Simulating shot-noise in 'real' electron bunches

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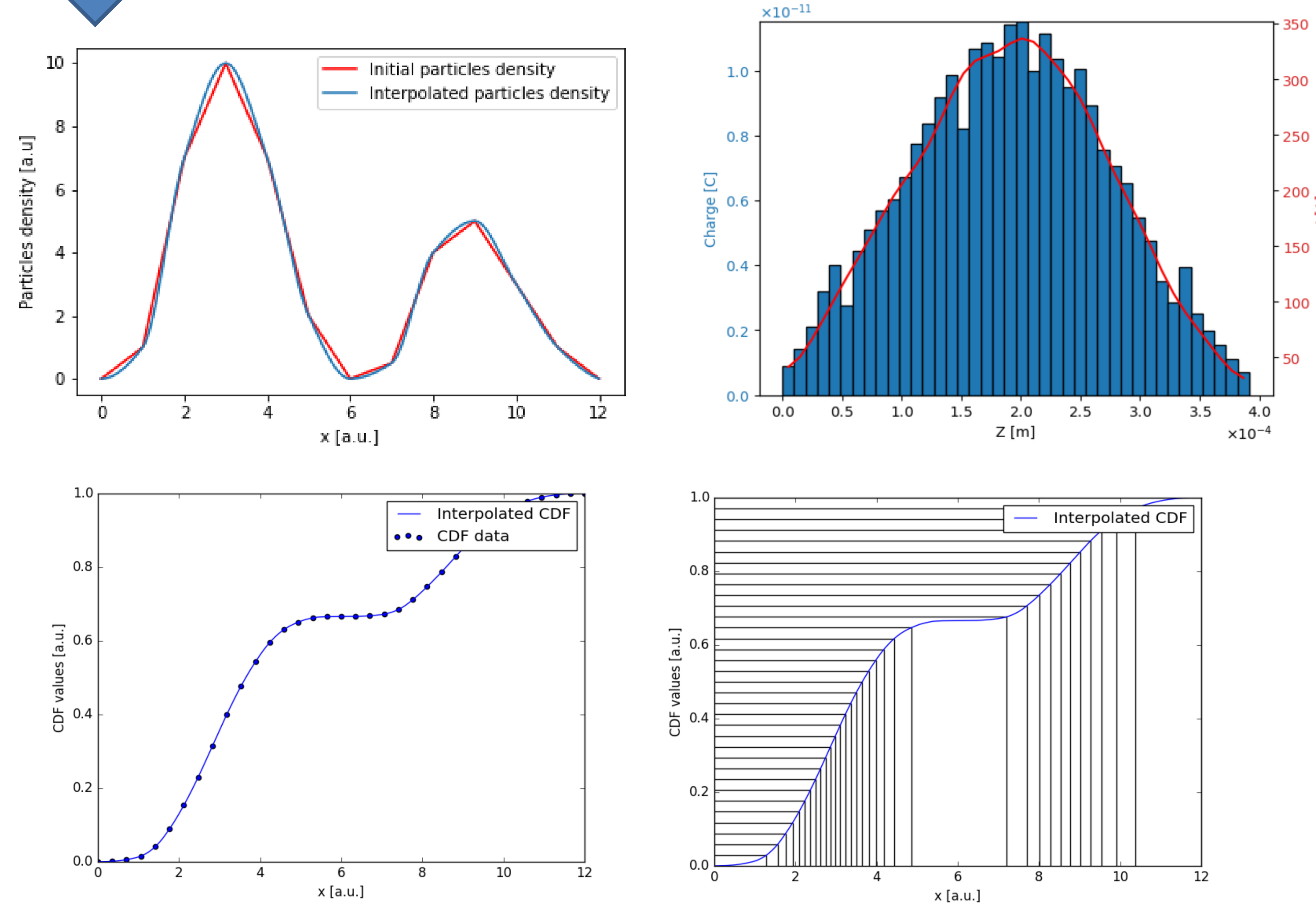
The shot-noise in Free Electron Laser

