ISBN: 978-3-95450-210-3 doi:10.18429/JACoW-FEL2019-TUP037

OPTIMIZATION OF THE TRANSVERSE GRADIENT UNDULATOR (TGU) FOR APPLICATION IN A STORAGE RING BASED XFELO*

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Abstract

The stringent energy spread requirement of the XFELO poses a challenge for its application in storage rings. One way to overcome this is by using a transverse gradient undulator (TGU) [1]. The TGU gain formula was discussed previously [2,3]. In this paper, we begin by reviewing the analytical 3D gain formula derived from the gain convolution formula. Following that, we apply numerical optimization to investigate the optimal beam and field parameters for maximal TGU gain. We found that a small emittance ratio (i.e. "flat beam" configuration) has a strong positive impact on TGU gain, as well as other patterns in the optimal parameters.

THEORY

An in-depth exploration of TGU physics can be found in [2]. Here, we provide an essential summary. A TGU scheme has two key ingredients. First, we introduce dispersion upstream of the TGU that correlates electron position and energy, i.e.

$$y = y_0 + D\eta, \tag{1}$$

where D is the dispersion strength and y_0, η are the initial vertical electron position and relative energy deviation respectively. (We assume here that the TGU acts in the y-direction.) Secondly, the TGU introduces a linear dependence in K on transverse displacement, i.e. $K(y) \approx (1 + \alpha y)K_0$, where K_0 is the on-resonance undulator parameter and α is the TGU magnetic gradient. Then, we can cancel out the energy spread by requiring

$$\alpha D = (2 + K_0^2)/K_0^2 \tag{2}$$

The TGU is only effective if $D\sigma_\eta \gg \sigma_y$, i.e. if beam size is primarily dominated by dispersion.

The 3D gain formula in this scenario is derived to be

$$G = \frac{G_0}{4\pi}\int_{-1/2}^{1/2} dz\, ds\, \frac{i(z-s)}{\sqrt{\mathfrak{D}_x \mathfrak{D}_y}} \exp\Bigg[-2ix(z-s) - \frac{2\tilde{\sigma}_\eta^2 (z-s)^2}{1+\Gamma^2} - \left(\frac{\Gamma}{1+\Gamma^2}\frac{\tilde{\sigma}_\eta}{\tilde{\beta}_y}\right)^2 \frac{(z^2-s^2)^2}{2}\frac{\mathfrak{d}_y}{\mathfrak{D}_y}\Bigg], \tag{3}$$

where

$$\Gamma = \frac{D\sigma_\eta}{\sigma_y}, \quad x = \pi N_u \Delta\nu, \tag{4}$$

$$\tilde{\sigma}_\eta = 2\pi N_u \sigma_\eta, \quad \tilde{\beta}_y = \beta_y / L_u, \tag{5}$$

$$\frac{G_0}{4\pi} = 4\pi\gamma_r \frac{I}{I_A}\frac{K_0^2[JJ]^2}{(1+K_0^2/2)^2} N_u^3 \lambda_1^2, \tag{6}$$

and $\sigma_{x,y}$ are the RMS electron beam sizes, L_u, N_u are the undulator length and number of undulator periods respectively, $\Delta\nu \equiv (1 - \lambda/\lambda_1)$ is the detuning factor based on resonant FEL wavelength λ_1, σ_η is the relative energy spread, $\beta_{x,y}$ is the respective betatron function, γ_r is the resonant Lorentz factor, I is the beam current, $I_A \approx 17\,\text{kA}$ is the Alfvén current, $[JJ] \equiv J_0[K_0^2/(4+2K_0^2)] - J_1[K_0^2/(4+2K_0^2)]$ is the Bessel factor, and

$$\mathfrak{D}_{x,y} = \Sigma_{x,y}^2 + sz\, L_u^2 \Sigma_{\phi x,y}^2 - iL_u(z-s)\left[\frac{1}{4k} + k\Sigma_{\phi x,y}^2 \Sigma_{x,y}^2\right], \tag{7}$$

