ISBN: 978-3-95450-210-3 doi:10.18429/JACoW-FEL2019-THP036

MICROBUNCH ROTATION FOR HARD X-RAY BEAM MULTIPLEXING*

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Abstract

Electron bunches in an undulator develop periodic density modulations, or microbunches, which enable the exponential gain of X-ray power in a SASE FEL. Many FEL applications could from the ability to preserve microbunching through a dipole kick. For example, X-ray beam multiplexing can be achieved if electron bunches are kicked into separate beamlines and allowed to lase in a undulator. The microbunches developed in upstream undulators, if properly rotated, will lase axis, producing radiation at an angle from the initial beam axis. Microbunch rotation with soft X-rays was previously published and demonstrated experimentally [1], multiplexing LCLS into three X-ray beams. Additional 2018 data demonstrated multiplexing of hard X-rays. Here we describe ts to reproduce these hard X-ray experiments using an analytical model and Genesis simulations. Our goal is to apply microbunch rotation to outcoupling from a cavity-based XFEL, (RAFEL/XFELO) [2].

ANALYTICAL MODEL

Microbunch rotation can be accomplished by providing a dipole kick to a bunch as it travels through a defocusing quadrupole. This dipole kick can be provided by either a dipole magnet or a transversely quadrupole. An analytical model for microbunch rotation within a defocusing quadrupole was developed previously by MacArthur et al. [1]. Here, we introduce a new analytical method for describing microbunch rotation using matrices, which is more intuitive to use with multiple quadrupoles.

Microbunch Rotation by Offset Thin Quadrupole

We begin by a thin quadrupole in 4D phase space (y, y', z, δ). The quadrupole has focal length f, and is in y by a distance o_1, producing a kick $\theta = -\frac{1}{f}(y_0 + o_1)$, where y_0 is the transverse beam position before the quadrupole. We treat an quadrupole as an on-axis quadrupole which also rotates the coordinate system in the y-z plane by θ to follow the beam trajectory. We assume a small kick angle ($sin(\theta) \approx \theta$, $cos(\theta) \approx 1$), and neglect order terms, ($\theta z \approx 0$, $-\theta y' \approx 0$) to simplify the rotation matrix.

$$M_\theta \approx \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \theta \\ -\theta & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

our initial microbunch size and divergence in the center of the quadrupole, we an quadrupole matrix with double the focal length of our full quadrupole

$$M_{Q\theta} = M_Q + M_\theta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{-1}{2f} & 1 & 0 & \theta \\ -\theta & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

Note that we still rotate the coordinate system by the full kick, θ, produced by the quadrupole. This is because the y-z plane (which we must rotate) is initially at the angle of the beam entering the quadrupole, $y'_0 = 0$, but our microbunch rotation analysis begins with particles at the center of the quadrupole, $(y_1, y'_1, z_1, \delta_1)$.

We apply a drift of length L to get our transfer matrix,

$$M = M_L M_{Q\theta} = \begin{bmatrix} 1 - \frac{L}{2f} & L & 0 & L\theta \\ \frac{-1}{2f} & 1 & 0 & \theta \\ -\theta & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (3)$$

We assume a single Gaussian microbunch, with a sigma matrix that is uncorrelated in the center of the quadrupole,

$$\Sigma_1 = \begin{bmatrix} \sigma^2_{y_1} & 0 & 0 & 0 \\ 0 & \sigma^2_{y'_1} & 0 & 0 \\ 0 & 0 & \sigma^2_{z_1} & 0 \\ 0 & 0 & 0 & \sigma^2_{\delta_1} \end{bmatrix}. \quad (4)$$

We then evolve the microbunch envelope using the transfer matrix, and obtain the second moments.

$$\Sigma = M \Sigma_1 M^T \quad (5)$$

$$\begin{aligned} \langle y^2 \rangle &= \left(1 - \frac{L}{2f}\right)^2 \sigma^2_{y_1} + L^2 \sigma^2_{y'_1} + L^2 \theta^2 \sigma^2_{\delta_1} \\ \langle z^2 \rangle &= \sigma^2_{z_1} + \theta^2 \sigma^2_{y_1} \\ \langle yz \rangle &= -\left(1 - \frac{L}{2f}\right) \theta \sigma^2_{y_1} \end{aligned} \quad (6)$$