- A numerical algorithm for applying electron beam shot noise in numerically created electron beams for free electron lasers (FELs) simulation software is presented
- Shot noise[2] is an important part of FEL numerical model, as it is a source of spontaneous emission that may be amplified in the self-amplified spontaneous emission (SASE) regime of operation
- The algorithm uses a phase-space distribution of macro-particles which comes from other software responsible for generating beams for injection into the FEL amplifier (e.g. ASTRA, VSim) and usually has relatively sparse macroparticle distribution without any correct noise statistics applied
- Appropriate conversion and noise is applied
- The statistical properties of the macroparticles are derived directly from the temporal Poisson statistical properties of the real electron distribution. The algorithm is used in the unaveraged 3D FEL numerical simulation code Puffin[1].
- The beam statistics (current, emittance, energy, energy spread) and dimensions (including shape) are kept very close to original (source) distribution.

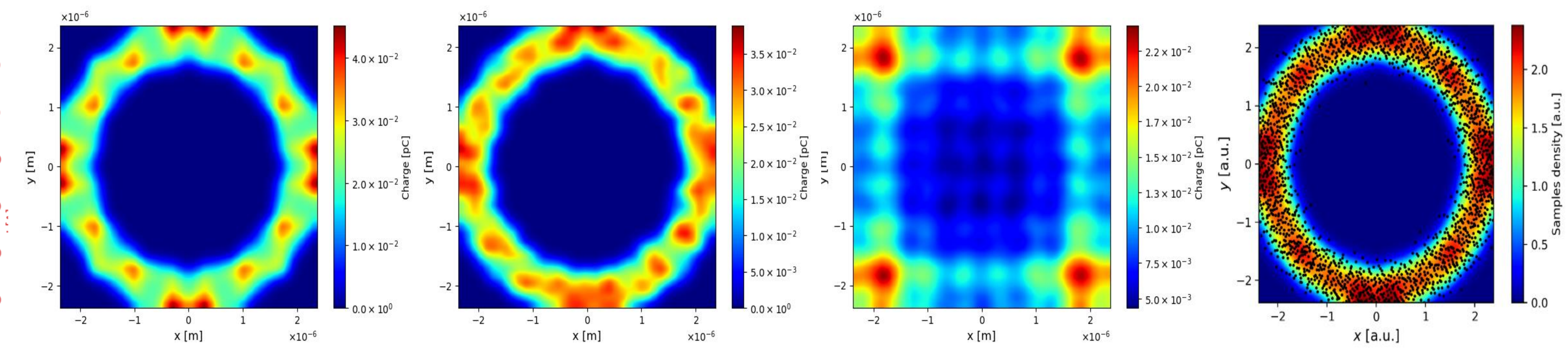
The algorithm

- The macroparticles of the accelerator modelling stage are used to create a discrete charge histogram of bin width along the longitudinal z-axis of beam propagation at a given time (e.g. on entering the FEL).
- A continuous longitudinal charge distribution function f(z) is then created by interpolating the histogram data.
- A transverse charge density function f(x,y,z) is then created for each longitudinal slice using a similar 2-D histogram-interpolation method.
- The longitudinal density function and the transverse density function can then be interpolated to create a continuous 3-D charge density function, f(x,y,z)
- The new set of microparticles is created in each longitudinal slice via a 2-D Cumulative Distribution Function approach in the transverse plane - also called a Joint Distribution Function (JDF).