$$\mathfrak{d}_y = \Sigma_y^2 + sz\, L_u^2 \sigma_{\phi y}^2 - iL_u(z-s)\left[\frac{1}{4k} + k\sigma_{\phi y}^2 \Sigma_y^2\right], \tag{8}$$

$$\Sigma_y^2 = \sigma_y^2 + \sigma_{ry}^2 + D^2\sigma_\eta^2, \tag{9}$$

$$\Sigma_x^2 = \sigma_x^2 + \sigma_{rx}^2, \tag{10}$$

$$\Sigma_{\phi x,y}^2 = \sigma_{px,y}^2 + \sigma_{\phi x,y}^2. \tag{11}$$

Here, $\sigma_{px,py}$ are the RMS electron beam divergences, $\sigma_{rx,ry}$ are the RMS radiation beam sizes, and $\sigma_{\phi x,\phi y}$ are the RMS radiation beam divergences.

In the context of a storage ring, there is a further constraint on the transverse emittances, namely $\epsilon_x + \epsilon_y = \epsilon_{x,0}$, with $\epsilon_{x,0}$ being the natural electron beam emittance [4]. We define the coupling constant k_c such that

$$\epsilon_x = \frac{\epsilon_{x,0}}{1+k_c}, \quad \epsilon_y = \frac{k_c \epsilon_{x,0}}{1+k_c}. \tag{12}$$

Modern storage rings are capable of operating in both round ($k_c = 1$) and flat ($k_c \ll 1$) beam configurations. Therefore, the coupling constant will be an important parameter in our investigation.

NUMERICAL OPTIMIZATION

We wish to optimize gain, as given by Eq. 3, with respect to electron and radiation beam parameters.In each optimization run, we fixed $\{\epsilon_{x,0}, \sigma_\eta\}$ and scan over k_c ranging from 0.001 to 1. At each value of k_c, the optimal parameters

* This work is supported by U.S. Dept. of Energy, Office of Science, Office of Basic Energy Sciences, under Contract No. DE-AC02-06CH11357, and U.S. National Science Foundation under Award No. PHY-1549132, the Center for Bright Beams.
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Table 1: Hypothetical Storage Ring Parameters (derived from PETRA-IV) [5]

Name	Symbol	Value
Electron beam		
Rel. gamma	γ_r	1.166×10^4
Beam current	I	40 A
Output radiation		
Res. energy	$\hbar\omega_1$	14.4 keV
Rad. emittance	ϵ_r	6.85 pm
Undulator		
Undulator period	λ_u	1.5 cm
Number of periods	N_u	2000
Undulator parameter	K_0	1.06

were found via the hill climber algorithm [6]. There are six "core" parameters which form our search space: the beta functions β_x, β_y which determine electron beam size, the Rayleigh ranges Z_{Rx}, Z_{Ry} which determinine radiation beam size, the dimensionless TGU factor Γ characterizing the strength of the TGU, and finally the dimensionless detuning $x = \pi N_u \Delta\nu$.

From the six core parameters we also investigate several "derived" parameters: radiation beam aspect ratio $\alpha_{\text{rad}} \equiv |E_y|^2/|E_x|^2$, dispersion D, and TGU magnetic gradient α, all of which may be practical values of interest. All other machine parameters are typical of a 4th generation light sources such as PETRA-IV or APS-U. Refer to Table 1.

Simulation Considerations

We explored a number of different optimization algorithms, including gradient descent, simulated annealing and simple hill climber. Ultimately, we chose the hill climber algorithm due to its simplicity and the (empirically observed) convex nature of the objective function. Provided reasonable starting parameters, a simple hill climber algorithm converged relatively quickly and reliably.

We also imposed the constraint $Z_{Rx} = \beta_x$ to simplify the search space. While they are in principle independent parameters, we found in practice that their optimal values are often equal. This result is unsurprising since we expect the largest gain when the radiation mode shape overlaps that of the electron beam [3].

