An Investigation of Possible Non-Standard Photon Statistics in a Free-Electron Laser II: Theory

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We explore whether we can at present find a theoretical basis for non-standard, sub-Poissonian photon statistics in the coherent spontaneous harmonic radiation of an FEL as was claimed to have been measured with the Mark III FEL [1]. We develop a one dimensional quantum FEL oscillator model of the harmonic radiation in the linear gain regime to calculate the photon statistics. According to our study, it seems unlikely that the photon statistics for an FEL oscillator starting from the noise could be sub-Poissonian.

Light emitted by randomly distributed electrons [3,4]

$$\langle(\delta n)^2\rangle = \langle(\delta n)^2\rangle_Q + \langle(\delta n)^2\rangle_C$$

$\langle(\delta n)^2\rangle_Q$: quantum origin, $\langle(\delta n)^2\rangle_C$: contribution of classical fluctuation of electrons

$$F_Q = \frac{\langle(\delta n)^2\rangle_Q}{\langle n\rangle} = 1,\ F_C = \frac{\langle n\rangle}{M}$$

$M = t_p/\tau_{coh} \gg 1$, and the photon statistics is super-Poissonian

(coherence time: τ_{coh}; pulse length: t_p)

A one dimensional quantum FEL model [2,7]

$$\mathfrak{a}_{1,n+1} = g\mathfrak{a}_{1,n} + \mathcal{F}_n$$

field operator for the fundamental mode at the nth pass ($\mathfrak{a}_{1,n}$); electron beam operator at the nth pass ($\mathcal{F}_n$)

$|g|^2$ is the gain per pass

$$g \equiv \frac{1}{3}\sum_\alpha e^{-i\mu_\alpha 2k_u\rho L_u}$$

undulator wave number (k_u); FEL pierce parameter (ρ); undulator length (L_u)

μ_α: three roots of $(\mu - \frac{\Delta\nu}{2\rho})(\mu^2 - \frac{q^2}{4}) = 1$

harmonic number's deviation from unity ($\Delta\nu$); quantum FEL parameter (q)

$$\mathcal{F}_n = -i\sum_\alpha \Upsilon_\alpha e^{-i\mu_\alpha 2k_u\rho L_u}\mathcal{B}_{n_0} - i\sum_\alpha \frac{\Upsilon_\alpha}{\mu_\alpha}e^{-i\mu_\alpha 2k_u\rho L_u}\mathcal{P}_{n_0}$$

$$\Upsilon_\alpha = \frac{\mu_\alpha}{(\mu_\alpha - \mu_\beta)(\mu_\alpha - \mu_\gamma)}$$

initial electron bunching at the nth pass ($\mathcal{B}_{n_0}$); initial collective momentum at the nth pass ($\mathcal{P}_{n_0}$)

Fundamental mode's photon statistics for electrons initially in the minimum noise state

- The radiation is chaotic
- The initial wave function of an electron is Gaussian with infinitesimal width in position space, which is centered at its classical position
- The classical position is randomly distributed
- Coherent radiation emitted by randomly distributed electrons

Fundamental mode's photon statistics in literature

- The radiation is chaotic for the initial electrons' wave function of momentum eigenstate (infinite width in position space, i. e., coherent radiation emitted by randomly distributed electrons) [8]
- Some initially entangled state of the electrons can result in the amplitude-squeezed radiation [9], which is interestingly not accompanied by the sub-Poissonian photon statistics

amplitude-squeezed radiation: fluctuation of the field variable in phase with the output field is reduced below the symmetrical vacuum fluctuation's quantum limit

Harmonic mode's photon statistics

classical expression: $a_{h,n+1} = a_{h,n} + k_h a^h_{1,n}$ — harmonic mode driven by the fundamental mode

$$\mathfrak{a}_{h,n+1} = \sqrt{\eta_h}e^{i\phi_{h,n}}\mathfrak{a}_{h,n} + k_h\mathfrak{a}^h_{1,n} + \sqrt{1-\eta_h}e^{i\phi_{h,u,n}}\mathfrak{a}_{h,u,n}$$

$$\mathfrak{a}_{h,n} = \sqrt{\eta_h}^{n-1}e^{i\Phi_{h,j}}\mathfrak{a}_{h,1} + \sum_{j=1}^{n-1}\sqrt{\eta_h}^{n-j-1}e^{i\Phi_{h,j+1}}\left(k_h\mathfrak{a}^h_{1,j} + \sqrt{1-\eta_h}e^{i\phi_{h,u,j}}\mathfrak{a}_{h,u,j}\right)$$

