PRODUCTION OF COHERENT X-RAYS WITH A FREE-ELECTRON LASER BASED ON OPTICAL WIGGLER

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Abstract

The interaction between high-brilliance electron beams and counter-propagating laser pulses produces X-rays via Thomson back-scattering. If the laser pulse is long and intense enough, the electrons of the beam can bunch on the scale of the emitted X-ray wavelength and collective effects can occur. These effects give rise to the FEL instability and the system behaves like a free-electron laser based on an optical undulator. Coherent X-rays can be irradiated, with a bandwidth much thinner than that of the corresponding incoherent emission. We analyse with a 3D code the transverse effects in the emission and give a generalized form of the Pellegrini criterion which is validated on numerical evidence.

INTRODUCTION

A Thomson back-scattering set-up can be considered in principle as a source of X-ray pulses at the same time easily tunable and highly monochromatic. Due to recent technological developments in the production of high brilliance electron beams and high power CPA laser pulses, it is now even conceivable to make steps toward their practical realisation [1,2]. The radiation generated in the Thomson back-scattering is traditionally considered incoherent and is calculated by summing at the collector the intensities of the fields radiated by each electron [3]. If the laser pulse is long enough, however, collective effects can take place and even become dominant. In this range of parameters the system behaves like a free-electron laser in which the usual static wiggler is substituted with the optical laser pulse. From the point of view of the theoretical description of the process, the generation of coherent X-radiation can be demonstrated on the basis of the same set of 1D equations that are used in the theory of the high-gain free-electron laser amplifier. However, many aspects of the process are connected with the finite transverse geometry of the electron beam and the laser pulse and, in order to give a more precise quantitative evaluation of the radiation efficiency, it is obviously necessary to consider 3D equations.

In this paper, we present some particularly interesting data relevant to the solution of a set of 3D equations with a discussion of their importance and the conditions that allow operating the Thomson back-scattering in a FEL mode[4,5].

BASIC EQUATIONS AND NUMERICAL RESULTS

We write the 3D equations we have used in the usual non-dimensional form (see Ref.[5] for details)

$$\frac{d}{d\bar{t}}\bar{\mathbf{r}}_j(\bar{t}) = \rho\,\frac{\mathbf{P}_j(\bar{t})}{\bar{\gamma}_j(\bar{t})} \tag{1}$$

$$\frac{d}{d\bar{t}}P_{jz}(\bar{t}) = -\frac{\bar{a}_{L0}^2}{2\rho\gamma_0^2}\frac{1}{\bar{\gamma}_j}\left[\frac{\partial}{\partial\bar{z}}|g|^2\right]_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j} - \frac{2}{\bar{\gamma}_j}\mathrm{Real}\left[(g^*\bar{A})_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j}e^{i\theta_j(\bar{t})}\right] \tag{2}$$

$$\frac{d}{d\bar{t}}\mathbf{P}_{j\perp}(\bar{t}) = -\frac{\bar{a}_{L0}^2}{2\rho\gamma_0^2}\frac{1}{\bar{\gamma}_j}\left[\bar{\nabla}_\perp|g|^2\right]_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j} - \frac{4\eta}{1+\frac{k_L}{k}}\frac{1}{\bar{\gamma}_j}\mathrm{Im}\left[(\nabla_\perp(g^*\bar{A}))_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j}e^{i\theta_j(\bar{t})}\right] \tag{3}$$

$$\left(\frac{\partial}{\partial\bar{t}}+\frac{\partial}{\partial\bar{z}}\right)\bar{A}(\bar{\mathbf{x}},\bar{t}) - i\eta\bar{\nabla}_\perp^2\bar{A} = b \tag{4}$$

where the scaled time and space variables are $\bar{t} = 2\rho\omega_L t$ and $\bar{x} = 2\rho k_L x$ and

$$b = \frac{1}{N_S}\frac{V_b}{V_b(t)}\sum_s\frac{g(\mathbf{r}_s(\bar{t}),\bar{t})}{\bar{\gamma}_s(\bar{t})}e^{-i\theta_s(\bar{t})} \tag{5}$$

$$\theta_j(\bar{t}) = \frac{k}{2\rho k_L}\left((1+\frac{k_L}{k})\bar{z}_j(\bar{t}) + (\frac{k_L}{k}-1)\bar{t}\right) \tag{6}$$

$$\gamma_j^2 = 1+\gamma_0^2\rho^2P_{jz}^2 + \bar{a}_{L0}^2(|g|^2)_{\bar{\mathbf{x}}=\bar{\mathbf{r}}_j(\bar{t})} \tag{7}$$

