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ANALYSIS OF UNDULATOR RADIATIONS WITH ASYMMETRIC BEAM AND NON-PERIODIC MAGNETIC FIELD

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Abstract

Harmonic Undulator radiations at third harmonics with non periodic constant magnetic field has been analysed. Symmetric and asymmetric electron beam with homogeneous spread has been used to present viable solution for the resonance shift inherited in undulator with constant magnetic field. The out coming radiation, recovers shifts in resonance and regain its intensity with asymmetric electron beam and harmonic field.

INTRODUCTION

Free electron lasers [FEL] generation is a cutting edge technology and has large numbers of research applications [1]. Tunablity and brilliance at lasing wave length in FEL are key parameters for number of applications. Lasing wavelength of FEL depends upon the value of undulator parameter, undulator wavelength and relativistic parameter of electron beam used. Recent works in FEL theory has emphasised the effect on non periodic magnetic field i.e. constant magnetic field component along or perpendicular or in both directions of the periodic magnetic field of planar undulator on the out coming undulator radiations. Quality of electron beam is also major concern in analysis of output undulator radiation. Partial compensation on the divergence of undulator radiation has been demonstrated by imposing weak constant magnetic component in the analytical form and all the major sources of homogeneous and inhomogeneous broadening have been accounted for the characteristics of the electrons beam by K. Zhukovsky [2]. The constant non-periodic magnetic constituents are studied to compensate the divergence of the electronic beam [3]. Dattoli *et al* has initially reported the effect on undulator radiation given by planar undulator with constant magnetic field component [4]. The later studies focus on higher harmonics generation by addition of additional harmonic field [5].

Harmonic planar undulator consists additional harmonic field along with sinusoidal planar magnetic field, uses modest electron beam energy and to increase the efficiency of FELs [6-9]. H jeevakhan et al have presented semi analytical results for the effect of perpendicular constant magnetic field on the gain of harmonic undulator at higher harmonics [10]. In the present paper we have analysed harmonic undulator with perpendicular non periodic magnetic field with asymmetric electron beam with energy spread and having shifted mean energy. The Harmonic field compensate the intensity loss and the asymmetric electron beam brings the undulator radiation back to resonance.

UNDULATOR FIELD

Planar undulator sinusoidal magnetic field encompass with a perpendicular constant magnetic field in present analysis and is given by

$$H = \left[H_0\kappa, b_0 H_0(sink_p + \Delta sink_l)z, 0\right] \qquad (1)$$

Where, $k_p = \frac{2\pi}{\lambda_p}$ and $k_l = \frac{2\pi}{\lambda_l}$ where k_p and k_l are undulator and harmonic undulator wave number respectively, λ_p is undulator wave length and $\lambda_l = l\lambda_p$, l is harmonic integer , H_0 is peak magnetic field, $\Delta = \frac{b_1}{b_0}$, b_0 and b_1 controls the amplitude of main undulator field and additional harmonic field and κ is the magnitude of constant non periodic magnetic field. The harmonic undulator can be designed by introducing shims in regular interval as per the design of harmonic undulator as shown in Fig. 1. The spacing between shims decides it, as 3rd or 5th harmonic undulator similarly as the harmonic number 'l' decides in Eq. (1).

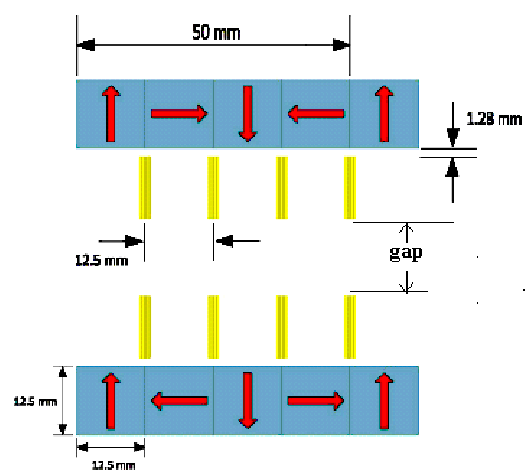

Figure 1: Schematic of one Period of harmonic Undulator*(l=3)*.

