

# ANALYSIS OF UNDULATOR RADIATION WITH ASYMMETRIC BEAM AND NON-PERIODIC MAGNETIC FIELD

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## Abstract

Harmonic undulator radiation at third harmonics with non periodic constant magnetic field has been analysed. Symmetric and asymmetric electron beam with homogeneous spread has been used to present viable solution for the resonance shift inherited in undulator with constant magnetic field. The out coming radiation, recovers shifts in resonance and regain its intensity with asymmetric electron beam and harmonic field.

**UNDULATOR FIELD**  $H = [H_0\kappa, b_0H_0(\text{sink}_p + \Delta\text{sink}_l)z, 0]$

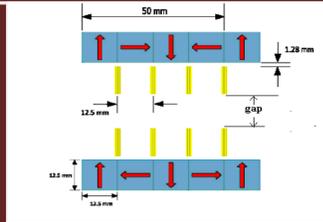
Velocity of electron

$$\beta_y = -\frac{K}{\gamma}\kappa\Omega_p t, \quad \beta_x = -\frac{K}{\gamma}\left[\cos(\Omega_p) + \Delta\frac{\cos(hl)}{l}\right]t$$

$$\beta_z = \beta^* - \frac{K^2}{2\gamma^2}\left[\frac{1}{2}\cos(2\Omega_p) + \frac{1}{2}\left(\frac{\Delta}{l}\right)^2\cos(2l\Omega_p) + \left(\frac{\Delta}{l}\right)\cos(1 \pm l)\Omega_p\right]t + (\kappa\Omega_p t)^2$$

Electron trajectory along z direction,

$$\frac{z}{c} = \beta^* t - \frac{K^2}{8\gamma^2\Omega_p}\sin(2\Omega_p t) - \frac{K_1^2}{8\gamma^2 l\Omega_p}\sin(2l\Omega_p t) - \frac{KK_1}{2\gamma^2(1 \pm l)\Omega_p}\sin(1 \pm l)\Omega_p t - \frac{K^2\kappa^2\Omega_p^2 t^3}{6\gamma^2}$$



Schematic of one Period of harmonic Undulator ( $l=3$ )

## Brightness expression

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2 T^2}{4\pi^2 c} \left\{ |I_x|^2 S(\vartheta, \varphi) + |I_y|^2 S'(\vartheta, \varphi) \right\}$$

$$I_x = \frac{K}{2\gamma} \left[ J_{m+1}(0, \xi_1) + J_{m-1}(0, \xi_1) \right] J_n(0, \xi_2) J_p(\xi_3) J_q(\xi_4) + \frac{\Delta}{h} J_{n+1}(0, \xi_2) + J_{n-1}(0, \xi_2) J_m(0, \xi_1) J_p(\xi_3) J_q(\xi_4)$$

$$I_y = \frac{2i\pi K \kappa N}{\gamma}$$

Line shape functions

$$S(\vartheta, \varphi) = \left| \int_0^1 e^{(\vartheta'\tau + \varphi'\tau^3)} d\tau \right|^2, \quad S'(\vartheta, \varphi) = \frac{\partial S(\vartheta, \varphi)}{\partial \vartheta} = \left| \int_0^1 \tau e^{(\vartheta'\tau + \varphi'\tau^3)} d\tau \right|^2$$

Emission frequency variables

$$\xi_1 = -\frac{\omega K^2}{8\gamma^2\Omega_p}, \quad \xi_2 = -\frac{\omega K_1^2}{8\gamma^2 l\Omega_p} \text{ and } \xi_{3,4} = -\frac{\omega K K_1 \kappa}{2\gamma^2(1 \pm l)\Omega_p}$$

Detuning parameter

$$\vartheta = \frac{\omega}{\omega_1} - \{m - nl - p(1-l) - q(1+l)\}\Omega_p, \quad \varphi = \frac{\omega K^2 \kappa^2 \Omega_p^2}{6\gamma^2}$$

$$\vartheta' = \vartheta T, \quad \varphi' = \varphi T^3 \text{ and } \tau = t/T$$

Coefficient of tertiary term in line shape function  
Detuning parameter with unit interaction time

## Electron beam with Gaussian distribution

Asymmetric electron beam

$$S(\vartheta + \delta\vartheta, \varphi) = \int_0^1 \exp i \left\{ (\vartheta' + 4\pi m N \varepsilon_1) \tau - \frac{m^2 \pi^2 \mu^2 \tau^2}{2} + \varphi' \tau^3 \right\} d\tau$$

$$S'(\vartheta + \delta\vartheta, \varphi) = \int_0^1 \tau \exp i \left\{ (\vartheta' + 4\pi m N \varepsilon_1) \tau - \frac{m^2 \pi^2 \mu^2 \tau^2}{2} + \varphi' \tau^3 \right\} d\tau$$