Furthermore

$$A_L(xyzt) = \frac{a_{L0}}{\sqrt{2}}(g(xyzt)e^{-i(k_Lz+\omega_Lt)}\hat{\mathbf{e}} + cc) + O(\frac{\lambda_L}{\sigma_L}) \tag{8}$$

is the circularly polarized laser field vector potential, $\lambda_L = 2\pi/k_L$ the laser wavelength, σ_L the r.m.s. spot radius, g(xyzt) the envelope, $\omega_L=ck_L$ the angular frequency and $\hat{e} = \frac{1}{\sqrt{2}}(e_x + ie_y)$. γ_0 is the average value of γ over all electrons of the beam at t=0, $\bar{\gamma}_j = \gamma_j/\gamma_0$, $\mathbf{P}_j = \mathbf{p}_j/\gamma_0\rho$, $\bar{a}_{L0} = \frac{e}{mc^2}a_{L0}$ is the laser parameter, $\eta = \frac{k_L}{k}\rho$,

$$A = -i\left(\frac{\omega_b^2 a_{L0}}{4\sqrt{2}\omega\omega_L\gamma_0\rho}\right)\frac{e\overline{A}}{mc^2}$$ is the collective field potential and the FEL parameter is

$$\rho = \frac{1}{\gamma_0}\left(\frac{\omega_b^2 a_{L0}^2}{16\omega_L^2}\left(1+\frac{\omega_L}{\omega}\right)\right)^{\frac{1}{3}} \tag{9}$$

We have developed a three-dimensional code that solves the set of equations (1)-(4) and is based on a fourth-order Runge-Kutta for the particles and a finite-difference scheme for the solution of the Schroedinger-like non homogeneous Eq. (4) for the radiation field. An example of solution is provided by a beam with the energy content of about 15 MeV (a factor of 2 lower than the typical Sparc-PlasmonX case), corresponding to $<\gamma>=30$, with a mean radius $\sigma_0=10$ μm, a total charge of 1 nC, a length $L_b= 200$ μm, corresponding to a beam current I=1.5 kA. The laser pulse considered in this case has a wavelength $\lambda_L=0.8$ μm. Furthermore, the focal spot radius w_0 is about 50 μm with $\bar{a}_{L0}=0.8$ so that the radiation wavelength is $\lambda= 3.64$ Angstrom and $\rho = 4.38\ 10^{-4}$. The gain length is about 145 μm, the collective effects saturating in 7-12 gain lengths, i.e. in a time interval of about 5 ps which is of the same order as the duration of the laser pulse. The quantum parameter q= 0.5, the energy spread $\Delta\gamma/\gamma=1.10^{-4}$ the initial normalized transverse emittance being varied from 0 up to 2 μm.

In Fig.1 the average values of the bunching factor and the collective potential amplitude are shown in time. In this calculation the laser profile is flat inside a region with $w_0=50$ μm, the laser parameter is $\bar{a}_{L0}=0.8$, the initial emittance $\varepsilon_n=0.88$ μm and the detuning $\Delta\omega/\omega= -2.10^{-4}$.

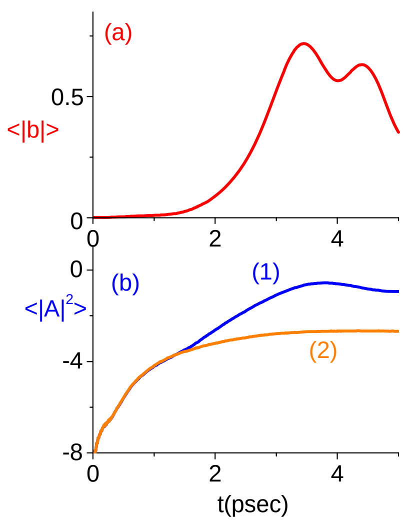

Figure 1: Averaged bunching factor (a) and logarithm of the radiation intensity (b) versus time in the coherent (1) and incoherent (2) cases and for $\lambda_L=0.8$μm , $\bar{a}_{L0}=0.8$, $\Delta\gamma/\gamma=10^{-4}$, $\Delta\omega/\omega=-2\ 10^{-4}$, $\varepsilon_n=0.88$.

