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A WAVEGUIDE-BASED HIGH EFFICIENCY SUPERRADIANT FEL OPERATING IN THE THz REGIME *

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Abstract

In this paper we describe a novel self-consistent 3D simulation approach for a waveguide FEL operating in the zero-slippage regime to generate high power THz radiation. In this interaction regime, the phase and group velocity of the radiation are matched to the relativistic beam traveling in the undulator achieving long interaction lengths. Our numerical approach is based on expanding the existing 3D particle tracking code GPT (General Particle Tracer) to follow the interaction of the particles in the beam with the electromagnetic field modes of the waveguide. We present two separate studies: one for a case which was benchmarked with experimental results and another one for a test case where, using a longer undulator and larger bunch charge, a sizable fraction of the input beam energy can be extracted and converted to THz radiation. The model presented here is an important step in the development of the zero-slippage FEL scheme as a source for high average and peak power THz radiation.

INTRODUCTION

Free-Electron Laser radiation sources have long been an attractive solution to generate large amounts of electromagnetic radiation in spectral ranges where solid-state based devices are less effective including THz, extreme UV, soft and hard X-rays[1]. The THz range is particularly interesting for an FEL due to the fact that only relatively modest energy (~ few up to tens of MeV) electron beams are required in order to match the resonance condition for efficient interaction in an undulator magnet.

In the THz range, fueled by the recent developments in laser-based generation of nearly single-cycle radiation pulses[2], much interest has been diverted towards broad bandwidth THz applications, for example in the study of non-linear response in materials. On the other hand, most THz FEL operate with fairly long electron beams and target the generation of narrowband THz radiation [3,4]. This is due to the intrinsic bandwidth of the FEL associated with the slippage effects which limits the number of undulator periods over which a strong interaction can be maintained.

Following earlier work on wave-guided FEL[5,6], we recently revisited a strong coupling scheme for electrons and electromagnetic wave co-propagating in an undulator, based on the zero-slippage interaction where the group velocity of the radiation in a waveguide is matched to the electron beam axial velocity [7]. This scheme has the potential for broadband interaction (see Fig. 1), extended interaction lengths and high efficiency energy conversion.

Two sets of experiments where completed using the same setup at UCLA Pegasus Laboratory. In the first one, we used a laser-generated THz pulse to impart energy modulation on an electron beam to show THz-based beam compression. In the second one, raising the charge of the e-beam we showed THz amplification and generation. [8,9]. In the design and analysis of the results for these experiments, an interesting issue we had to face was the lack of adequate simulation tools to model this interaction. Standard wide-spread use FEL codes such as GENESIS or GINGER typically employ the slice-model and suffer from limitations related to period-averaging and slowly varying envelope approximation which fail to capture the dispersion dominated radiation evolution and the broad bandwidth of the interaction in a waveguide FEL.

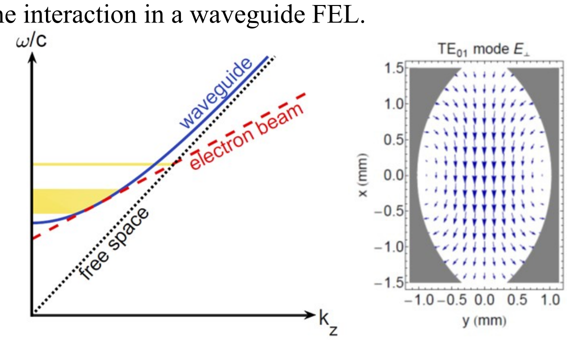

Figure 1: Left) Schematics of the waveguide coupling scheme. Strong interaction occurs where phase synchronicity between the radiation and the electron occurs. In the free space case, this intersection is limited to a single point. In the waveguide an entire range of frequencies can interact with the beam (due to the tangential crossing). Right) Spatial profile of the TE_{01} fundamental curved parallel plate waveguide mode which dominates the interaction.

In order to understand our experiments we used a mixture of a GPT module [10] to follow the particle evolution assuming a frozen-field approximation (radiation fields propagating unperturbed by the presence of the e-beam), and a 1D self-consistent modal-decomposition based FEL simulation wafFEL [9] where the full dispersion relation for the waveguide was used. In this paper we combine the two approaches and use GPT to calculate the spectrum of the radiation along the undulator. We will first describe the

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approach and use it to validate the simulation in the cases already benchmarked with experimental results. We will then explore the scientific opportunities outlined at the end of [9] to use this scheme for the generation of high power THz radiation.

DESCRIPTION OF THE CODE

The frequency-domain modal decomposition is a natural solution to describe the evolution of the field in our system as it allows complete freedom in setting the dispersion relation for the different modes. This solution provided the basis of the UCLA wafFEL code, which on the other hand had the strong limitation of one dimensional particle dynamics and completely neglected the effects of emittance and undulator focusing.

While exploring various methods to extend the simulations to three spatial dimensions, it was pointed out that more than a decade ago GPT developed an element suitable to simulate the interaction of the beam with free-space resonator modes [11,12]. It then became a natural solution to extend this model to calculate the interaction with the curved parallel plate waveguide modes.