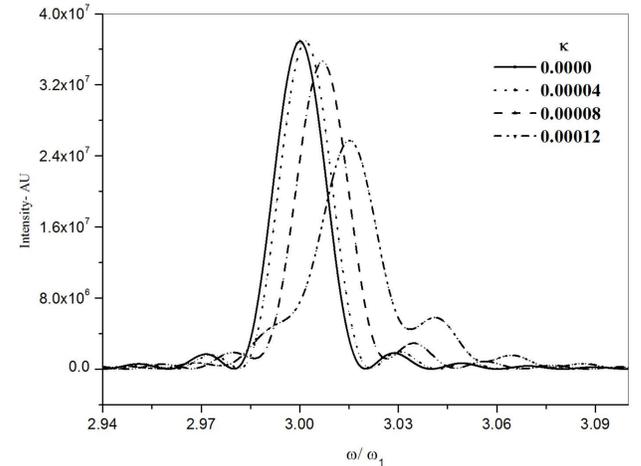
Symmetric electron beam-  $\varepsilon_1=0$

$$S(\vartheta + \delta\vartheta, \varphi) = \int_0^1 \exp i \left\{ (\vartheta') \tau - \frac{m^2 \pi^2 \mu^2 \tau^2}{2} + \varphi' \tau^3 \right\} d\tau$$

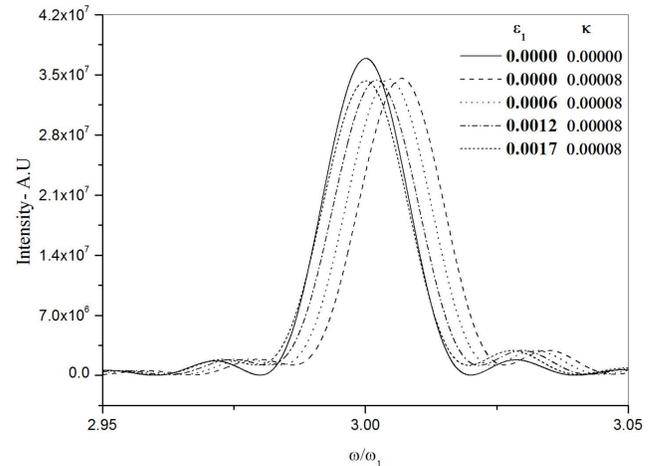
$$S'(\vartheta + \delta\vartheta, \varphi) = \int_0^1 \tau \exp i \left\{ (\vartheta') \tau - \frac{m^2 \pi^2 \mu^2 \tau^2}{2} + \varphi' \tau^3 \right\} d\tau$$

Parameters	Value/ Symbol	Parameters	Value/ Symbol
Undulator parameter	$K = \frac{b_0 e H_0}{\Omega_p m_0 c} = 1$	Number of periods	N=100
Electron beam relativistic parameter	$\gamma = 20$	Total time for transverse	$\frac{2N\pi}{\Omega_u}$
Undulator wavelength	$\lambda = 5 \text{ cm}$	Undulator frequency	$\Omega_p, \Omega_l$
Additional periodic harmonic field number	$l=3$	Unit Interaction Time	$\tau = t/T$
Energy spread parameter	$\mu = 4N\sigma = 0.01$	Emission frequency	$\omega$
Harmonic field parameter	$K_1 = 0 - 0.075$	Mean Energy	$\varepsilon_1 = 0 - 0.0017$
Magnitude of constant magnetic field	$\kappa = 0 - 0.00012$	rms energy spread	$\sigma$

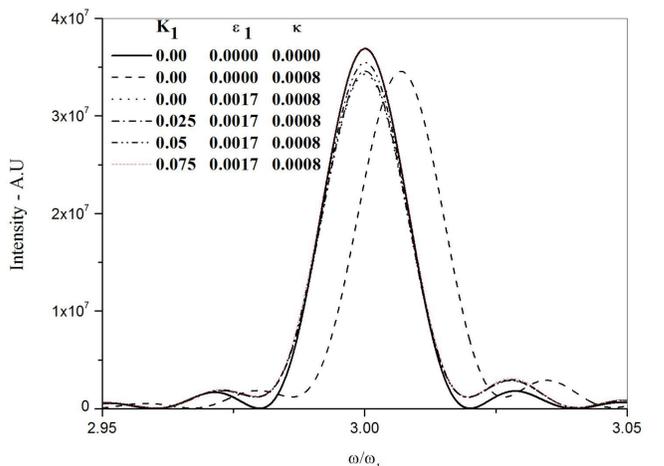
- The present analysis gives solution both for intensity enhancement along with resonance shift of radiation simultaneously.
- The harmonic field can be generated by the addition of shims in the planar undulator structure.
- The constant magnetic field inherently present due to earth's magnetic field or error in the design of the undulator modifies spectrum of which can be compensated by using asymmetric electron beam



Frequency Spectrum at third harmonic varying constant magnetic field parameter  $\kappa$



Frequency Spectrum at third harmonic varying  $\kappa$  and mean energy  $\varepsilon_1$



Frequency Spectrum at third harmonic varying  $\kappa$ ,  $\varepsilon_1$  and Harmonic field amplitude  $K_1$

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