The tilt in the rotated coordinate system, t_{yz}, can then be geometrically found from,

$$tan(2t_{yz}) = \frac{2\langle yz \rangle}{\langle y^2 \rangle - \langle z^2 \rangle}, \quad (7)$$

$$t_{yz} \approx \frac{\langle yz \rangle}{\langle y^2 \rangle - \langle z^2 \rangle}. \quad (8)$$

In the limit of pancake microbunches, $\langle z^2 \rangle << \langle y^2 \rangle$ and if $\theta^2 \approx 0$, we write the tilt in terms of the Beta function, β_1.

$$t_{yz} \approx \frac{\langle yz \rangle}{\langle y^2 \rangle} = \frac{-\left(1 - \frac{L}{2f}\right)\theta}{\left(1 - \frac{L}{2f}\right)^2 + \frac{L^2}{\beta_1^2}} \quad (9)$$

* This work was supported by the Department of Energy, Laboratory Directed Research and Development program at SLAC National Accelerator Laboratory, under contract DE-AC02-76SF00515.
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For relatively large kick angles and energy spread, we found $\theta^2 \approx 0$ was not always a safe assumption, and used Eq. (8) for our analysis. However, Eq. (9) is useful for building intuition that the microbunch rotation direction is determined by the sign of f. Eq. (9) we

$$\left.\frac{\partial t_{yz}}{\partial L}\right|_{L=0} = -\frac{\theta}{2f}. \tag{10}$$

This result shows that a defocusing quadrupole ($f < 0$) will rotate the microbunch in the direction of the kick, θ.

Microbunch Rotation by Thin Quadrupole Triplet

Next, we want to analyze the rotation of a microbunch through three quadrupoles with focal lengths f_1,f_2,f_3; in y by distances o_1,o_2,o_3; separated by drifts L_1,L_2; and followed by a drift L_3. Subsequent simulations will consider a defocusing-focusing-defocusing triplet, $f_1 < 0$,$f_2 > 0$,$f_3 < 0$, as this triplet achieves a given rotation with the smallest kick angles, but the analytical method can be applied to quadrupoles of arbitrary focal length.

We the transfer matrix for each quadrupole,

$$M_{Q\theta 1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{-1}{2f_1} & 1 & 0 & \theta_1 \\ -\theta_1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, M_{Q\theta 2,3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{-1}{f_{2,3}} & 1 & 0 & \theta_{2,3} \\ -\theta_{2,3} & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{11}$$

The kick angles can be readily solved using a matrix method. If $\langle y_0 \rangle = \langle y_0' \rangle = 0$, the kick angles are,

$$\begin{aligned} \theta_1 &= -\frac{o_1}{f_1}, \\ \theta_2 &= -\frac{1}{f_1 f_2}(f_1 o_2 - L_1 o_1), \\ \theta_3 &= -\frac{1}{f_1 f_2 f_3}(f_1 f_2 o_3 + f_2 f_3 o_1 - f_2 L_1 o_1 - f_1 L_2 o_2 \\ &\qquad - f_2 L_2 o_1 - L_1 L_2 o_1). \end{aligned} \tag{12}$$

Using these, we build our transfer matrix,

$$M = M_{L3} M_{Q\theta 3} M_{L2} M_{Q\theta 2} M_{L1} M_{Q\theta 1}. \tag{13}$$

To the tilt, one can propagate a sigma matrix at the center of the t quadrupole, Σ_1, through this transfer matrix using Eq. (5). An expression for the microbunch tilt, too long to write out here, can be obtained using Eq. (8).

For arbitrary quadrupole the microbunch tilt will continue to evolve with additional drift distance, L_3. To achieve microbunch rotation, we need to lock the microbunch tilt into the direction of beam travel, eg. $t_{yz} = 0$ and $\frac{\partial t_{yz}}{\partial L_3} = 0$. We found this is equivalent to requiring the matrix be achromatic, eg. $D = 0$ and $D' = 0$. D and D' are the dispersion and change in dispersion (R36 and R46) of M, read directly from the transfer matrix. For a given beam trajectory angle α, we solve this system of equations,

$$\begin{aligned} D &= 0, \\ D' &= 0, \\ \alpha &= \langle y_0' \rangle + \theta_1 + \theta_2 + \theta_3. \end{aligned} \tag{14}$$

