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XCOS SCILAB MODEL FOR SIMULATION OF INTENSITY AND GAIN OF PLANAR UNDULATOR RADIATION

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Abstract

Scilab Xcos based model has been designed to simulate the intensity and gain of planar undulator radiation. Numerical approach has been used to determine the trajectories of an electron along 'x' and 'z' directions, traversing through a planar undulator. The present paper describes the technical details of the different blocks, parameters and possibility of combined model used for trajectories and intensity. Simulation results are compared with the previous conventional syntax based codes.

INTRODUCTION

Free electron laser (FEL), attosecond pulses can be used in attosecond interferometry to explain the nature of materials [1-2], design of Q-bits [3] and in other cutting age multidisciplinary researches. FEL is coherent light source generated due to passage of relativistic electron beam through a periodic magnetic structure in presence of resonating electromagnetic radiation. Trajectory of electron beam in undulator, Intensity of out coming radiation and the transfer of energy from electron beam to radiation i.e. overall gain in FEL system are the parameters of interest in FEL technology and applications. Simulation software's are used to calculate the intensity and gain of real FEL system before actual experimentation.

Scilab is an open source software with numerous applications in engineering and research[4]. Scilab Xcos based simulation models for analytical solutions of electron trajectory equations have been presented with some limitations by H Jeevakhan *et al*. [5]. An improved analytical Xcos Model has been used to find the trajectory of the electron moving in the Harmonic undulator and variation of magnetic field along the axis of undulator [6]. This paper has also presented a model to find trajectory by second integral of the undulator magnetic field profile given by real measurements.

In the present paper, Scilab Xcos model has been presented based on numerical approach for trajectory of electron along undulator along 'x' and 'z' direction. The intensity of the out coming spontaneous radiation can also be simultaneously calculated by numerical method, by using this same model. The gain can further be simulated by employing Madey's Theorem for a given FEL system. The results are also compared with the previous program results based on Fortran.

PLANAR UNDULATOR FIELD

For present simulation, planar undulator magnetic field is considered and is given by [7-8]:

$$B = [0, a_0 B_0 (sin k_u z), 0] \tag{1}$$

Where, $k_u = \frac{2\pi}{\lambda_u}$ and where k_u is undulator wave number, λ_u is undulator wave length, B_0 is peak magnetic field.

The equation of motion of relativistic free electron moving in undulator field is given by

$$ma = -\frac{e}{c}\left[\vec{v} \times \vec{B}\right] \tag{2}$$

For numerical calculation, the acceleration along 'x' and 'z' direction can be written as

$$m\ddot{x} = \frac{K}{\gamma}\Omega_u v_z\, sin(\Omega_u \beta_*)t \tag{3}$$

$$m\ddot{z} = \frac{K}{\gamma}\Omega_u v_x\, sin(\Omega_u \beta_*)t \tag{4}$$

The component of β along x' and 'z' direction can be evaluated analytically as,

$$\beta_x = -\frac{K}{\gamma}[\cos(\Omega_u)t] \tag{5}$$

$$\beta_z = \beta_* - \frac{K^2}{4\gamma^2}[\cos(2\Omega_u)t] \tag{6}$$

corresponding initial values read as,

$$\beta_{x=0} = -\frac{K}{\gamma} \tag{7}$$

$$\beta_{z=0} = -\frac{K^2}{4\gamma^2} \tag{8}$$

Where $\beta_* = 1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$, γ is relativistic parameter, $K = \frac{a_0 e B_0}{\Omega_u m_0 c}$ is the undulator parameter, m_0 is rest mass of electron, and $\Omega_u = k_u c$

The Intensity of radiation can be evaluated from Lienard - Wiechart integral [9],

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\{\hat{n} \times (\hat{n} \times \hat{\beta}\} exp\left[i\omega(t - \frac{z}{c})\right] dt\right|^2 \tag{9}$$

For Numerical simulation of Intensity along the axis the Eq. (9) can be re written as

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_0^T \{\widehat{\beta_x}\} exp\left[i\omega(t - \frac{z}{c})\right] dt\right|^2 \tag{10}$$

SIMULATION MODEL AND RESULTS

The model design based on numerical analysis is given in Fig. 1. It is based on execution of equations. Single execution of this model in Xcos window gives the numerical simulation of trajectory along 'x' and 'z' direction for a particular value of 'ω'. Intensity and gain can be calculated by running the model on console for various values of 'ω'. The present model comprises of three sections.

