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A GENERAL OPTIMIZATION METHOD FOR HIGH HARMONIC GENERATION BEAMLINE

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Abstract

Shorter bunches produce a more coherent radiation and contain higher harmonic components. Here, based on transverse and longitudinal phase space coupling, a general method for analyzing the production of very short beam and searching for compression beamline is presented. With this method, several beamlines are found and optimized. The electron beam can be compressed to tens of nanometers, generating coherent high harmonic radiation.

INTRODUCTION

The coherence of synchrotron radiation depends strongly on beam length and has been enormously demanded by many applications. Generally, a beam will radiate coherently while its length is comparable to the radiation wavelength [1]. For this reason, shortening bunch length in storage ring has been studied widely, but the length is still on the order of tens femtoseconds [1–4]. On the other hand, free electron laser (FEL), benefiting from the natural microbunching process, holds the shortest length achieved. Combining these short bunches with storage rings, and uniting the high brightness of FEL with the high repetition rate of synchrotron radiation will create a new light source with excellent properties. However, this combination requires to compress the beam to a very short length before injecting it into the undulator. Several compression methods and beamlines have been proposed [5–12]. But the optimization methods vary. In this paper, we present a general method for analyzing the production of very short beam and searching for compression beamlines. Based on this method, several beamlinems are found and optimized. The electron beam can be compressed to tens of nanometers, and very coherent high harmonic radiation can be generated.

GENERAL METHOD

Transfer matrix is used to analyze the compression problem. For simplicity, the electron six-dimensional phase space is shortened to four, $(x,\ x',\ z,\ \delta)^T$. Here, the symbol x can be either horizontal or vertical direction. Under this simplification, a general matrix form for both dipole and quadrupole magnets is

$$a = \begin{pmatrix} a_{11} & a_{12} & 0 & a_{14} \\ a_{21} & a_{22} & 0 & a_{24} \\ a_{31} & a_{32} & 1 & a_{34} \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{1}$$

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The symplectic condition tells that only six elements in the matrix are free. Besides, it can be proved that the product of any number of matrix in this type has the same form. Hence, this matrix is actually a general expression for an arbitrary beamline consisting of only dipoles and/or quadrupoles. Now, we use symbols A (with elements A_{ij}), B (with elements B_{ij}) to express two such beamlines.

Another critical component is the modulator, where an undulator and a laser cooperate to act like a radio frequency (RF) cavity. Considering the case that laser incidents with a small angle t, the beam energy modulation can be expressed as

$$\delta = \delta_0 + \tilde{\delta}\sin(kz + tkx), \tag{2}$$

with $\tilde{\delta} = \frac{\Delta\gamma}{\gamma_c}$ the maximum absorbed energy, and γ_c the beam central energy. k is laser wave number. Panofsky-Wentzel theorem [13] indicates that this energy modulation will be accompanied by an angular modulation

$$x' = x'_0 + t\tilde{\delta}\sin(kz + tkx). \tag{3}$$

Thus, the linearized transfer matrix for a modulator is

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ ht^2 & 1 & ht & 0 \\ 0 & 0 & 1 & 0 \\ ht & 0 & h & 1 \end{pmatrix}. \tag{4}$$

Here, $h = k\tilde{\delta}$, is a parameter representing modulation strength. With $t = 0$, M degenerates into a standard modulator, which changes electron energy only.

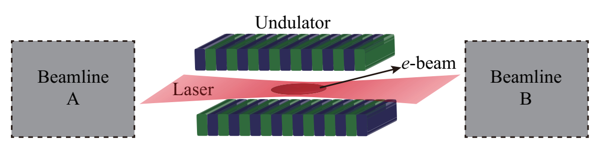

Figure 1: Structure of a compression beamline.