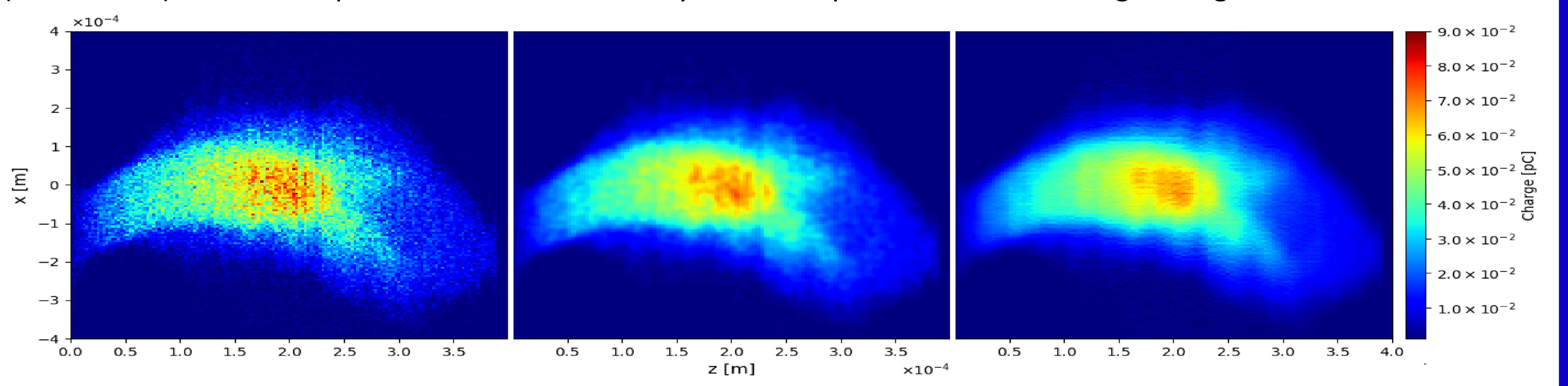
This method **allows to model any cavities and irregular shapes** in the initial beam – it is achieved due the joint cumulative distribution function is not dependant on one dimension, e.g. f(x), only but is based on joint probability f(x,y,z). Example of structure with cavity is presented on the figures below showing a transverse image of a 'tube' type electron beam. Approach where one will first create cumulative distribution function for f(x) and then f(y) separately will result in not obtaining the proper image as the f(x) or f(y) function would not recognize that there is void in the middle – results are presented below.



The plots above show various stages of going from initial particle distribution into a new one with desired number of particles. The plots show only one dimension (cumulative distribution function) approach – e.g. probability density function is used for x or y axis only. The method presented on this poster is extended to use a joint probability function for x,y.



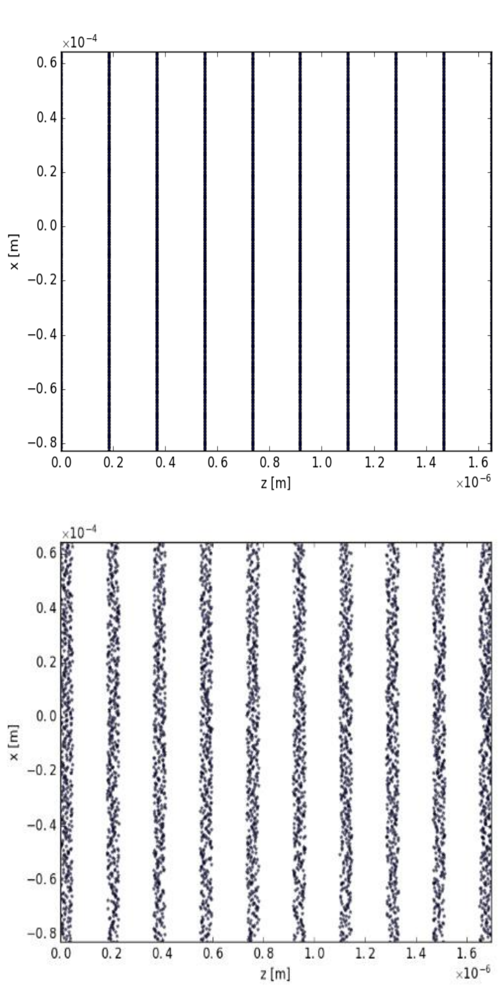
The three pictures above present: the initial particles density map, the density map after the JDF algorithm is applied and the density map after algorithm with separate cumulative distribution function for x and y is used (obvious fail). The fourth picture shows both density and microparticles created using JDF algorithm.



