

# INFLUENCE OF ENERGY CHIRP IN THE ELECTRON BEAM AND UNDULATOR TAPERING ON SPATIAL PROPERTIES OF THE RADIATION FROM SEEDED AND SASE FEL

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## Abstract

In this report we present analysis of the spatial properties of the radiation from an FEL amplifier for the case of energy chirp in the electron beam and undulator tapering. Two configurations, seeded FEL amplifier, and SASE FEL are under consideration. Studies are performed with numerical simulations using time-dependent FEL simulation code FAST. Evolution of the amplification process is traced along the undulator. It is shown that spatial properties of the radiation may be significantly distorted by the effect of energy chirp in the electron beam and undulator tapering.

## INTRODUCTION

FEL amplification process is sensitive to the detuning from the FEL resonance. Energy chirp in the electron beam and undulator tapering are responsible for the change of the resonance condition. Undulator tapering is widely used in practical devices. Well known examples are post-saturation undulator tapering for radiation power increase, reverse undulator tapering for effective operation of afterburners, and application of linear undulator tapering for compensation of energy chirp effect [1–3]. These are essentially one dimensional effects [3–5]. Detuning effects significantly influence on the spatial properties of the radiation from FEL amplifier [6]. It has been shown in [7] in the framework of 3D FEL theory that effects of energy chirp and undulator tapering will result in an increase of the angular divergence and decrease of the degree of transverse coherence of the radiation from SASE FEL.

In this paper we trace evolution of amplification process in an FEL amplifier from start-up to the deep nonlinear regime. The case of linear frequency detuning (along undulator or along electron bunch) is under study. We compare two configurations: seeded FEL amplifier, and SASE FEL. We found that for positive linear detuning there is optimum value of chirp (tapering rate) when maximum output radiation power is achieved. Increase of the detuning in the positive direction leads to significant increase of the angular divergence of the radiation. On the other hand, radiation power significantly drop when increasing strength of the detuning in the negative direction (the power of the effect is similar to that given by 1D approximation). Stronger negative tapering results in shrinking of the angular divergence of the radiation.

## BASIC DEFINITIONS

Detuning from the FEL resonance is defined as:

$$C = \frac{2\pi}{\lambda_w} - \frac{\pi(1 + K^2)}{\lambda\gamma^2},$$

where  $\lambda_w$  is undulator period,  $\lambda$  is radiation wavelength,  $K$  is rms value of the undulator parameter,  $\gamma$  is Lorentz factor. Figure of merit of the detuning effect on the FEL performance is detuning parameter  $\hat{C} = C/\Gamma$  with the gain parameter  $\Gamma$  given by [6]:

$$\Gamma = \left[ \frac{I}{I_A} \frac{16\pi^2 K^2 A_{JJ}^2}{(1 + K^2) \lambda_w^2 \gamma} \right]^{1/2}.$$

3D FEL parameter relates to the FEL gain parameter as  $\bar{\rho} = \lambda_w \Gamma / (4\pi)$ .

For the linear law of the undulator tapering the detuning parameter is

$$\hat{C}(\hat{z}) = \beta \hat{z}, \quad \beta = -\frac{\lambda_w}{4\pi \bar{\rho}^2} \frac{K_0}{1 + K_0^2} \frac{dK}{dz}.$$

Here longitudinal coordinate is normalized as  $\hat{z} = \Gamma z$ , and  $K_0$  refers to the initial value of the undulator parameter.

The chirp parameter  $\alpha$  is the figure of merit for an effect on FEL amplification process of the energy chirp along the electron bunch:

$$\alpha = -\frac{d\gamma}{dt} \frac{1}{\gamma \omega \bar{\rho}^2},$$

where  $\omega = 2\pi c/\lambda$ .