Generally, a full compression cell is a modulator sandwiched by two beamlines, assuming the upstream one A and downstream one B, shown as Fig. 1. The complete transfer matrix can be written into

$$T = BMA \tag{5}$$

Hence, the final longitudinal position of an electron is

$$z = T_{31}x_0 + T_{32}x'_0 + T_{33}z_0 + T_{34}\delta_0, \tag{6}$$

with

$$\begin{aligned} T_{31} &= A_{21}B_{32} + A_{31}(1 + B_{34}h + B_{32}ht) \\ &\quad + A_{11}[B_{31} + ht(B_{34} + B_{32}t)] \\ T_{32} &= A_{22}B_{32} + A_{32}(1 + B_{34}h + B_{32}ht) \\ &\quad + A_{12}[B_{31} + ht(B_{34} + B_{32}t)] \\ T_{33} &= 1 + B_{34}h + B_{32}ht \\ T_{34} &= A_{24}B_{32} + B_{34} + A_{34}(1 + B_{34}h + B_{32}ht) \\ &\quad + A_{14}[B_{31} + ht(B_{34} + B_{32}t)]. \end{aligned}$$

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This equation gives the bunch length after passing through copression beamline, $\sigma_s^2 = T_{31}^2\sigma_{x0}^2 + T_{32}^2\sigma_{x'0}^2 + T_{33}^2\sigma_{s0}^2 + T_{34}^2\sigma_{\delta 0}^2$. The size in each dimension will contribute to final beam length. A straightforward way to realize bunch compression is keeping only one contribution while evanishing others.

THREE CASES OF BEAM COMPRESSION

Based on this method, three cases of beam compression will be discussed in this section.

Pure Dispersive Compression

Pure dispersive compression has the longest and most successful history. It only utilizes a dispersive section to change energy modulation into density distribution. Perfect examples are the coherent harmonic generation (CHG) or high gain harmonic generation (HGHG) [6, 14–21], where by increasing the modulation amplitude h, a small B_{34}, hence T_{34}, is achieved, leading to a compression ratio of $R = \frac{\sigma_{s0}}{\sigma_s} = \frac{h\sigma_{s0}}{\sigma_{\delta 0}}$. With a small energy spread $\sigma_{\gamma 0}$, the beam can be compressed at a significant ratio. However, it is of great difficulty to minify $\sigma_{\delta 0}$ down to 10^{-4}. Thus, a large compression requires a very great modulation amplitude, which indicates a powerful laser.

T_{33} indicates an alternative way for pure dispersive compression, which may ease the requirements of laser power. The idea is to use an obliquely incoming laser.

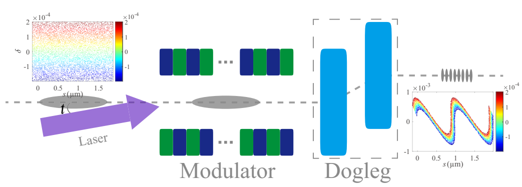

Figure 2: A variant of CHG.

When laser incidents with an angle, the energy and divergence modulation will be generated. With an x' and z coupling in beamline, the extra term $B_{32}ht$ will contribute to a smaller B_{34}. This means a shorter bunch. Figure 2 presents a simple beamline arrangement of this idea, a modulator followed by a dogleg. The two subfigures at upper-left and bottom-right corners are the initial and final beam longitudinal phase space respectively, with both colorbars representing initial energy deviation. The initially uniform distributed beam is micro-bunched after passing through the whole beamline. And final length of a microbunch is determined by initial energy spread and modulation amplitude.

The compression ratio of this idea is

$$R = \frac{h\sigma_{s0}}{(1 - |B_{32}ht|)\sigma_{\delta 0}}. \qquad (7)$$

It is of great similarity with the CHG case. But under the same energy modulation amplitude, the final length will be $|B_{32}ht|$ times shorter in this case. However, at fixed laser power, h shrinks with the growth of t, as Fig. 3 shows. This fact restricts the compression of bunch length greatly.