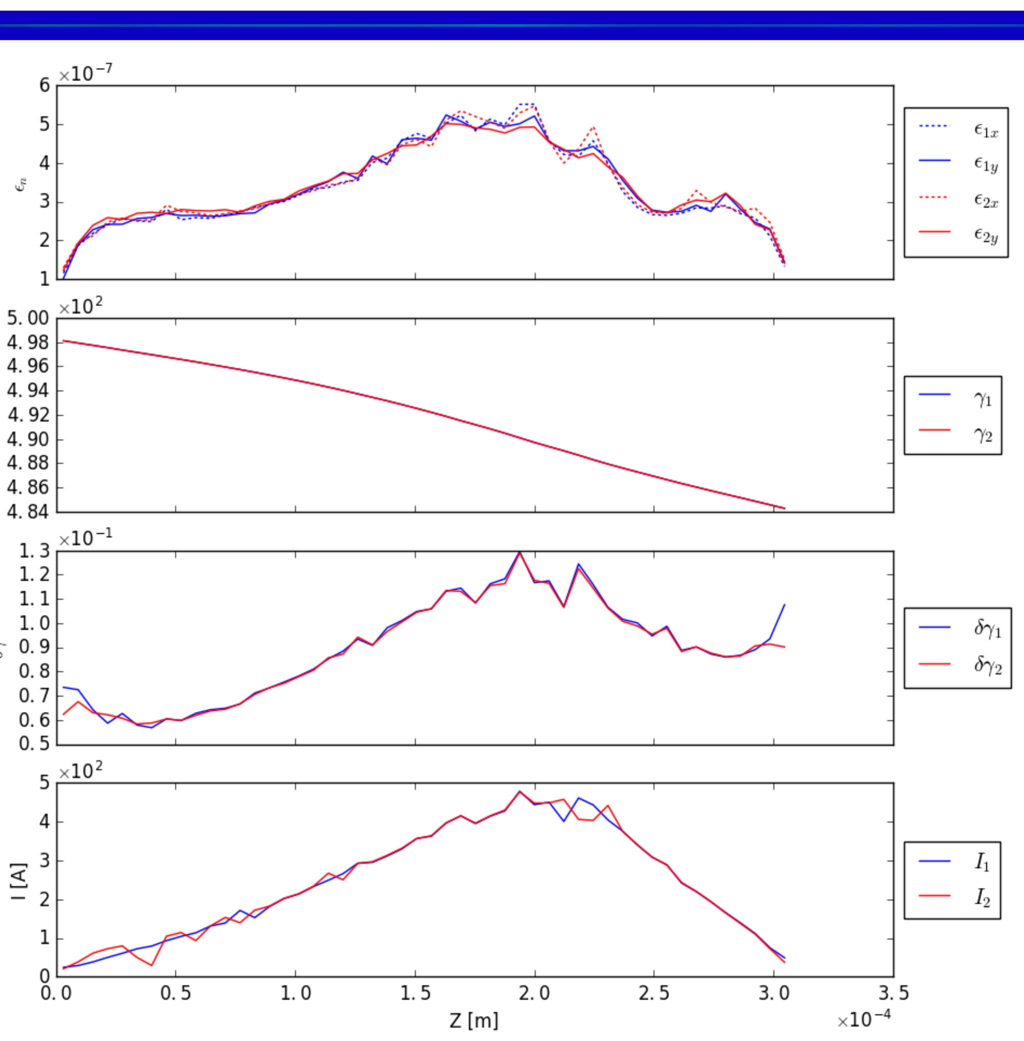
The plots above show three stages of the data processing. First picture on the left shows initial macroparticle density distribution. The middle picture is density distribution with *SciPy ndimage* Gaussian smoothing applied. The right picture is the final microparticle density distribution after JDF routine is applied. In all 3 cases the data was sampled over 150x150 grid.