We obtain the ideal for microbunch rotation through three thin quadrupoles. For $\langle y_0 \rangle = \langle y_0' \rangle = 0$,

$$\begin{aligned} o_1 &= \frac{-\alpha f_1 f_2}{L_1}, \\ o_2 &= \frac{-\alpha f_1 (f_2 L_1 + f_2 L_2 - L_1 L_2)}{L_1 L_2}, \\ o_3 &= \frac{-\alpha f_1 (L_2^2 + f_2 f_3)}{L_2}. \end{aligned} \tag{15}$$

Microbunch Rotation by Thick Quadrupole

We also pursued a thick lens approach, with quadrupole length L_Q and $k^2 = \frac{1}{|f_{thinlens}| L_Q}$. A shifted defocusing and focusing quadrupole can be respectively,

$$\begin{aligned} M_{Q_D\theta} &= \begin{bmatrix} cosh(kL_Q) & \frac{1}{k} sinh(kL_Q) & 0 & 0 \\ ksinh(kL_Q) & cosh(kL_Q) & 0 & \theta \\ -\theta & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\ M_{Q_F\theta} &= \begin{bmatrix} cos(kL_Q) & \frac{1}{k} sin(kL_Q) & 0 & 0 \\ -ksin(kL_Q) & cos(kL_Q) & 0 & \theta \\ -\theta & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \end{aligned} \tag{16}$$

For the defocusing-focusing-defocusing case, we again build our transfer matrix similarly to Eq. (13), shortening L_Q by half in the t quadrupole and using the full L_Q in the second two. θ_1,θ_2,θ_3 are also recalculated using thick quadrupoles. We solve for the optimal using the system of equations in Eq. (14), obtaining a lengthy analytical solution for the optimal used in subsequent sections.

COMPARISON OF ANALYTICAL MODEL TO GENESIS SIMULATIONS

We supported our analytical model with Genesis [3] simulations, summarized in Fig. 1 and Table 1.

Table 1: Genesis Simulation Parameters

Parameters Common to All Triplets			
B_1'	−65.267 T/m	L_1	4.05 m
B_2'	65.600 T/m	L_2	3.9 m
B_3'	−65.267 T/m	L_Q	6 cm
a_w	2.4749	$L_{Undulator}$	3.3 m
$E_{electron}$	14.535 GeV	E_{λ_r}	9.3858 keV
σ_{y_1}	1.745×10^{-5} m	$\sigma_{y_1'}$	8.059×10^{-7} rad
σ_{z_1}	3.457×10^{-11} m	σ_{δ_1}	2.378×10^{-4}

Triplet Quadrupole Offsets (μm)					
α	−5 μrad		−10 μrad		Exp
Model:	Thin	Thick	Thin	Thick	
o_1	-190.94	-193.84	-381.89	-387.68	-150
o_2	-258.23	-262.66	-516.46	-525.33	-263
o_3	-178.79	-180.54	-357.58	-361.08	-206

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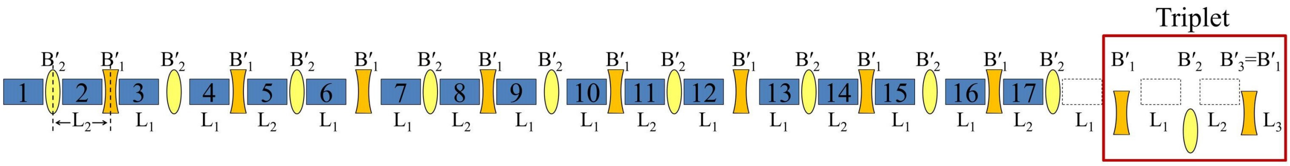

Figure 1: Genesis lattice. The beam was pre-bunched in 17 undulators (blue), then sent through an quadrupole triplet.