RESULTS

Varying Natural Emittance

Figure 1 (top) shows the optimal gain with respect to k_c for fixed $\sigma_\eta = 0.15\%$ and different values of $\epsilon_{x,0}$. There are two key observations. First, with smaller k_c (i.e. flatter electron beam), gain increases dramatically, by up to an order of magnitude. Secondly, gain also increases with decreasing $\epsilon_{x,0}$ (perhaps unsurprisingly). In this case, the highest gain was obtained when $\epsilon_{x,0} = \epsilon_r = 6.85$ pm. Figure 2 shows the optimal beta functions. With smaller k_c, the optimal

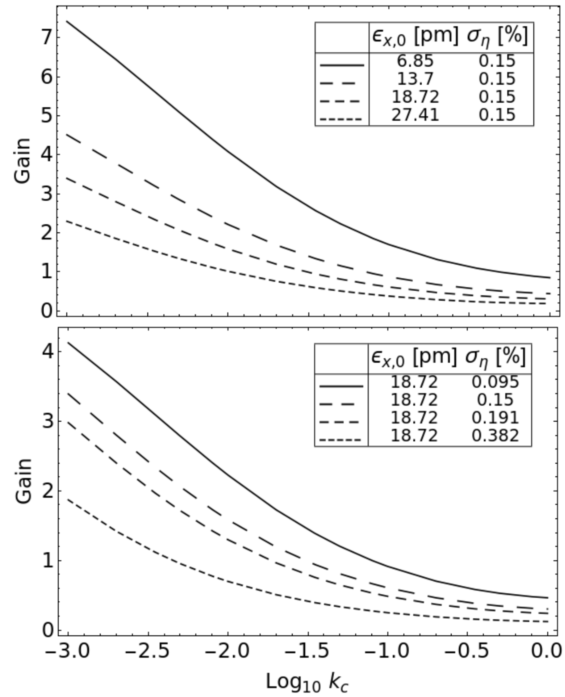

Figure 1: Optimal gain vs k_c for varying $\epsilon_{x,0}$ (top) and σ_η (bottom).

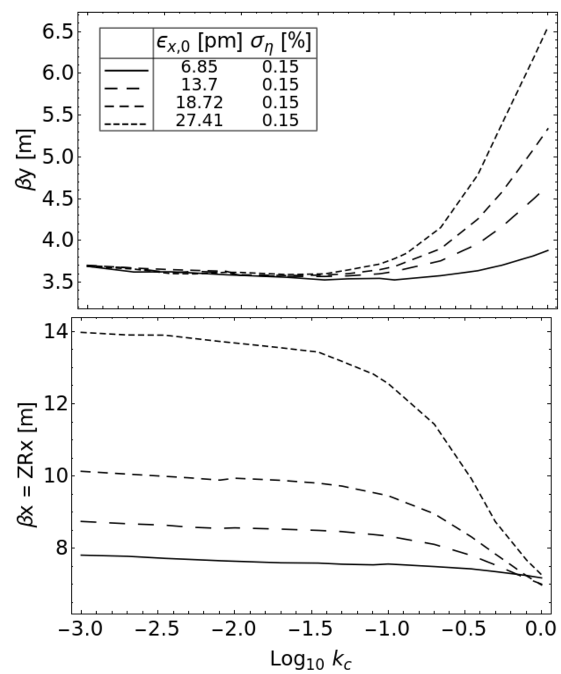

Figure 2: Optimal β_y (top) and β_x (bottom) vs k_c for varying $\epsilon_{x,0}$.

β_y decreases and so does its sensitivity to $\epsilon_{x,0}$. In fact, for $\log_{10} k_c \lesssim -1.5$, the optimal β_y is apparently independent of $\epsilon_{x,0}$. The opposite behavior is observed in β_x. We attribute this behavior to the constraint $\epsilon_x + \epsilon_y = \epsilon_{x,0}$, whence decreasing k_c decreases ϵ_y and increases ϵ_x simultaneously. Thus changing $\epsilon_{x,0}$ affects a particular direction less if its

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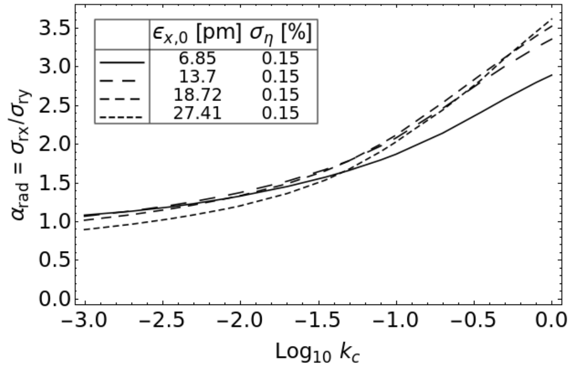

Figure 3: Optimal beam aspect ratio α_{rad} vs k_c for varying $\epsilon_{x,0}$.