The saturation level of the radiation is reached at about t=4 ps with the value $<|A|^2>_{peak}=0.275$ and a total number of photons 1.86 10^{10}, against the 2.10^8 provided by the incoherent process. The peak brilliance, for this example, is 3.7 10^{25} photons/(sec mm^2 mr^{2}0.1%), while the coherent power is 15.5 MW.

In Fig.2 the spectrum of the radiation is reported versus $\Delta\omega/(\omega\rho)$ for $\varepsilon_n=0.44$ and 0.88 μm.

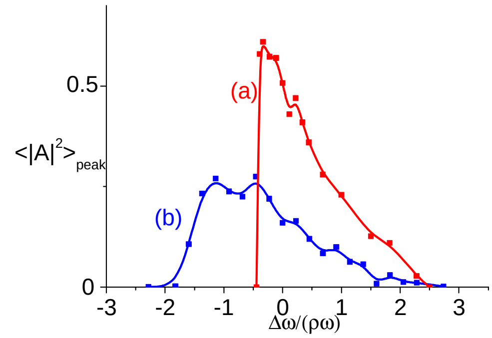

Figure 2: $<|A|^2>_{peak}$ versus $\Delta\omega/(\omega\rho)$ for the case of Fig.1 and $\varepsilon_n=0.44$ μm (a) and $\varepsilon_n=0.88$ μm (b).

We may note a broadening of the emission line of about a factor of two with increasing emittance.

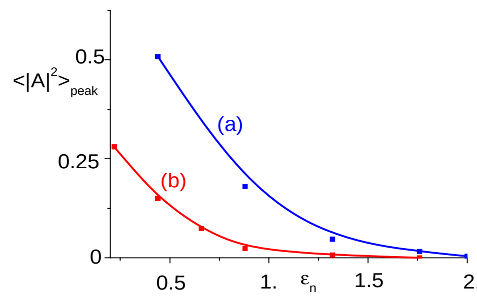

Figure 3: $<|A|^2>_{peak}$ versus ε_n for the case of Fig.5, with $\Delta\omega/\omega=0.$ and for a flat laser profile with $w_0=50$ μm and $a_{L0}=0.8$ (curve (a)) and a Gaussian laser profile with $a_{L0}=0.8$ and $\sigma_L=106$ μm (curve (b)).

Fig.3 shows the dependence of the radiation intensity at saturation on the emittance. Curve (a) is relevant to the case of a flat laser pulse with $w_0=50$ μm, while curve (b) shows the more critical situation when the laser has a Gaussian profile. In this last case the quantity $\sigma_L= 106$ μm with $\bar{a}_{L0}=0.8$, leading to a corresponding increase in the laser power.

We have considerable emission in violation of the Pellegrini criterion [6] for a static wiggler. In fact, in case (a) of Fig. 3, for instance, the emittance largely exceed the value $\gamma\lambda/4\pi$, which is, in this case, about 9 10^{-4} μm. We can justify this result by considering that the line width in a situation dominated by emittance effects can be written as

$$\frac{\Delta\lambda}{\lambda} \approx \frac{\gamma^2\theta^2}{1+\bar{a}_{L0}^2} \approx \frac{\varepsilon_n^2}{\sigma_0^{\ 2}}\ . \tag{10}$$

In order to have emission of radiation at the planned wavelength, we must assume that the linewidth $\Delta\lambda/\lambda < \alpha\rho$, with α a numerical factor not much larger than 1. Hence, we can write for the emittance

$$\varepsilon_n \leq \sqrt{\alpha\rho}\ \sigma_0 . \qquad (11)$$

Considering the definitions of the gain length $Lg = \lambda_L/(4\pi\rho)$ and of the radiation Rayleigh length $Z_R = 2\pi\sigma_R^2/\lambda$, we can express the factor ρ in terms of the ratio Z_R/L_g, obtaining $\rho = \frac{Z_R}{L_g}\frac{\lambda\lambda_L}{8\pi^2\sigma_R^2}$. Supposing that the electron beam and the radiation overlap, i.e., that $\sigma_0 = \sigma_R$, and remembering the resonance relation in its simpler form $\lambda_L = 4\gamma^2\lambda$, we obtain for an optical undulator

