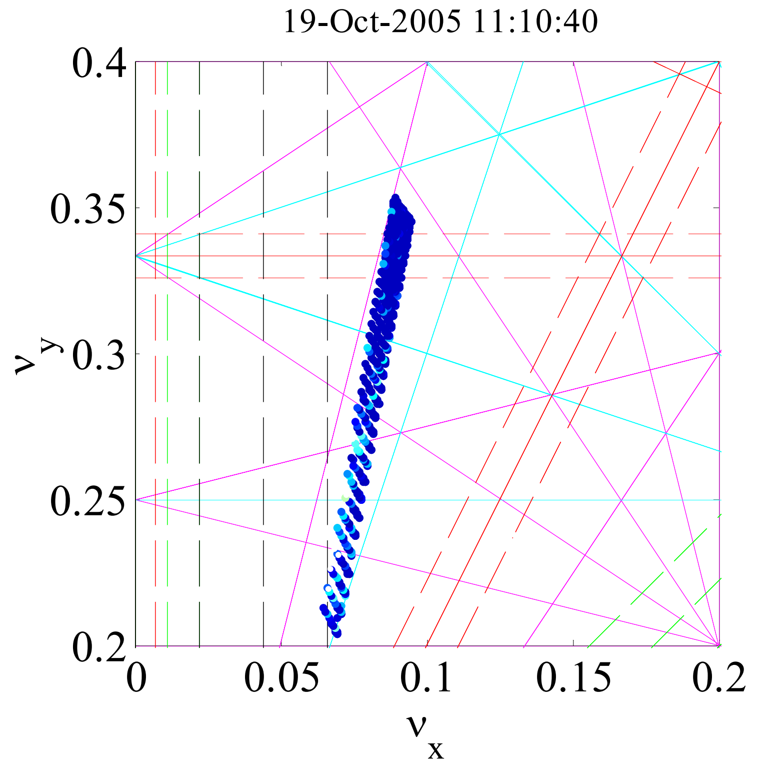
- Simulation code is modified version SIXTRACK (W. Decking and Y. Li)
- Simulation including
  - Damping wigglers (numerical GF);
  - 13 insertion devices with typical parameters (analytical GF);
  - Multipole errors of all elements (1-10th order thin-length kicks, ~0.5mm orbit distortion and ~10% beta-beat);
  - 7mm physical aperture limits of undulators gap.