The velocity of electron traversing through undulator is derived by using Lorentz force equation:

$$\frac{dv}{dt} = -\frac{e}{\gamma mc}\left(\vec{v} \times \vec{H}\right) \qquad (2)$$

This gives

$$\beta_x = -\frac{K}{\gamma}\left[\cos(\Omega_p) + \Delta\frac{\cos(l\Omega_p)}{l}\right]t$$

$$\beta_y = -\frac{K}{\gamma}\kappa\Omega_p t \qquad (3)$$

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$$\beta_z = \beta^* - \frac{K^2}{2\gamma^2}\left[\left\{\frac{1}{2}\cos(2\Omega_p) + \frac{1}{2}\left(\frac{\Delta}{l}\right)^2\cos(2l\Omega_p) + \left(\frac{\Delta}{l}\right)\cos(1 \pm l)\Omega_p\right\}t + (\kappa\Omega_p t)^2\right] \quad (4)$$

Where m and m_0 are relativistic and rest mass of electron respectively and value of m is governed by the relativistic parameter γ , $K = \frac{b_0 e H_0}{\Omega_p m_0 c}$ is the undulator parameter and $\beta^* = 1 - \frac{1}{2\gamma^2}\left[1 + \frac{K^2+K_1^2}{2}\right]$ with $K_1 = \frac{\Delta K}{l}$ and $\Omega_p = k_p c$. The solution of Eq. 4 gives the electron trajectory along z direction,

$$\frac{z}{c} = \beta^* t - \frac{K^2}{8\gamma^2\Omega_p}\sin(2\Omega_p t) - \frac{K_1^2}{8\gamma^2 l\Omega_p}\sin(2l\Omega_p t) - \frac{KK_1}{2\gamma^2(1\pm l)\Omega_p}\sin(1 \pm l)\,\Omega_p t - \frac{K^2\kappa^2{\Omega_p}^2 t^3}{6\gamma^2} \quad (5)$$

The spectral properties of radiation can be evaluated from Lienard - Wiechart integral [11],

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\{\hat{n}\times(\hat{n}\times\hat{\beta}\}exp\left[i\omega(t-\frac{z}{c})\right]dt\right| \quad (6)$$

when integrated over undulator length, $T = \frac{2N\pi}{\Omega_u}$ and ω is the emission frequency with variables as

$\xi_1 = -\frac{\omega K^2}{8\gamma^2\Omega_p}$, $\xi_2 = -\frac{\omega K_1^2}{8\gamma^2 l\Omega_p}$ and $\xi_{3,4} = -\frac{\omega K K_1 \kappa}{2\gamma^2(1\pm l)\Omega_p}$

The brightness expression read as

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left(\frac{K}{\gamma}\right)^2\left[\left|\hat{\imath}\int_0^T dt\left\{cos(\Omega_p t) + \frac{\Delta}{l}cos(l\Omega_p t)\right\}expi(\vartheta t + \varphi t^3)J_m(0,\xi_1)J_n(0,\xi_2)J_p(\xi_3)J_q(\xi_4)\right|^2 + \left|\hat{\jmath}\int_0^T dt\{\kappa\Omega_u t\}expi(\vartheta t + \varphi t^3)J_m(0,\xi_1)J_n(0,\xi_2)J_p(\xi_3)J_q(\xi_4)\right|^2\right] \quad (7)$$

Where,

$$\vartheta = \frac{\omega}{\omega_1} - \{m - nl - p(1-l) - q(1+l)\}\Omega_p$$

$$\varphi = \frac{\omega K^2\kappa^2{\Omega_p}^2}{6\gamma^2}$$

And Eq. (7) can be further reduced to

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2T^2}{4\pi^2 c}\left\{|I_x|^2\,S(\vartheta,\varphi) + |I_y|^2 S'(\vartheta,\varphi)\right\} \quad (8)$$

With

$$I_x = \frac{K}{2\gamma}\Big[\{J_{m+1}(0,\xi_1) + J_{m-1}(0,\xi_1)\}J_n(0,\xi_2)J_p(\xi_3)J_q(\xi_4) + \frac{\Delta}{h}\{J_{n+1}(0,\xi_2) + J_{n-1}(0,\xi_2)\}J_m(0,\xi_1)J_p(\xi_3)J_q(\xi_4)\Big]$$

$$I_y = \frac{2i\pi K\kappa N}{\gamma}$$

$$S(\vartheta,\varphi) = \left|\int_0^1 e^{(\vartheta'\tau+\varphi'\tau^3)}d\tau\right|^2$$

$$S'(\vartheta,\varphi) = \frac{\partial S(\vartheta,\varphi)}{\partial\vartheta} = \left|\int_0^1 \tau e^{(\vartheta'\tau+\varphi'\tau^3)}d\tau\right|^2$$

$\vartheta' = \vartheta T, \varphi' = \varphi T^3$ and $\tau = {t}/{T}$ is unit interaction time.

