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MICROBUNCHING ENHANCEMENT BY ADIABATIC TRAPPING

Xiujie Deng*, Alex Chao[1], Wenhui Huang, Chuanxiang Tang,
Tsinghua University, Beijing, China
[1] also at SLAC, Menlo Park, USA

Abstract

Storage ring based concept called steady-state microbunching (SSMB) was proposed years ago for generating high average power narrowband coherent radiation. There are now active efforts on-going by the SSMB collaboration established among Tsinghua University and several other institutes. In this paper we study the particle trap and filamentation process of beam in RF or Micro Bucket which is useful for understanding the injection beam dynamics of SSMB. One remarkable result is the steady-state current distribution after full filamentation has little dependence on the bucket height as long as it is several times larger than the energy spread. A discrete increase of bucket height after filamentation can boost the bunching, with the sacrifice of emittance growth. An adiabatic change of bucket height from a smaller value to the final desired value is then proposed to boost the bunching while preserving the longitudinal emittance.

INTRODUCTION

Microbunching is and will continue to be in the coming years one of the accelerator physics research focus. Storage ring based concept called steady-state microbunching (SSMB) [1] was proposed years ago for high average power narrowband coherent radiation generation, digging the potential of longitudinal coherence of beam which is parallel to the efforts on diffraction limited rings emphasizing on the transverse dimension. The idea of SSMB is using laser modulator to play the role of RF focusing in traditional rings as shown in Fig. 1 and 2. By precise longitudinal phase space manipulations (laser modulator cooperated with dedicated lattice), microbunching is formed and maintained turn-by-turn each time passing through the radiator. The high peak power coherent radiation from microbunches and high repetition rate from storage ring combined lead to a high average power facility.

There are now active research efforts on-going for different issues of SSMB by a collaboration established among Tsinghua University and several other institutes [2–5]. In this paper we investigate the particle trap and filamentation process of beams in RF or Micro Bucket which is useful for understanding the dynamics of SSMB beam injection.

PARTICLE TRAP AND FILAMENTATION IN RF OR MICRO BUCKET

To simplify the analysis, we restrict ourselves to longitudinal dimension only and ignore the various noises from laser and lattice. Quantum excitation and radiation damping

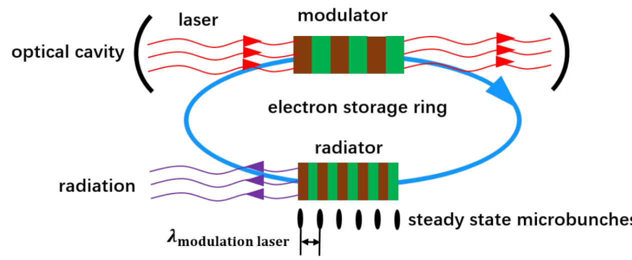

Figure 1: Schematic layout of SSMB.

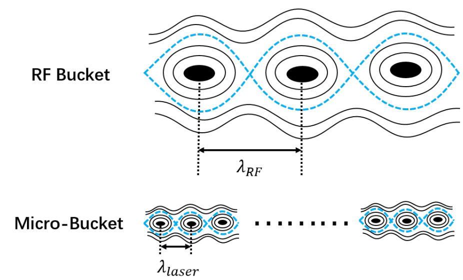

Figure 2: From RF Bucket to Micro-Bucket.

is also neglected as this paper focus on the first stage after the beam injection. The symplectic longitudinal dynamics of a particle in a storage ring with a single RF cavity (laser modulator) can be modeled by the well-known "standard map" [6]

$$\begin{aligned} I_1 &= I_0 + K\sin\theta_0, \\ \theta_1 &= \theta_0 + I_1, \end{aligned} \tag{1}$$

in which

$$\theta = kz,\ I = R_{56}k\delta,\ K = VR_{56}k. \tag{2}$$

Equation 1 can be described with the pendulum Hamiltonian driven by a periodic perturbation

$$H(I,\theta,t) = \frac{1}{2}I^2 + K\cos\theta\sum_{n=-\infty}^{\infty}\cos(2\pi nt). \tag{3}$$

For small K and I, which is the case for usual storage rings, the differences in Eq. 1 can be replaced approximately by derivatives and the Hamiltonian by a pendulum Hamiltonain

$$H = \frac{1}{2}I^2 + K\cos\theta. \tag{4}$$

The separatrix is $H = K$ with a bucket half-height of $2\sqrt{K}$. Chaos is beyond the discussion of this paper.

The longitudinal phase space evolution of a mono-energetic beam after injection into RF bucket described by Eq. 1 is shown in Fig 3. As can bee seen in Fig. 4, there is

* dxj11@tsinghua.org.cn

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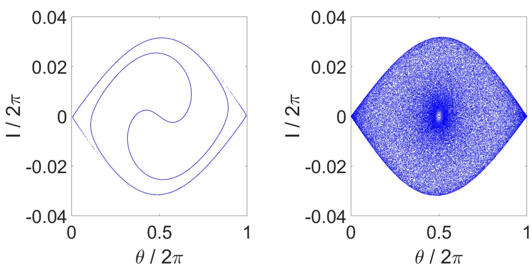

Figure 3: Filamentation in RF or Micro Bucket.