Ideal machine with
Qx=36.095, Qy=32.345

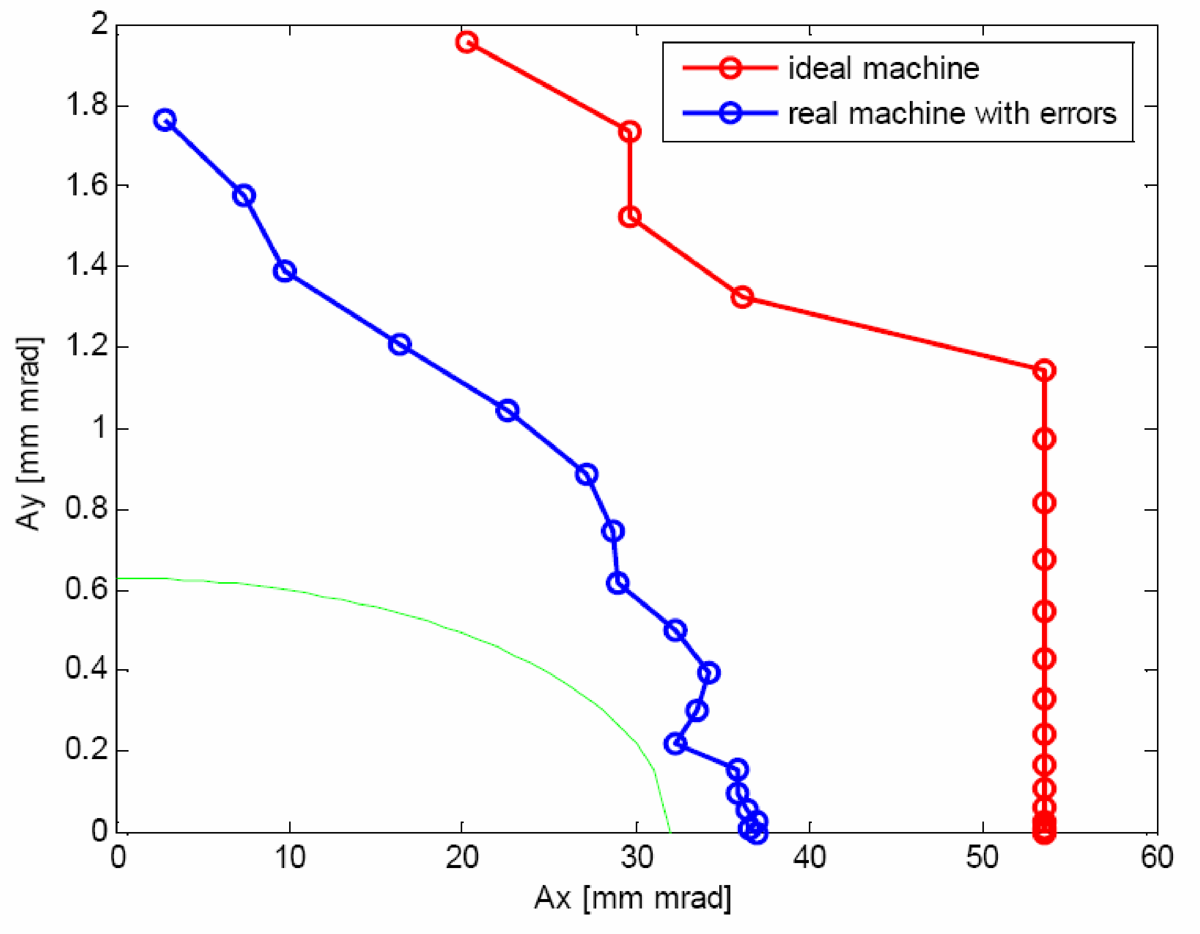




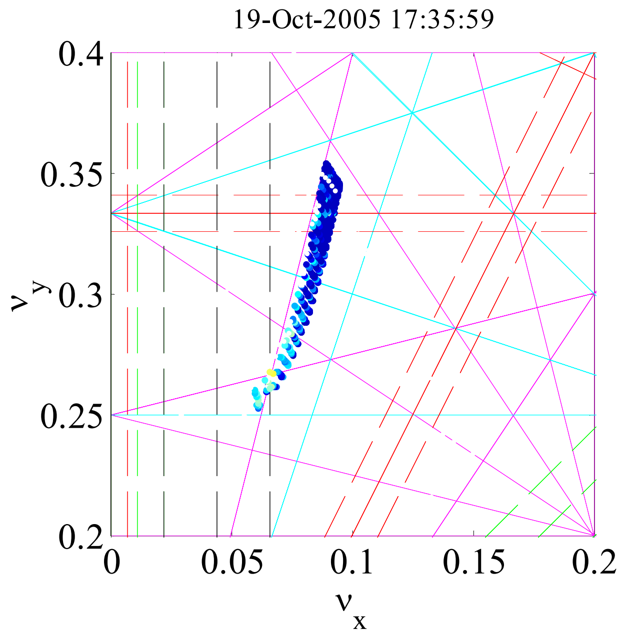
Frequency map

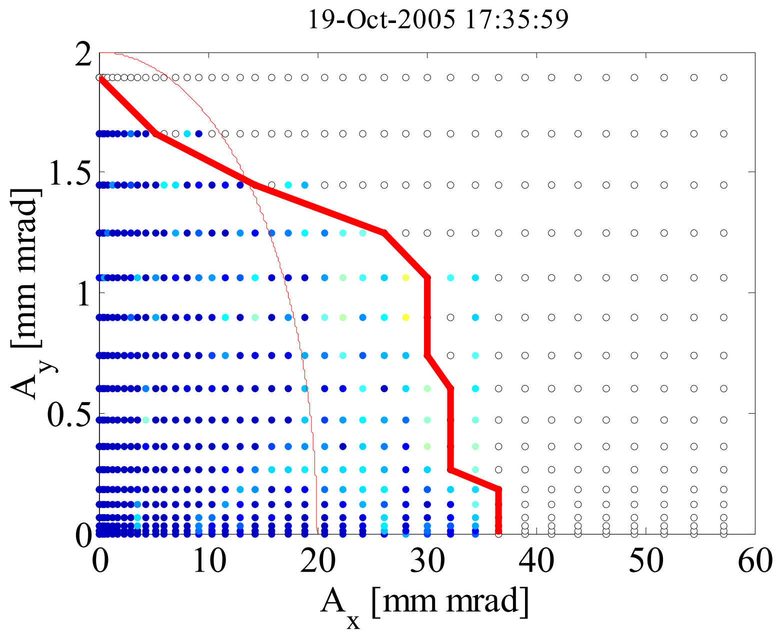


Dynamic aperture of PETRA III ring



One random seed





Thank you for your attention!