The transverse and longitudinal wavenumbers for the modes in a curved parallel plate waveguide with radius R and separation b are

$$k_{mn} = \frac{1}{b}\left(n\pi + (2m+1)\tan^{-1}\frac{b}{\sqrt{2Rb-b^2}}\right)$$

$$k_z(\omega) = \sqrt{(\omega/c)^2 - k_{mn}^2}$$

The TM (TE) mode profiles can be derived from the expression for the electric (magnetic) longitudinal field which can be written [13]

$$\Phi_{mn} = \frac{e^{-\frac{\beta_{mn}^2 x^2}{\alpha_{mn}(y)}}}{\sqrt[4]{\alpha_{mn}(y)}} He_m\left(\frac{2\beta_{mn}x}{\sqrt{\alpha_{mn}(y)}}\right)\binom{cos}{sin}\left[k_{mn}y + \frac{2\beta_{mn}^4 y x^2}{k_{mn}\alpha_{mn}(y)} - \left(m+\frac{1}{2}\right)\tan^{-1}\frac{2\beta_{mn}^2 y}{k_{mn}}\right] e^{\pm i k_z z}$$

where He_m are the Hermite polynomials of order m and

$$\alpha_{mn}(y) = 1 + 4\frac{\beta_{mn}^4 y^2}{k_{mn}^2} \quad \text{and} \quad \beta_{mn} = \sqrt{\frac{k_{mn}}{\sqrt{2Rb-b^2}}}$$

The transverse E-field complex amplitudes for each mode are then given by

$$\mathbf{T}_{(x,y)}(x,y)e^{\pm i k_z z} = \frac{-i}{k_{mn}^2}\left(k_z\frac{\partial E_z}{\partial(x,y)} \mp \omega\mu_0\frac{\partial H_z}{\partial(y,x)}\right)$$

Just like in wafFEL and any other frequency based FEL code, we model the electric field as a sum over different modes:

$$\boldsymbol{E}(\boldsymbol{r},t) = Re\sum_j c_j \mathbf{T}_j(x,y)e^{-i(k_j z - \omega_j t)} \quad (1)$$

where we limited the description to forward propagating modes (no backward wave interactions), index j runs both over the different frequencies ω_j and the different mode numbers *(m,n)* and the longitudinal wavenumber is $k_j = k_{z,m,n}(\omega_j)$. The complex coefficients c_j encode the spectral content of the radiation field as it evolves along the interaction. The spectral interval between the different frequencies ΔN is the simulated bandwidth $(f_{max} - f_{min})$ divided by the number of modes N_{freq} and normalized by the frequency resolution $2L/c$ where L is the interaction length.

There are two different equivalent ways to derive the equations for the evolution of the mode (amplitude and phase). One option is to start from Maxwell equations, substitute Eq. (1) and project the source term onto the mode basis. Alternatively, we can also observe that energy conservation implies that the energy given by a mode to any one particle needs to be removed from the energy in that mode. This second view is completely equivalent, and offers an intuitive approach to the calculation of the evolution of the mode profile.

$$\frac{dW_j}{dt} = -\Delta N \sum_i Q_i \boldsymbol{v_i} \cdot \boldsymbol{E_j}(\boldsymbol{r_i},t) \quad (2)$$

where the sum is over all macroparticles of charge Q_i and velocity $\boldsymbol{v_i}$ and $W_j = \frac{1}{2}\varepsilon_0|c_j|^2 V$ is the energy of the mode j integrated over the frequency interval ΔN. $V \sim L\pi b^2/3$ is the mode volume and is related to the normalization factor of the mode basis $\mathrm{T}_{\boldsymbol{j}}(x,y)$.

Starting from Eq. (2) we can derive the differential equations for the real and imaginary parts of the amplitude coefficients for each mode j. For the interaction with the TE modes (no E_z) at each time step we will have

$$\frac{dc_j}{dt} = -\sum_i \frac{Q_i}{\varepsilon_0 V}\left(v_{x,i}\mathrm{T}_{x,j}(x_i,y_i) + v_{y,i}\mathrm{T}_{y,j}(x_i,y_i)\right)e^{-i(k_j z_i - \omega_j t)}$$

where (x_i, y_i, z_i) represent the instantaneous particle positions. The complex initial conditions $c_{0,j}$ can be specified externally if the initial electric field is known. For example, a simple initial gaussian spectrum pulse with no spectral phase can be initialized.

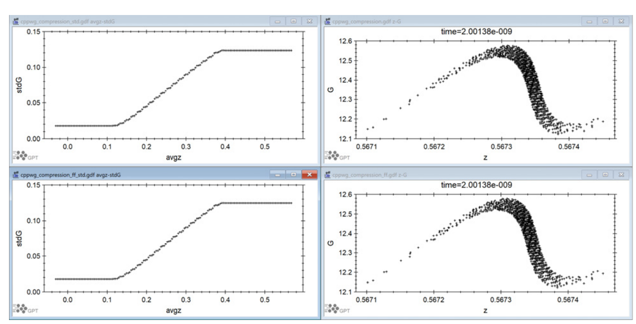

Figure 2: Comparison between GPT results from cppwg01mf.c and code benchmarked with experimental results. Left) Energy spread vs. z. Right) Longitudinal phase space.