$$\Phi_{h,j} = \sum_{k=j}^{n-1}\phi_{h,k}$$

$$\langle\mathfrak{n}_h\rangle_n = |k_h|^2\sum_{j,k=1}^{n-1}\sqrt{\eta_h}^{2(n-1)-(j+k)}\langle\mathfrak{a}'^{\dagger h}_{1,n_j}\mathfrak{a}'^h_{1,n_k}\rangle \qquad \langle\mathfrak{n}^2_h\rangle_n = \langle\mathfrak{n}_h\rangle_n + \sum_{j,k,l,m=1}^{n-1}\sqrt{\eta_h}^{4(n-1)-(j+k+l+m)}|k_h|^4\left\langle\mathfrak{a}'^{\dagger h}_{1,n_j}\mathfrak{a}'^h_{1,n_k}\mathfrak{a}'^{\dagger h}_{1,n_l}\mathfrak{a}'^h_{1,n_m}\right\rangle$$

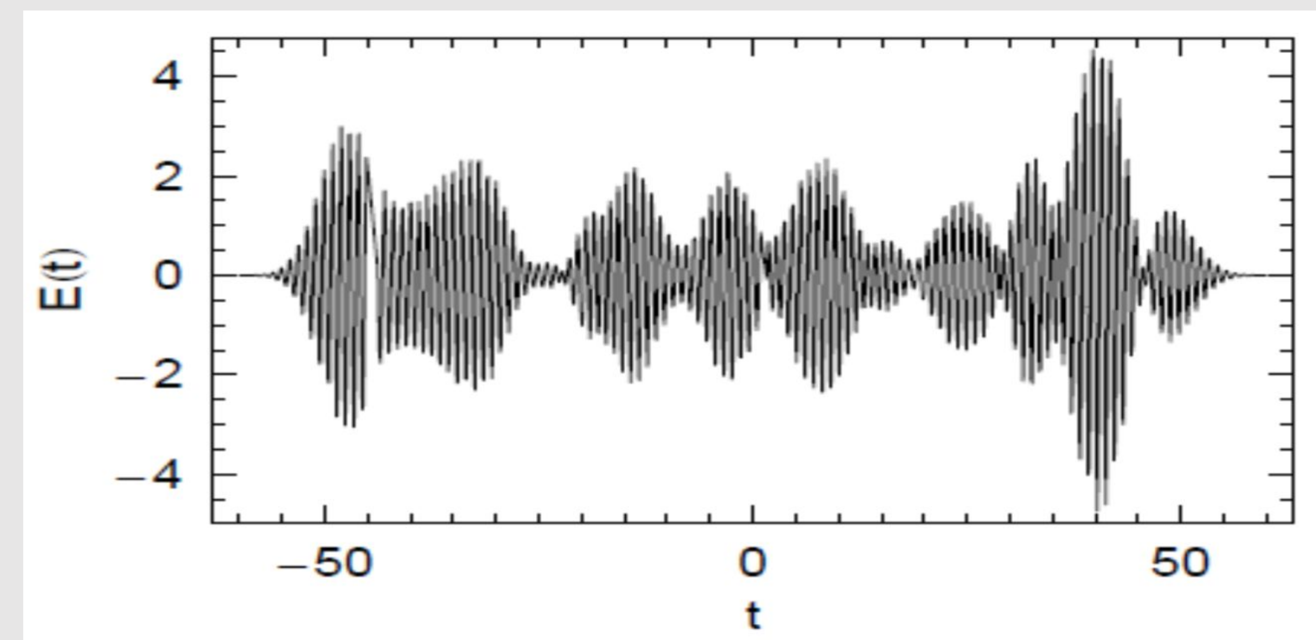
$$\mathfrak{a}'_{1,n_j} \equiv \mathfrak{a}_{1,j}e^{i\Phi_{h,j+1}/h}$$

$$F_{h,n} = \frac{\langle(\delta\mathfrak{C}_{h,n})^2\rangle}{\langle\mathfrak{n}_h\rangle_n} + 1 \geq 1$$

$$\mathfrak{C}_{h,n} = |k_h|^2\sum_{j,k=1}^{n-1}\sqrt{\eta_h}^{2(n-1)-(j+k)}\mathfrak{a}'^{\dagger h}_{1,n_j}\mathfrak{a}'^h_{1,n_k}$$