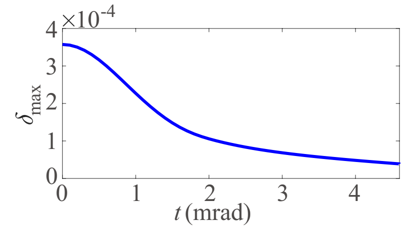

Figure 3: Modulation amplitude at various incident angle. Laser parameter: $P = 5$ MW, $w_0 = 398$ µm, $\lambda_l = 1.0$ µm.

For both CHG and its variant, limited by laser power, the modulation depth $D = \frac{\tilde{\delta}}{\sigma_{\delta 0}}$ is typically below 10. And the harmonic bunching factor

$$b_n = e^{-\frac{1}{2}(nkT_{34}\sigma_{\delta 0})^2} J_n[-nk(B_{32}t + B_{34})\tilde{\delta}] \qquad (8)$$

is also hard to improve for a significant $\sigma_{\delta 0}$. To reach a higher harmonic, one more complicated cascaded scheme, putting two or more beamlines in sequence, is proposed. But this one is beyond bunch compression, readers interested can refer to [22–30].

x-z Coupling Compression

To get a larger R with a moderate laser power, taking advantages of some beam properties helps. For example, under the absence of vertical dispersion and horizontal-vertical coupling, the vertical beam size in storage ring may be extremely small [31].

Utilizing this small-size feature means to keep only T_{31} nonzero in Eq. 6. Then, the final beam length will be $\sigma_s = T_{31}\sigma_{x0}$. A small T_{31} results to a good compression. Combining $T_{32} = T_{33} = T_{34} = 0$ with the symplectic condition, we get

$$T_{31} = 1/(A_{32}h + A_{12}ht). \qquad (9)$$

It is obvious that bunch stretching caused by A_{32} contradicts with compressing. By inclining the incident laser, this contradiction can be alleviated.

Equation 9 indicates a compression ratio of $R = (A_{32}h + A_{12}ht)\frac{\sigma_{s0}}{\sigma_{x0}}$. And final beam bunching factor

$$b_n = e^{-\frac{1}{2}(nkT_{31}\sigma_{x0})^2} J_n[-nk(B_{32}t + B_{34})\tilde{\delta}]. \qquad (10)$$

A small transverse size contributes to a large compression ratio and a high harmonic number.

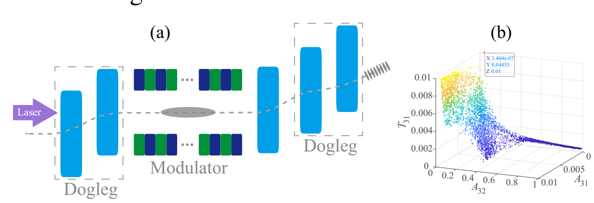

Figure 4: Compression beamline using *x*-*z* coupling.

Figure 4(a) presents a compression beamline making use of *x*-*z* coupling. The normally incoming laser means an inversely proportional relation between T_{31} and $A_{32}h$. Optimized result of a multi-objective genetic algorithm (MOGA) code, given in Fig. 4(b), shows clear, too. Great compression and little bunch stretching require huge dipoles and long drifts.

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One working point (red dot in Fig. 4(b)) is selected for test, where beam stretching is well controlled, while well compression is sacrificed. Figure 5 is the final electron distribution. The significant difference of beam length results from various initial horizontal beam size. The left one is 100 times greater than the right.

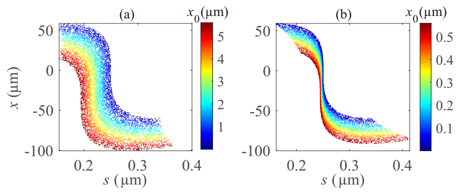

Figure 5: Electron distribution after compression.

When laser incidents with an angle, the co-work with a transversely dispersion section can also compress beam well [7]. Besides, another effective method called the Off-Resonance Laser Modulation or Phase-merging Enhanced Harmonic Generation [8,9], where an undulator with a transverse gradient (called TGU) [10] is used, also takes advantage of small transverse size.

x'-z Coupling Compression

If the beam angular spread is orders smaller than beam size, one alternative coupling —x'-z— can be much better for beam compression.