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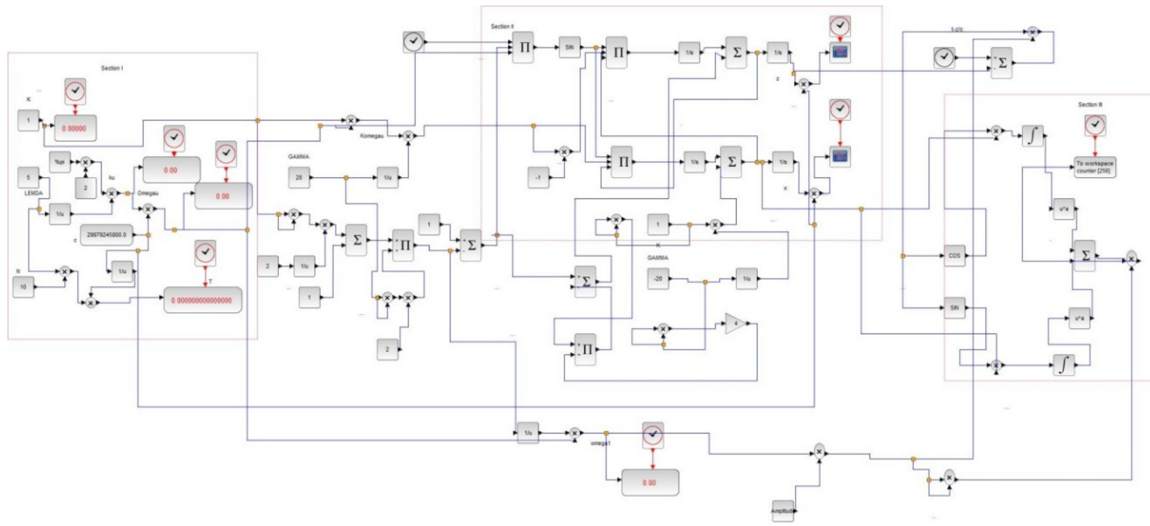

Figure 1: Simulation model for Trajectory and Intensity at particular value of 'ω' (Numerical Method).

Section I

This section consists of the desired primary parameters, natural constants and secondary parameters used in simulation and tabulated in Table 1. The primary parameters such as undulator wavelength undulator parameter, number of undulator period and relativistic factors are fixed to tune the wave length of out coming radiation.

Table 1:Parameters Used for Simulation

Primary Parameter	Symbol	Values
Undulator parameter	K	1
Number of Undulator periods	N	10
Undulator wavelength	λ	5cm
Relativistic parameter	γ	20

The values of natural constants are taken in CGS units. The magnitude of the secondary parameters (Table 2) depends on the values of the primary parameters and utilized in further simulation process. The Total time for traversing of electron 'T' in undulator will decides the final integration time and steps of integration.

Table 2: Secondary Parameters for Simulation

Secondary Parameters	Symbol	Values
Total time for traverse of electron in undulator	T	1.66782×10^{-9} s
Undulator wave number	k_u	1.26 cm^{-1}
Undulator frequency	Ω_u	3.76×10^{10} s^{-1}
Radiation frequency	ω	2.009×10^{13} rad/s

Section II

In section II, Eqs. (3) and (4) is executed and represnts numerical model for velocity and trajectoery along 'x' and 'z' direction. As in numerical computation, the output of one equation has to be feeded to the input of other for further calcutations. Equations (3) and (6) & Eqs. (4) and (5) are interdependent. In conventional programing loop is used for such calulations. In present case, it is done by giving feed back of the output of one block to the input of other and vice versa, as shown in Fig. 1.

The integration block is used for integration of Eqs. (3) and (4) & Eqs. (7) and (8) are used for initial values of Eqs. (5) and (6) respectively. The results of trajectory calulated along 'x' and 'z' directions is presented in the Fig. 2. This figure also displays the result obtained from the Fortran code based on Runge-kutta method.