The photon statistics is not sub-Poissonian regardless of the initial electrons' state

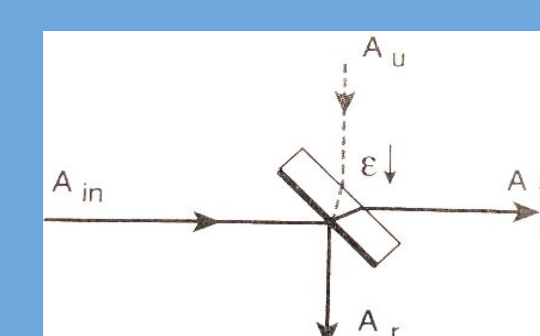
A sum of coherent wave trains emitted by randomly distributed electrons [2]



Cavity loss described by a beam splitter model [5,6]

$$A_t e^{i\phi_t} = \sqrt{1-\epsilon}A_{in}e^{i\phi_{in,t}} + \sqrt{\epsilon}A_u e^{i\phi_{u,t}}$$

$$A_r e^{i\phi_r} = \sqrt{\epsilon}A_{in}e^{i\phi_{in,r}} + \sqrt{1-\epsilon}A_u e^{i\phi_{u,r}}$$



Fundamental mode's photon statistics

$$\mathfrak{a}_{1,n+1} = g\sqrt{\eta_1}e^{i\phi_{1,n}}\mathfrak{a}_{1,n} + \mathcal{F}_n + g\sqrt{1-\eta_1}e^{i\phi_{1,u,n}}\mathfrak{a}_{1,u,n}$$

$$\mathfrak{a}_{1,n} = (g\sqrt{\eta_1})^{n-1}e^{i\Phi_j}\mathfrak{a}_{1,1} + \sum_{j=1}^{n-1}(g\sqrt{\eta_1})^{n-j-1}e^{i\Phi_{j+1}}\left[\mathcal{F}_j + g\sqrt{1-\eta_1}e^{i\phi_{1,u,j}}\mathfrak{a}_{1,u,j}\right]$$

$$\Phi_j = \sum_{k=j}^{n-1}\phi_{1,k}$$

$$\mathcal{F}'_{n_j} \equiv e^{i[\Phi_{j+1} - j\mathrm{Arg}(g)]}\mathcal{F}_j$$

$$\langle\mathfrak{n}\rangle_{1,n} = \sum_{j,k=1}^{n-1}(|g|\sqrt{\eta_1})^{2(n-1)-(j+k)}\langle\mathcal{F}'^{\dagger}_{n_j}\mathcal{F}'_{n_k}\rangle$$

$$\langle\mathfrak{n}^2\rangle_{1,n} = \frac{(|g|^2\eta_1)^{n-1}(|g|^2-1) - |g|^2(1-\eta_1)}{|g|^2\eta_1 - 1}\langle\mathfrak{n}\rangle_{1,n} + \sum_{j,k,l,m=1}^{n-1}(|g|\sqrt{\eta_1})^{4(n-1)-(j+k+l+m)}\langle\mathcal{F}'^{\dagger}_{n_j}\mathcal{F}'_{n_k}\mathcal{F}'^{\dagger}_{n_l}\mathcal{F}'_{n_m}\rangle$$

Fano factor at the nth pass:

$$F_{1,n} = \frac{\{(|g|^2\eta_1)^{n-1} - 1\}(|g|^2-1)}{|g|^2\eta_1 - 1} + \frac{\langle(\delta\mathfrak{A}_n)^2\rangle}{\langle\mathfrak{n}_1\rangle_n} + 1 \geq 1$$

$$\mathfrak{A}_n = \sum_{j,k=1}^{n-1}(|g|\sqrt{\eta_1})^{2(n-1)-(j+k)}\mathcal{F}'^{\dagger}_{n_j}\mathcal{F}'_{n_k}$$

The photon statistics is not sub-Poissonian regardless of the initial electrons' state

Conclusion

- It does not seem probable that the photon statistics in the linear gain regime of the FEL starting from the noise is sub-Poissonian, regardless of the harmonic number or the initial state of the electrons
- If a sub-Poissonian FEL light is experimentally observed contradicting our finding, a theory beyond the standard one may be necessary to explain the observation

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