To get a full analysis of this idea, we follow the path in Sec . Again, utilizing x'-z coupling means to keep only T_{32} in Eq. 6. And the coupling coefficient is

$$T_{32} = -1/(A_{31}h + A_{11}ht). \tag{11}$$

Upon this, the simplest design is given by X. F. Wang, which uses the tilt term along [11]. The beamline is shown in Fig. 6(a). Because upstream beamline does not exits, there is also no bunch stretching. For a long beam, and considering a sinusoidal energy modulation, the bunching factor of this case is

$$b_n = e^{-\frac{\left(nk_l\eta_b\sigma_{x'}\right)^2}{2}} J_n(nh\xi_b), \tag{12}$$

where $\eta_b = -\frac{1+\xi_b h}{ht}$ is the dispersion of dipole, with ξ_b dipole momentum compaction. Owing to the tiny angular spread $\sigma_{x'}$, the bunching factor at harmonic number n still has a significant value.

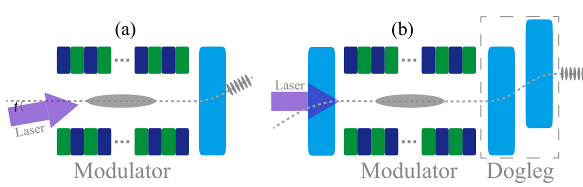

Figure 6: Two beamlines using x'-z coupling.

Another ingenious design reported by C. Feng is presented in Fig. 6(b) [12]. The idea lies in using a dipole to create a x'-z or y'-z coupling, and then utilizing energy modulation and momentum compaction to neutralize original z value, during which the dispersion introduced by first dipole is also canceled by the dogleg.

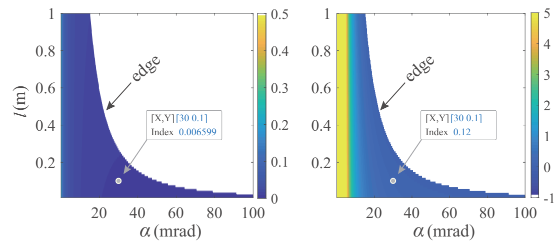

Figure 7: Matched η_d(left) and L(right) of dogleg at various bending angle α and length l.

Here, a well compression requires a smaller η_d. But achieving this needs to increase bending angle of the upstream dipole, which will cause a significant beam stretching. Figure 7 shows the parameter space of dogleg. At a fixed modulator, a smaller η_d means the working point moves closer to the edge in Fig. 7, which corresponds to smaller distance between the two dopiles of dogleg. With the working point chosen in Fig. 7, one beam can be greatly compressed, and up to 35th harmonic can be achieved. Figure 8 gives this result.

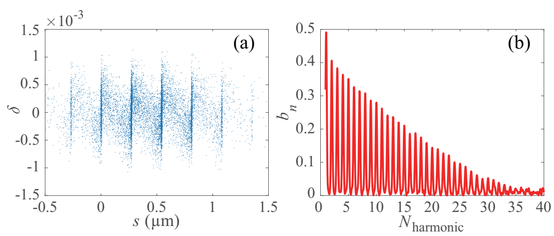

Figure 8: (a) Beam final phase space and (b) bunching factor.

Typically, the angular spread is over ten times smaller than the corresponding beam size. Thus it is very helpful to take advantage of angular spread, and the beam can be greatly compressed, leading to a very slow decreasing of high harmonic bunching factor.

SUMMARY

In this paper, based on transverse-longitudinal coupling, a general method for optimizing and searching for compression beamline is presented. With the optimized result, electron beam can be compressed at large ratio and up to 35th harmonic can be generated. However, only a beam with initial length smaller than $\lambda_l/2$ works, longer one will be splited into several bunches. And only a linear modulation can be well compressed, but the energy modulated is sinusoidal. This may be relieved by using a two-color-laser modulator.