There is a symmetry between the linear energy chirp and the undulator tapering [3]. Indeed, if we look at the radiation field acting on some test electron from an electron behind it, this field was emitted at a retarded time. In the first case a radiating electron has a detuning due to an energy offset, in the second case it has the same detuning because undulator parameters were different at a retarded time. The effect of the energy chirp is compensated by the undulator tapering when

$$\frac{1}{H_{w0}} \frac{dH_w}{dz} = -\frac{1}{2} \frac{(1 + K_0^2)^2}{K_0^2} \frac{1}{\gamma_0^3} \frac{d\gamma}{cdt}.$$

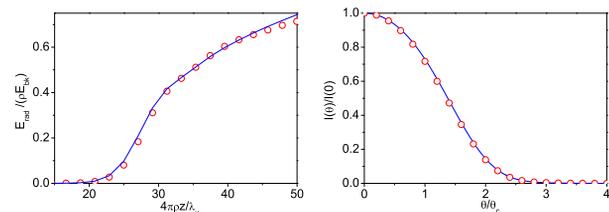


Figure 1: Equivalence of energy chirp and undulator tapering. Left: Average radiation pulse energy along undulator. Right: Average angular divergence in the far zone in the saturation. Solid curve:  $\alpha = 0.07$ ,  $\beta = 0$ . Circles:  $\alpha = 0$ ,  $\beta = 0.035$ .

## FEL PERFORMANCE

In the following we illustrate operation of the FEL amplifier with linear chirp (tapering) using numerical simulations with code FAST [8]. To be specific, we used baseline parameters of the electron beam at the European XFEL: charge 20 pC, rms pulse duration 1.2 fs, normalized rms emittance 0.32 mm-mrad, rms energy spread 4.1 MeV [9, 10]. Undulator period is 4 cm, and energy of electrons is 8.5 GeV, radiation wavelength is 0.62 nm. The value of diffraction parameter is equal to 5 for this parameter set. Output results are presented in normalized form, and can be scaled to a wider range of the physical parameters.

We start with demonstration of compensation of the energy chirp by the undulator tapering [3]. We see from Fig. 1 that compensation takes place for both values, energy in the radiation pulse and angular divergence of the radiation. Some small difference is connected with not sufficient statistics.

Taking into account equivalence of linear energy chirp and undulator tapering, we present results only for wide range of the energy chirp values covering the whole range of practical interest. We trace amplification process along the undulator up to deep nonlinear regime and compare characteristics of SASE FEL and seeded FEL. Different color codes on the plots correspond to different values of the energy chirp parameter  $\alpha$ . Color codes are: black for  $\alpha = 0$ , red for  $\alpha = 0.025$ , green for  $\alpha = 0.07$ , blue for  $\alpha = 0.15$ , turquoise for  $\alpha = 0.2$ , brown for  $\alpha = -0.025$ , violet for  $\alpha = -0.05$ . To help visual identification of the sign of the energy chirp, we use solid curve type for positive, and dashed curve type for negative values of  $\alpha$ . Plots on the left and the right hand side refer to SASE FEL and seeded FEL, respectively.

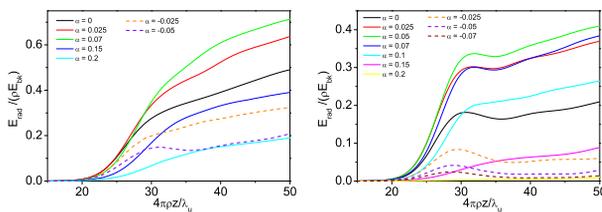


Figure 2: Energy in the radiation pulse along the undulator for different values of the energy chirp parameter  $\alpha$ . Left plot: SASE FEL. Right plot: seeded FEL.

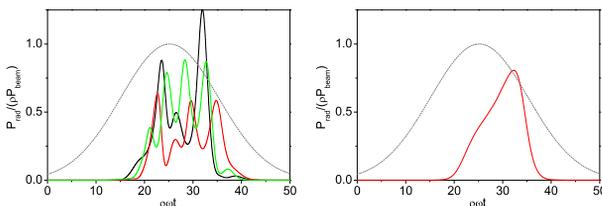


Figure 3: Temporal profile of the radiation pulse in the saturation regime.  $\alpha = 0.07$ . Left plot: three shots from SASE FEL. Right plot: seeded FEL. Grey line shows axial profile of the electron bunch.