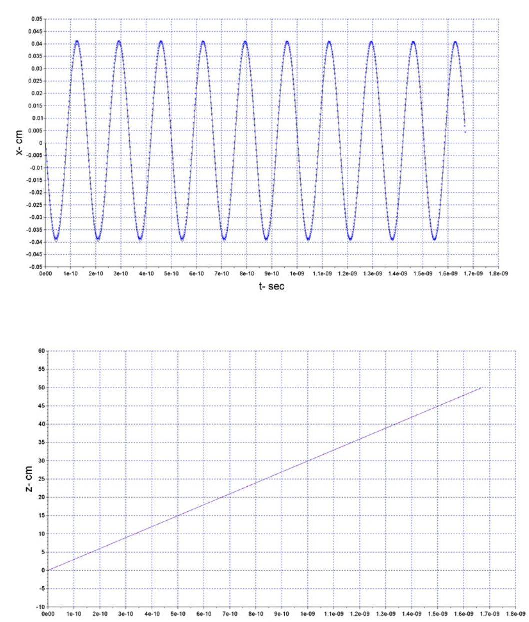

Figure 2: Trajectory of electron along ‘x’ and 'z in Planar undulator (Numerical calculation).

Section III

This section, finally executes Eq. (10) and represents the Numerical model for on axis radiation intensity calculation. The value 'z' and the $'\beta_x'$ is required for the calculation of intensity at particular value of "ω". The solver kind used for simulation is RK45- Rungekutta 4(5).

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The complete model has to be simulated for different values of 'ω'. To calculate the intensity for different values of 'ω' seperate code written on console.

The model desinged on Xcos window has been called and executed for different values of 'ω' as per requirement, by using the 'set context' feature available in simulation tab in menu bar of Xcos window. The FEL gain is simulated by applying the derivative command on the intensity data results from present model, as per Madey's Theorom. The intensity and gain curve for present simulation is displayed in Figs. 3 and 4 respectively.The Intensity graph also displays the result obtained from the Fortran code for Eq. (10).

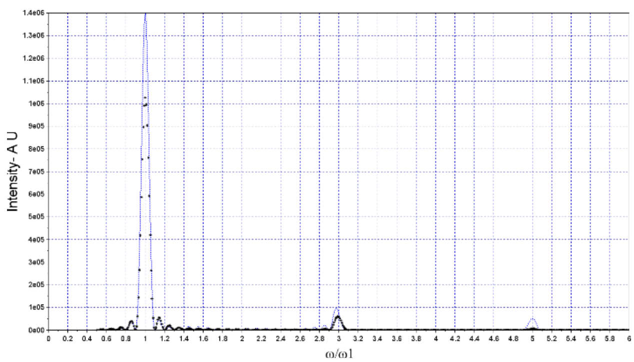

Figure 3: Simulation results for Intensity for various value of 'ω' given by Model in Fig. 1.

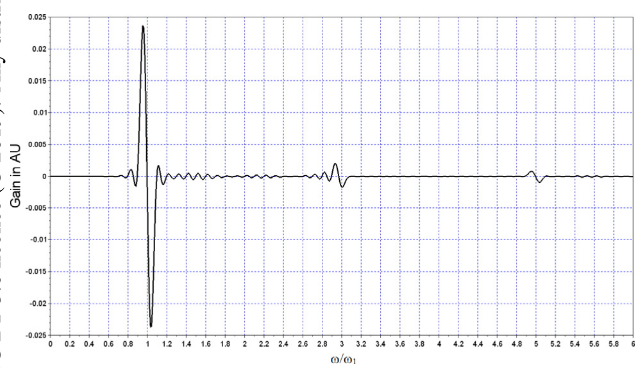

Figure 4: Simulation results for Gain for various value of 'ω'.

CONCLUSION

In the present paper Scilab Xcos based model has been designed and utilised for simulation of electron trajectories, Intensity and gain by applying algorithm of numerical method. In present simulation, Xcos and Console window has been simultaneously used for intensity calculations. The Xcos model does not require a separate plot command for trajectory plot, however intensity and gain is plotted from separate command in console window. The present work can be extended as a numerical model for trajectory, intensity and gain for FEL system having constant magnetic field component. The GUI model for present simulation is also a future scope of work.

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