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REFERENCES

[1] M. Abo-Bakr, J. Feikes *et al.*, "Brilliant, Coherent Far-Infrared (THz) Synchrotron Radiation", *Phys. Rev. Lett.* **90**, 094801 (2003). doi:10.1103/PhysRevLett.90.094801

[2] H. Hama, S. Takano *et al.*, "Control of the bunch length on an electron storage ring", *Nucl. Instrum. Methods Phys. Research A* **329**, 29 (1993). doi:10.1016/0168-9002(93)90919-9

[3] R. W. Schoenlein, S. Chattopadhyay *et al.*,"Generation of Femtosecond Pulses of Synchrotron Radiation", *Science* **287**, 2237 (2000). doi:10.1126/science.287.5461.2237

[4] P. Beaud, S. L. Johnson *et al.*, "Spatiotemporal Stability of a Femtosecond Hard–X-Ray Undulator Source Studied by Control of Coherent Optical Phonons", *Phys. Rev. Lett.* **99**, 174801 (2007). doi:10.1103/PhysRevLett.99.174801

[5] P. L. Csonka, "Enhancement of Synchrotron Radiation by Beam Modulation", *Part. Accel.* **8**, 225 (1978).

[6] L. H. Yu, "Generation of intense uv radiation by subharmonically seeded single-pass free-electron lasers", *Phys. Rev. A* **44**, 5178 (1991). doi:10.1103/PhysRevA.44.5178

[7] C. Feng and T. Zhang *et al.*, "Design of a 4.8m ring for inverse Compton scattering x-ray source", *Phys. Rev. ST Accel. Beams* **17**, 070701 (2014). doi:10.1103/PhysRevSTAB.17.070101

[8] H. X. Deng and C. Feng, "Using Off-Resonance Laser Modulation for Beam-Energy-Spread Cooling in Generation of Short-Wavelength Radiation", *Phys. Rev. Lett.* **111**, 084801 (2013). doi:10.1103/PhysRevLett.111.084801

[9] C. Feng and H. X. Deng *et al.*, "Phase-merging enhanced harmonic generation free-electron laser", *New J. of Phys.* **16**, 043021 (2014). doi:10.1088/1367-2630/16/4/043021

[10] Z. R. Huang, Y. T. Ding *et al.*, "Compact X-ray Free-Electron Laser from a Laser-Plasma Accelerator Using a Transverse-Gradient Undulator", *Phys. Rev. Lett.* **109**, 204801 (2012). doi:10.1103/PhysRevLett.109.204801

[11] X. F. Wang, C. Feng *et al.*, "Angular dispersion enhanced prebunch for seeding ultrashort and coherent EUV and soft X-ray free-electron laser in storage rings", *J. Synchrotron Rad.* **26**, 677 (2019). doi:10.1107/S1600577519002674

[12] C. Feng and Z. T. Zhao, "A Storage Ring Based Free-Electron Laser for Generating Ultrashort Coherent EUV and X-ray Radiation", *Sci. Rep.* **7**, 4724 (2017). doi:10.1038/s41598-017-04962-5

[13] W. Panofsky and W. Wenzel, "Some considerations concerning the transverse deflection of charged particles in radiofrequency fields", *Rev. Sci. Instrum.* **27**, 967 (1956). doi:10.1063/1.1715427

[14] B. Girard, Y. Lapierre *et al.*, "3D simulation of high gain FELs in an optical klystron configuration", *Phys. Rev. Lett.* **53**, 2405 (1984). doi:10.1016/0168-9002(92)91138-Y

[15] M. Labat, M. Hosaka *et al.*, "Coherent harmonic generation on UVSOR-II storage ring", *Eur. Phys. J. D* **44**, 187 (2007). DOI:10.1140/epjd/e2007-00177-6

[16] G. De Ninno, E. Allaria *et al.*, "Generation of Ultrashort Coherent Vacuum Ultraviolet Pulses Using Electron Storage Rings: A New Bright Light Source for Experiments", *Phys. Rev. Lett.* **101**, 053902 (2008). doi:10.1103/PhysRevLett.101.053902