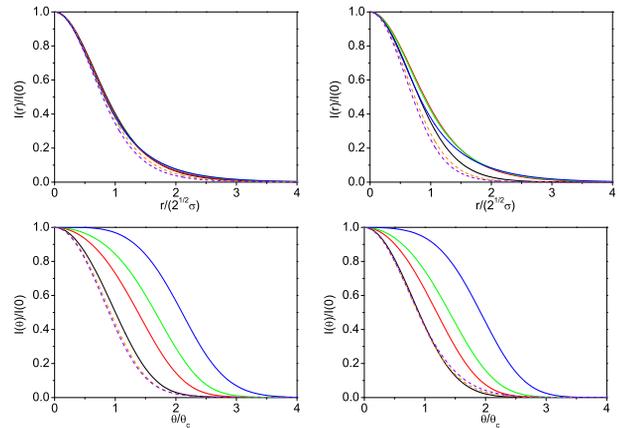


Figure 4: Average distribution of the radiation intensity in the near zone (top row) and in the far zone (bottom row) in the saturation for different values of the energy chirp parameter  $\alpha$ . Left column: SASE FEL. Right column: seeded FEL.

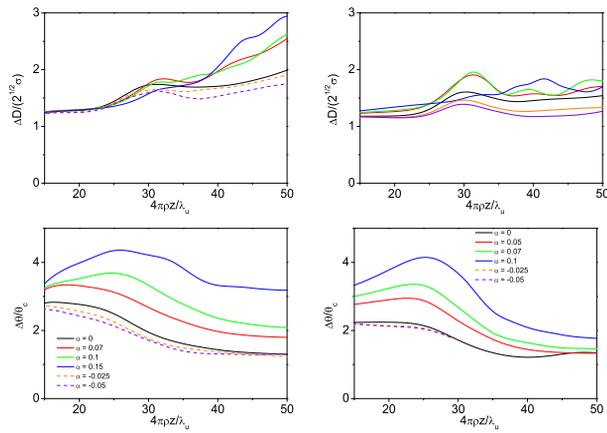


Figure 5: Evolution along the undulator of FWHM spot size of the radiation in the near zone (top row) and FWHM angular divergence of the radiation in the far zone (bottom row) different values of the energy chirp parameter  $\alpha$ . Left column: SASE FEL. Right column: seeded FEL.

Overview of the results shows that both, SASE and seeded FEL exhibit pretty similar performance. Figure 2 shows evolution of the FEL efficiency along the undulator. Longitudinal coordinate is normalized to the gain parameter as  $\tilde{z} = \Gamma z$ , and FEL efficiency is defined as  $\tilde{\eta} = E_{rad} / (\tilde{\rho} E_{bk})$ , where  $E_{bk} = N_e \gamma mc^2$  is kinetic energy of the electron bunch. We find that FEL efficiency rapidly decreases for negative values of the energy chirp. Situation is different for positive values of  $\alpha$ . First, efficiency grows with  $\alpha$ , reaches maximum value at  $\alpha \approx 0.07$ , and then starts to drop. Figure 3 shows temporal profile of the radiation pulse in the saturation regime for  $\alpha = 0.07$  corresponding to maximum FEL efficiency. Time is normalize as  $\tilde{\rho} \omega t$ .

Plots in Fig. 4 present intensity distributions of the radiation in the near and far zone in the saturation. We see that angular divergence of the radiation shrinks for negative values of  $\alpha$ , and gradually expands for positive values. Plots

in Fig. 5 trace relevant FWHM values along the undulator. Spot size in the near zone and angular divergence of the radiation in the far zone are always wider for the case of SASE FEL which is a signature of the degradation of transverse coherence for the frequency chirped SASE FEL [7]. However, by the time we can not put a number for the drop of the degree of transverse coherence, more comprehensive study is required.

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