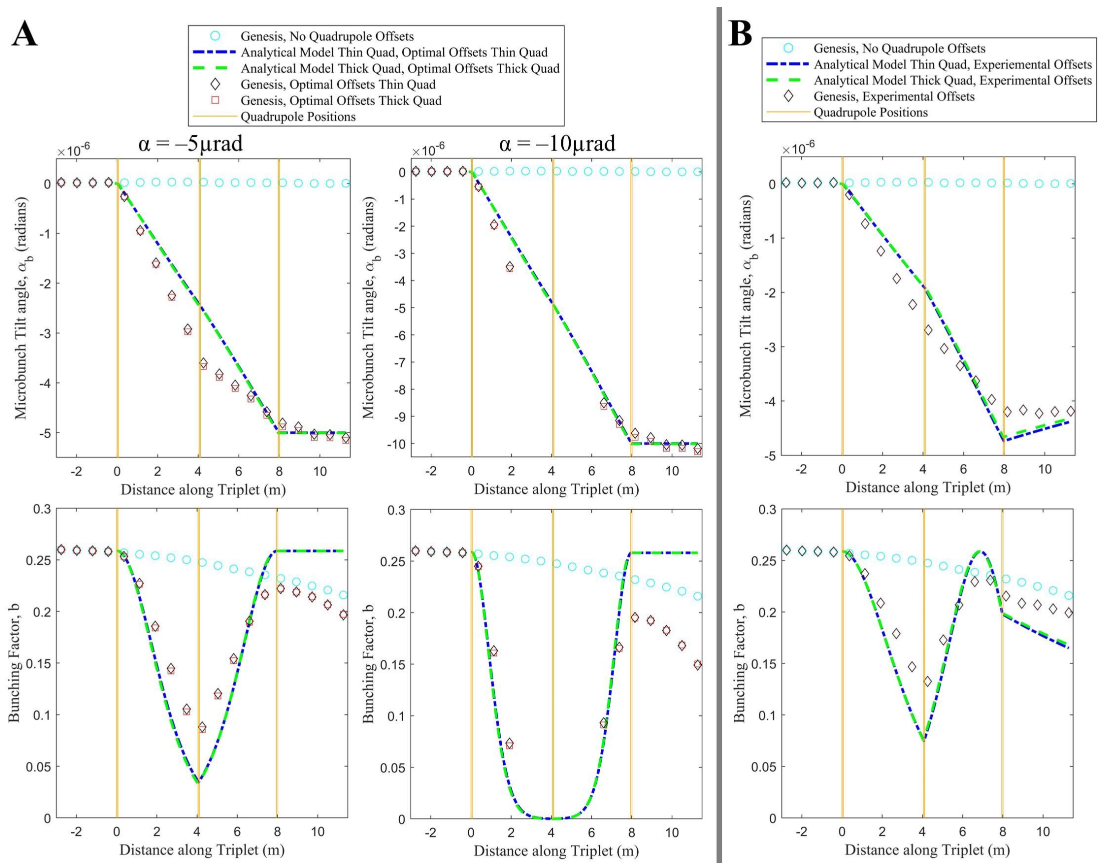

Figure 2: Microbunch tilt angle in nonrotated frame, α_b and bunching factor, b, through an quadrupole triplet. are A) analytical model optimal using thin or thick quadrupoles, or B) replicating a 2018 experiment.

We calculated optimal for $\alpha = -5\,\mu$rad and $\alpha = -10\,\mu$rad using thin and thick quadrupole models, and simulated each. As shown in Fig. 2A, both models achieved similar results. The analytical models predict 99.98% or 99.7% recovery of b after a $-5\,\mu$rad or $-10\,\mu$rad rotation. For these angles at $L_3 = 15$ cm, Genesis predicts 85.9% or 75% recovery. Some loss can be explained by the degradation of b to 89.7% over the long length of the triplet. While this recovery is sizable, we suspect there are still unaccounted for in our matrix model which limit our ability to choose optimal Genesis shows the microbunch angle continues to evolve slowly with distance, instead of locking into the beam travel angle. We suspect this is due to second-order (microbunch smearing) [1] which Genesis accounts for, or $\langle yz \rangle$ correlations in the quadrupole center caused by imperfect lattice matching.

We would like to use Genesis and our analytical model to explain the microbunch rotation observed in an experiment at LCLS in 2018. A successful microbunch rotation, which lased at angles between −2 to $-7\,\mu$rad was achieved through a combination of quadrupole and corrector dipole magnets, producing e quadrupole given in Table 1. For these our analytical model and Genesis give results shown in Fig. 2B.

The analytical model predicts only a 76% recovery, while Genesis shows 83.2% recovery of b. Interestingly, Genesis shows these lock the microbunch rotation into an average angle of $-4.19\,\mu$rad, close to $\alpha = -4.11\,\mu$rad. While these generally behave better than the analytical model would predict, the model does correctly predict a maximum in bunching between the last two quadrupoles.

CONCLUSION

We have developed an analytical model for microbunch rotation using matrices which gives a t-order estimate of microbunch rotation through an quadrupole triplet.

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