Applying the shot-noise

The shot-noise is applied only to the longitudinal axis (Z) in a way proposed in [2] – therefore the ordered (sliced) structure has been generated in previous steps. The shot-noise is applied via the macroparticle charge weight and a randomness to the macroparticle. In first instance the noise is applied to Z-position of the particle, following the equation: $Z_{noise} = \frac{Step*(RND)}{\sqrt{Ne}}$, where $Step$ is the distance between nearest macroparticles, Ne is the macroparticle weight and RND is random number ranging from *-0.5* to *+0.5*. This value is calculated and applied individually to each macroparticle. Finally, a random Poisson noise is applied to the macroparticles charge weights.

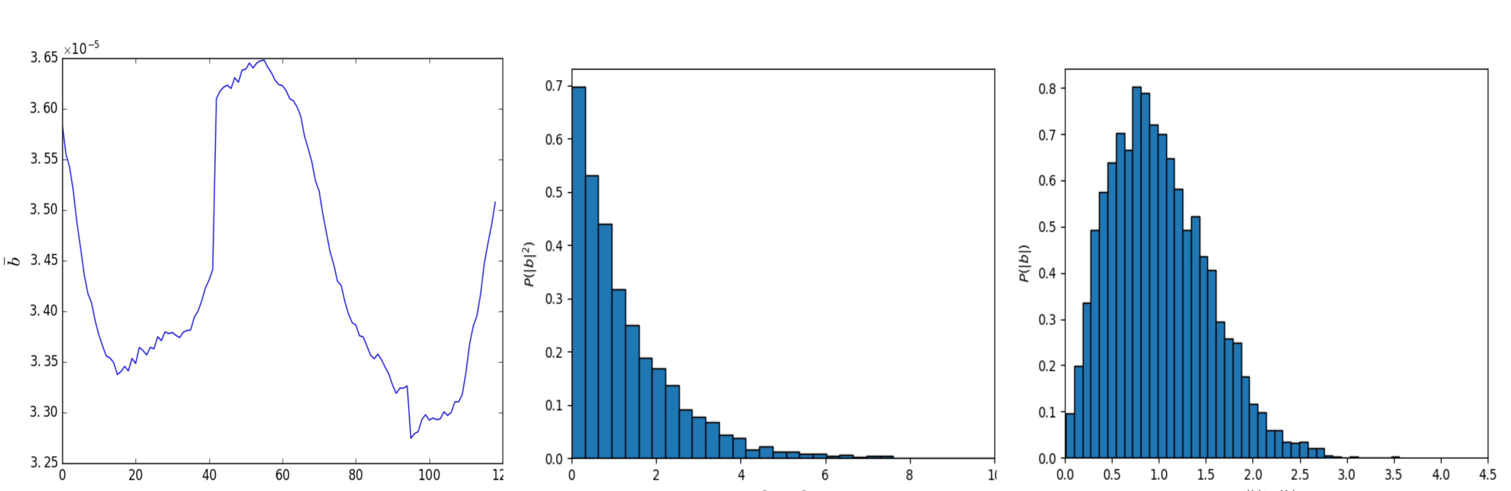


The plots on the left show X/Z projection of the beam before (upper) and after applying the noise. On both pictures one can clearly observe that the beam has an ordered structure with macroparticles equally separated along the longitudinal Z-axis.



Results application and verification

The results were tested by analysing the bunching statistics and other beam statistics. The bunching statistics (below) show good agreement with theoretical model by McNeil et. al. [2]. Beam parameters (left) remain unchanged – the parameters crucial for FEL operation (emittance and energy spread) remained at a similar level with the initial distribution.



References

[1] LT Campbell & BWJ McNeil, '*Puffin: A three dimensional, unaveraged free electron laser simulation code*', Phys. Plasmas **19**, 093119 (2012)

[2] Brian WJ McNeil, MW Poole & GRM Robb, '*Unified model of electron beam shot noise and coherent spontaneous emission in the helical wiggler free electron laser*', Physical Review Special Topics – Accelerators and Beams Vol **6**, 070701 (2003)