EFFECT OF ENERGY SPREAD

To introduce energy spread in the electron beam ,the line shape functions $S(\vartheta,\varphi)$ and $S'(\vartheta,\varphi)$ in Eq. 8 will be modified as

$$S(\vartheta+\delta\vartheta,\varphi) = \int_{-\infty}^{\infty} S(\vartheta,\varphi)\;f(\varepsilon)\,d\varepsilon \quad (9)$$

$$S'(\vartheta+\delta\vartheta,\varphi) = \int_{-\infty}^{\infty} S'(\vartheta,\varphi)\;f(\varepsilon)\,d\varepsilon \quad (10)$$

Where, $f(\varepsilon)$ represents Gaussian type energy distribution [12] . Eq. (8) can be solved to get modified line shape function for the values as $m = 1,3,5\ldots, with\ \mu = 4N\sigma$

For Non symmetric electron beam

$$S(\vartheta+\delta\vartheta,\varphi) = \int_0^1 \exp i\left\{\left(\vartheta' + 4\pi m\,N\,\varepsilon_1\right)\tau - \frac{m^2\pi^2\mu^2\tau^2}{2} + \varphi'\tau^3\right\}\;d\tau \quad (11)$$

$$S'(\vartheta+\delta\vartheta,\varphi) = \int_0^1 \tau\,exp\,i\left\{(\vartheta' + 4\pi mN\varepsilon_1)\tau - \frac{m^2\pi^2\mu^2\tau^2}{2} + \varphi'\tau^3\right\}\;d\tau \quad (12)$$

ε_1 is mean energy spread, σ is the r.m.s relative energy spread

For symmetric electron bean i.e. ε_1=0, than Eq. (11) and (12) can be re written as

$$S(\vartheta+\delta\vartheta,\varphi) = \int_0^1 \exp i\left\{(\vartheta')\tau - \frac{m^2\pi^2\mu^2\tau^2}{2} + \varphi'\tau^3\right\}\;d\tau \quad (13)$$

$$S'(\vartheta+\delta\vartheta,\varphi) = \int_0^1 \tau\,exp\,i\left\{(\vartheta')\tau - \frac{m^2\pi^2\mu^2\tau^2}{2} + \varphi'\tau^3\right\}\;d\tau \quad (14)$$

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RESULTS AND DISCUSSION

Equation (8) reads the intensity of spontaneous radiations extracting from harmonic planar undulator with non periodic magnetic field specified by the value of κ. The line shape functions $S(\vartheta,\varphi), S'(\vartheta,\varphi)$ in Eq. (8) from asymmetric electron beam is given by Eqs. (11) and (12) and for symmetric electron beam is given by Eqs. (13) and (14). In earlier reported works [4,5,10] the second term in Eq. (8) consisting I_y has been neglected due to diminishing value of κ. In our analysis we have included the second term in numerical integration and its effect on the line shape function . The parameters used for simulation are listed in Table 1.

Table 1: Simulation Parameters

S. No	Parameters	Value
1.	Undulator parameter	K=1
2.	Electron bean relativistic parameter	$\gamma = 100$
3.	Undulator wavelength	λ_u=5 cm
4.	Addition periodic harmonic field number	$l = 3$
5.	Energy spread parameter	$\mu = 0.01$
6.	Harmonic field parameter	K_1=0

Figure 2 illustrates the intensity distribution of radiations by symmetric electron beam at third harmonic in arbitrary units with selection of parameters given in Table 1 and different values of κ. The line shape functions $S(\vartheta,\varphi)$ *and* $S'(\vartheta,\varphi)$ are read from Eqs. (13) and (14) respectively. There is a shift in resonance and reduction in intensity with effect of non periodic constant field contribution. At κ= 0.00012 there is distortion of Gaussian line shape function. In Fig. 2 the shift in resonance at third harmonic in terms of normalised frequency (ω/ω_1) is 0.0069 for value of κ =0.00008. The shift in resonance at third harmonic is more as compare to previous reported works [5,10]. For same simulation parameters it was shown that for the same value of κ =0.00008 the resonance shift is 0.0045 and the intensity reduction is nearly 12 % whereas in present case the intensity reduction is 6.2 %, as the second term in Eq. (8) is included in analysis along with energy spread.