$$\varepsilon_n \leq \sqrt{\alpha}\sqrt{\frac{Z_R}{L_g}\frac{\lambda\ \gamma}{\sqrt{2}\pi}} \qquad (12)$$

where $\alpha = \frac{\delta\omega}{\omega\rho}$. The usual form of the Pellegrini criterion for a static wiggler can be recovered by assuming that $Z_R = L_g$ and inserting the resonance condition for the static undulator. If we take into account the fact that in our case $Z_R/L_g = 1.18\ 10^4$ and estimating $\alpha=2$, we can predict considerable emission up to an emittance value of $\varepsilon_n = 0.3$ µm (corresponding to a value of $\varepsilon_{nx} = 0.15$ mm) which is not far from the result of Fig. 3.

The last Fig.4 shows the most critical effect, i.e., the dependence of the signal growth on the transverse energy distribution of the laser in the case of a Gaussian pulse and for $\varepsilon_n = 0.44$ µm, $\Delta\omega/\omega = -1.10^{-4}$, $\bar{a}_{L0} = 0.8$. In this case, in fact, if the Gaussian laser has a spot size with a radius

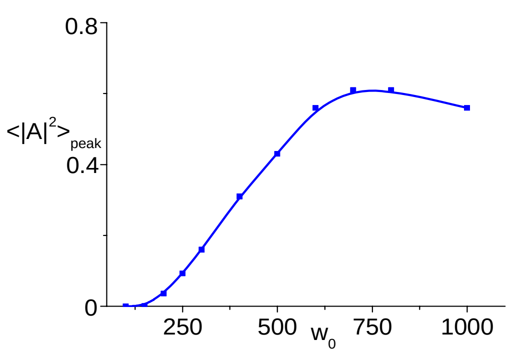

Figure 4: $<|A|^2>_{peak}$ versus w_0 for the case of a Gaussian laser profile for $\varepsilon_n = 0.44$ µm, $\Delta\omega/\omega = -1.10^{-4}$, $a_{L0} = 0.8$.

smaller than 75 µm, the FEL instability does not develop. This example is also characterized by a choice of the electron beam with an initial emittance larger than in the first case and actually not far from the best experimental values, but with a larger current. However, one can see that the requirements on the total energy of the laser and stability of the energy transverse profile are in this case particularly demanding.

Another critical issue is the variation in the laser intensity $\Delta = \Delta a_{L0}/a_{L0}$, that leads to a broadening in the spectrum of the order $\Delta\lambda/\lambda = 2\bar{a}_{L0}^2\Delta/(1.+\bar{a}_{L0}^2)$. For an FEL mode operation of the Thomson source, the condition $\Delta \leq \rho\frac{(1+\bar{a}_{L0}^2)}{\bar{a}_{L0}^2}$ must therefore be added. Assuming a laser pulse duration of $c\tau = 10\ L_g$, we can derive a further threshold condition on the laser pulse energy U of the form $U > 18.6\ \lambda_L/\Delta^2$, which leads to $U > 0.15/\ \Delta^2$ J.

CONCLUSIONS

Considerable coherent X-rays radiation is possible as a result of the collective interaction between an electron beam and a counter-propagating laser pulse that take place in a Thomson scattering. If the laser pulse is sufficiently long the FEL instability can develop and a regime of collective effects is established. The result is an emission at least two order of magnitudes larger than the usual incoherent emission, with a more narrow and peaked spectrum. However, the brilliance and the power delivered are a few orders of magnitude smaller than in the case of an FEL in the X-ray range operating with a static wiggler. Other critical issues for the appearance of collective effects are related with: (i) the current density carried by the electron beam which has to be large enough, (ii) the initial emittance of the electron bunch which must be not much larger than indicated by the generalized Pellegrini criterion in (12), (iii) the transverse distribution of the laser pulse which must be sufficiently homogeneous within the region occupied by the electrons, (iv) the fluctuations of the laser intensity and (v) the large laser energy content needed .

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