Frozen-field approximation

By neglecting the back action of the particles on the fields, we can study the evolution of the beam distribution

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in the so-called frozen field approximation. We use a simple THz amplification case for this comparison similar to what presented in [9], but without any undulator tapering.

The results of this comparison are excellent as shown in Fig. 2 where we simulate the first of the UCLA experiments and show the phase space of the beam and the evolution of the energy spread along the undulator. The parameters for this simulations are given in the first column in Table 1.

Table 1: Parameters Used in the Simulation

Parameter	PEGASUS	THz FEL
Beam energy	6.3 MeV	8.8 MeV
Bunch charge	0-8 pC	200 pC
Beam energy spread	0.5 %	0.5 %
RMS Bunch length	200 fs	2 ps
THz central frequency	0.84 THz	0.8 THz
Seed energy	1 uJ	1 uJ
Undulator period	3 cm	3.6 cm
Undulator magnetic field	0.45 T	0.6 T
Waveguide spacing	2.06 mm	2.4 mm
Waveguide radius	2.0 mm	2.0 mm

Self-Consistent Solution

We then move on to simulate the behavior of the system self-consistently (*i.e.* allowing for feedback of the beam on the radiation). The case we consider has similar parameters as described in [9], with an injected peak electron beam current of 20 Amp and a long (60 cm) untapered undulator. We compare in Fig.3 the results of the 1D and 3D simulations. In the 1D wafFEL simulation, one of the main issues is the estimate of the overlap integral between the electron beam distribution and the mode profile which enters in the coupling factor. This is the main reason for the difference in the output energy.

The output options have been maintained similar to the original GPT module and we can look at the field profile as a function of time (oversampled in a time-window consistent with N_{freq}), or in the frequency domain at the evolution of amplitude and phase of each mode. The code runs fast on a high-end quad-core laptop processor taking 5 mins to push 10000 particles for the undulator length with $N_{freq} = 51$ frequency modes. Computation time is expected to scale linearly with number of particles and number of modes.

FUTURE OUTLOOK

There are two main outcomes of the work presented in this paper. First we will continue working on modal decomposition with GPT as this tool offers a unique opportunity to study in detail cases where standard FEL codes can only reach using approximations. For example in this case we can use real undulator field maps and no period-averaging approximation is needed to follow the beam dynamics. For simulation starting with unbunched beam and very low signal, it will be important to implement a quiet-start algorithm. In wafFEL this was done using a charge-weight. This is easily extendable to GPT, but further tests and benchmarks will be required to assess how to mplement it in 3D. Longitudinal space charge (not at the wavelength scale, but at the bunch length scale) and beam rearranging between the slices is also naturally taken into account in this model. The use of supercomputers can enable for a large number of modes to be simulated.

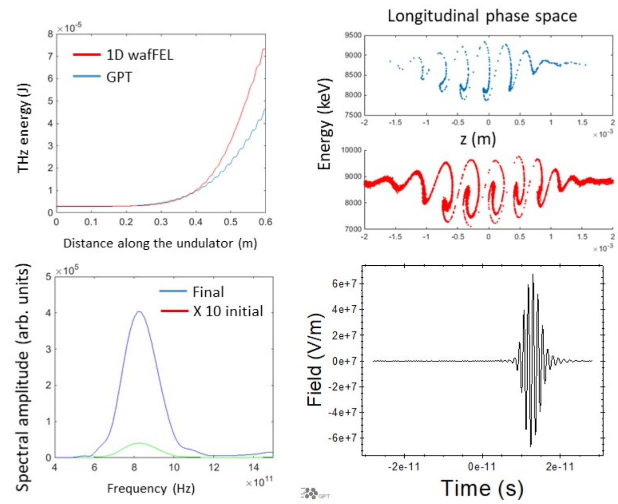

Figure 3: Top left) Comparison of THz energy vs. undulator distance between wafFEL and GPT for parameter set given in second column of Table 1. Top right) Output longitudinal phase spaces. Bottom left) Initial and final spectral amplitudes in GPT. Bottom right) Final THz electric field pulse profile vs. time in GPT.

At this regard one interesting possibility that we will be pursuing is to take advantage of source-dependent expansion (SDE) initially proposed by Sprangle in the '80s [14] to limit the number of modes required to describe the radiation. Especially in cases where a strong seed is used (such as in TESSA amplifiers [15]), the mode will be described for the entire interaction with a gaussian profile with changing waist and radius of curvature. By following how the radiation waist and curvature evolve, it is possible to limit the numbers of modes required to be solved.

The other direction is to directly exploit the outcome of the paper to study high efficiency THz FELs. Starting with higher energy electron beams or higher brightness electron beams and strongly tapering the undulator (or the waveguides) can enable large efficiency. Further studies are certainly required, but the goal of achieving 10 uJ at 10 THz within 10 % bandwidth seems within the reach of a super-radiant tapering-enhanced THz FEL.

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