In Fig. 3 we have presented a solution to bring back the radiation at resonance. The electron beam has been given a shift in the mean energy position and an asymmetric electron beam is used for analysis. All the parameters are kept same as used in Fig. 1 and for constant magnetic field κ =0.00008 the mean energy spread ε_1 has been varied. As the mean energy parameter changes the peak of intensity of radiation shifted in opposite direction. The radiation regains its original position for $\varepsilon_1 = 0.0017$ with slight intensity degradation.

In Fig. 3, it has been also illustrated that shift in resonance has been compensated with asymmetric electron beam with non zero value of $\varepsilon_1 = 0.0017$. The intensity degradation mainly occurs due to energy spread in the electron beam, can be accommodated by additional harmonic field.

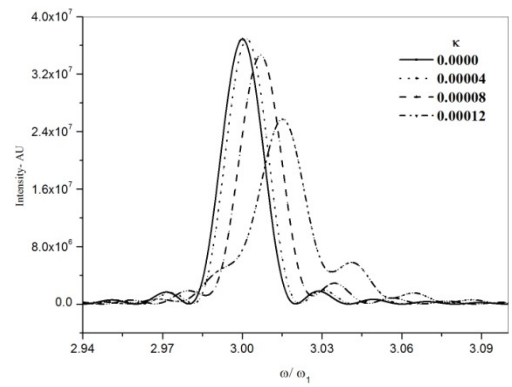

Figure 2: Frequency Spectrum at third harmonic with parameter given in Table 1 varying constant magnetic field parameter κ from 0 to 12 x 10^{-5}.

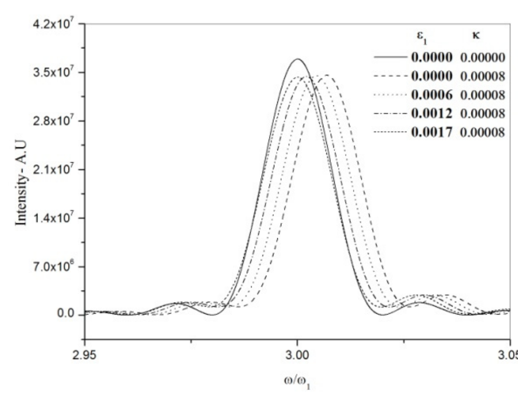

Figure 3: Frequency Spectrum at third harmonics with parameters same given in Table 1, κ = 8 x 10^{-5} and varying ε_1 from 0 to 17 x 10^{-4}.

The harmonic field enhances intensity and compensate the loss by energy spread parameter μ and due to constant magnetic field. The effect of energy spread with variation of energy spread parameter μ =0.01 on the intensity at third harmonics with constant magnetic field as κ = 0.00008 and variation harmonic field parameter as K_1 = 0.0 to 0.075 is shown in Fig. 3 and keeping all the remaining parameter as given in Table 1.

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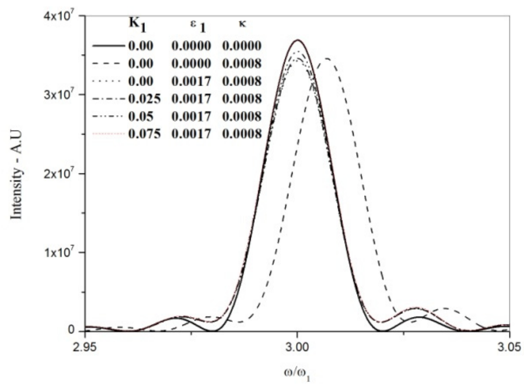

Figure 4: Frequency spectrum at third harmonics with varying harmonic Field amplitude as K1=0 and 0.075, κ =0.00008, $\varepsilon_1 = 0.0017$ and rest parameters same as in Fig. 1

As a particular case intensity reduction by μ=0.01 and κ = 0.00008 can be compensated by additional harmonic field K_1 = 0.075 and shift in resonance is compensated by the selection of $\varepsilon_1 = 0.0017$ as manifested in Fig. 4.

In conclusion, there is as an intensity enhancement at third harmonics due to additional harmonic field where as shift in resonance remains unaltered in previous reported results. The present analysis gives solution both for intensity enhancement along with resonance shift of radiation simultaneously. The harmonic field can be generated by the addition of shims in the planar undulator structure. The constant magnetic field inherently present due to earth's magnetic field or error in the design of the undulator modifies spectrum of which can be compensated by using asymmetric electron beam.

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