[17] G. De Ninno, E. Allaria *et al.*, "Self-Induced Harmonic Generation in a Storage-Ring Free-Electron Laser", *Phys. Rev. Lett.* **100**, 104801 (2008). doi:10.1103/PhysRevLett.100.104801

[18] D. Xiang and W. Wan, "Generating Ultrashort Coherent Soft X-Ray Radiation in Storage Rings Using Angular-Modulated Electron Beams", *Phys. Rev. Lett.* **104**, 084803 (2010). doi:10.1103/PhysRevLett.104.084803

[19] L. H. Yu, M. Babzien *et al.*, "Ultrafast Electron Localization Dynamics Following Photo-Induced Charge Transfer", *Science* **289**, 932 (2000). doi:10.1126/science.289.5481.93

[20] L. H. Yu, L. DiMauro *et al.*, "First Ultraviolet High-Gain Harmonic-Generation Free-Electron Laser", *Phys. Rev. Lett.* **91**, 074801 (2003). doi:10.1103/PhysRevLett.91.074801

[21] E. Allaria, R. Appio *et al.*, "Highly coherent and stable pulses from the FERMI seeded free-electron laser in the extreme ultraviolet", *Nature Photon.* **6**, 699 (2012). doi:10.1038/nphoton.2012.233

[22] J. H. Wu, L. H. Yu, "Coherent hard X-ray production by cascading stages of High Gain Harmonic Generation", *Nucl. Instrum. Methods Phys. Research* A **475**, 104 (2001). doi:10.1016/S0168-9002(01)01552-2

[23] B. Liu, W. B. Li *et al.*, "Demonstration of a widely-tunable and fully-coherent high-gain harmonic-generation free-electron laser", *Phys. Rev. ST Accel. Beams* **16**, 020704 (2013). doi:10.1103/PhysRevSTAB.16.020704

[24] E. Allaria, D. Castronovo *et al.*, "Two-stage seeded soft-X-ray free-electron laser", *Nature Photon.* **7**, 913 (2013). doi:10.1038/nphoton.2013.277

[25] D. Xiang and G. Stupakov, "Echo-enabled harmonic generation free electron laser", *Phys. Rev. ST Accel. Beams* **12**, 030702 (2009). doi:10.1103/PhysRevSTAB.12.030702

[26] G. Stupakov, "Using the Beam-Echo Effect for Generation of Short-Wavelength Radiation", *Phys. Rev. Lett.* **102**, 074801 (2009). doi:10.1103/PhysRevLett.102.074801

ISBN: 978-3-95450-210-3 doi:10.18429/JACoW-FEL2019-THP022

[27] D. Xiang, E. Colby *et al.*, "Demonstration of the Echo-Enabled Harmonic Generation Technique for Short-Wavelength Seeded Free Electron Lasers", *Phys. Rev. Lett.* **105**, 114801 (2010). doi:10.1103/PhysRevLett.105.114801

[28] D. Xiang, E. Colby *et al.*, "Evidence of High Harmonics from Echo-Enabled Harmonic Generation for Seeding X-Ray Free Electron Lasers", *Phys. Rev. Lett.* **108**, 024802 (2012). doi:10.1103/PhysRevLett.108.024802

[29] Z. T. Zhao, D. Wang *et al.*, "First lasing of an echo-enabled harmonic generation free-electron laser", *Nature Photon.* **6**, 360 (2012). doi:10.1038/nphoton.2012.105

[30] E. Hemsing, M. Dunning *et al.*, "Echo-enabled harmonics up to the 75th order from precisely tailored electron beams", *Nature Photon.* **10**, 512 (2016). doi:10.1038/nphoton.2016.101

[31] J. B. Murphy, C. Pellegrini, "Generation of high-intensity coherent radiation in the soft-x-ray and vacuum-ultraviolet region", *J. Opt. Soc. Am.* B **2**, 259 (1985). doi:10.1364/JOSAB.2.000259