"share" of the natural emittance is smaller. In addition, if we decrease $\epsilon_{x,0}$ such that it is comparable to ϵ_r, the optimal $\beta_{x,y}$ becomes relatively independent of k_c.

Figure 3 shows the optimal radiation beam aspect ratio. Interestingly, with smaller k_c the optimal aspect ratio tends to 1. In other words, the optimal radiation mode shape is approximately round.

Varying Energy Spread

Figure 1 (bottom) shows the optimal gain for different σ_η. In addition to observations made in the previous subsection, we also see that smaller σ_η leads to higher gain, as expected.

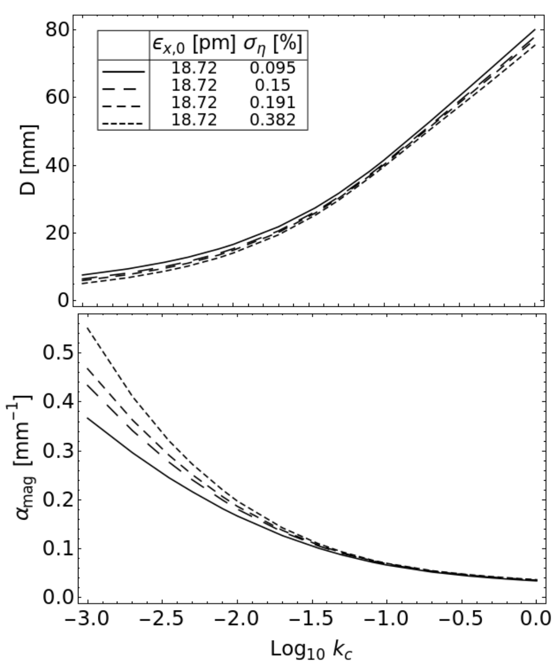

Figure 4: Optimal disperson (top) and magnetic gradient (bottom) vs k_c for varying σ_η.

Interestingly, most optimal parameters are relatively independent of σ_η. These include the beta functions, beam dispersion D and TGU magnetic gradient α (Figure 4).

CONCLUSION

The results clearly indicate that a flat-beam configuration (i.e. $k_c \ll 1$) greatly enhances gain in a TGU-enabled storage ring. This is easily satisfied in a storage ring, where the vertical emittance contribution primarily comes from magnet misalignments or coupling [4]. Typical values of k_c in modern storage rings can approach $\sim 10^2$ or smaller. From Figure 1, we see that large gain (often greater than 1) can be achieved in this regime, even with generously large $\epsilon_{x,0}$ and σ_η.

There are also several empirical observations to be made regarding optimal beam and machine parameters. First, for small k_c, the optimal β_y becomes independent of $\epsilon_{x,0}$ and σ_η (Figure 2). Second, for small k_c, the output radiation tends towards an aspect ratio of unity (Figure 3). This could prove useful in applications where a round radiation mode is desired. It is unclear at the moment whether this behavior is truly asymptotic, and how it could be explained from an analytical standpoint. Finally, we observed that σ_η does not significantly impact any of the optimal parameters within the range tested. At first glance, this is surprising given that σ_η shows up prominently in the numerators of the latter two terms in the exponent of Equation 3. However, it is also "hidden" in the definitions of Γ and $\mathfrak{D}_y$, belying its complicated role in the gain equation.

ACKNOWLEDGEMENTS

This work is supported by U.S. Dept. of Energy, Office of Science, Office of Basic Energy Sciences, under Contract No. DE-AC02-06CH11357, and U.S. National Science Foundation under Award No. PHY-1549132, the Center for Bright Beams.

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