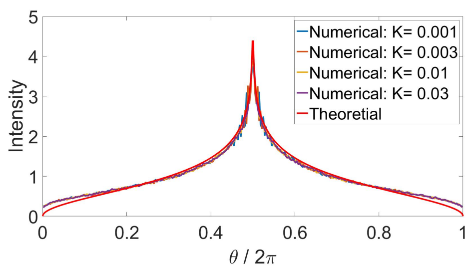

Figure 4: Steady-state current distribution of different K (bucket height) after full filamentation.

a steady-state current distribution due to filmantation (phase mixing). One thing remarkable is that the steady-state distribution has little dependence on the bucket height which is count-intuitive at the first sight. Now we try to give some semi-rigorous analysis.

In the following derivations, the pendulum approximation described above is made. For convenience, we shift the center of bucket to the origin, which means $\theta - \pi \to \theta$. Since the electron bunch is usually much longer than the laser wavelength and to get the main physics, we assume an initial bunch with uniform distribution of θ in $[0, \pi]$ and $I = 0$ (mono-energetic beam). What we want to know is the steady-state distribution $f(\theta, I, t \to \infty)$.

In action-angle coordinates (ϕ, J), the distribution function evolves according to

$$f(\phi, J, t) = f(\phi - \omega(J)t, J, 0). \tag{5}$$

When there is a tune dependence $\omega(J)$ on J and in the limit of $t \to \infty$, the steady-state distribution depends only on the initial action distribution as a result of phase mixing

$$f(\phi, J, t \to \infty) = \frac{1}{2\pi}\int_0^{2\pi} f(\phi, J, 0)d\phi = \frac{1}{2\pi}f(J, 0). \tag{6}$$

For $0 < x < \pi$, the probability of $\theta \le x$ is

$$P(\theta \le x) = \frac{x}{\pi} + \int_x^{\pi}\left(1 - \frac{\phi(x, I(x, \beta))}{\pi}\right)\frac{1}{\pi}d\beta, \tag{7}$$

in which $I(x, \beta)$ represents the I coordinate of a point on the phase space trajectory traversing $(\beta, 0)$ with a θ coordinate of x. The current distribution can then be got according to

$$f(\theta) = \left.\frac{\partial P}{\partial x}\right|_{x=\theta}. \tag{8}$$

However, $\phi(x, I(x, \beta))$ has a complex form, so it is hard to get a simple analytical expression for $f(\theta)$. Below we made a great simplification by approximating all the phase space trajectories in the bucket by ellipses to arrive at an analytical formula of $f(\theta)$.

For ellipse, we have

$$\phi(x, I(x, \beta)) = \arccos\frac{x}{\beta} \tag{9}$$

Note the result has no dependence on K. For a real RF bucket, there is a dependence of $\phi(x, I(x, \beta))$ on K. The dependence is however weak, especially for trajectories close to the origin. Substituting Eq. 9 into Eq. 7 and 8, we have

$$f(\theta) = \frac{1}{\pi^2}\ln\left|\frac{\pi + \sqrt{\pi^2 - \theta^2}}{\theta}\right| \tag{10}$$

Note the steady-state current distribution is independent of K. It is actually the ellipse approximation of phase space trajectory leading to this result. The simplified theoretically calculated current distribution $f(\theta)$ and its comparison with "standard-map" numerical simulation is shown in Fig. 4. As can be seen the theory agrees well with numerical results close to the stable fixed point while deviate more closer to the unstable fixed point, which fits with our expectation as the ellipse approximation is more valid around the stable fixed point.

The simulation and analysis presented in this section reveals a remarkable feature of filamentation in RF or Micro Bucket: the steady-state current distribution after full filamentation has little dependence on bucket height as long as most of the particles can be trapped in the bucket. This is helpful for the Quasi-SSMB [7] experiment focusing on the period after beam injection and before arriving at the true steady-state as it means the requirement on laser power is not that demanding. Other interesting ideas could also be invented by taking advantage of this characteristic.

VARIATION OF BUCKET HEIGHT

Disctrete Change of K

Although the steady-state current distribution has little dependence on the bucket height, it can be anticipated that more particles will bunch closer to the stable fixed point phase when we increase K after the beam reach its steady-state distribution after filamentation as shown in the numerical simulation in Fig. 5. A similar steps to the above section can be performed for calculating the new steady-state current distribution. A transformation of action when K changed is all that extra needed.

Adiabatic Trapping

An discrete change of K can boost bunching (Fig. 5). However, it is not without sacrifice, the filamentation process will result in beam emittance growth. This emittance increase may be harmful in some cases. As well-studied in RF gymnastics [8], an adiabatic change of RF voltage or

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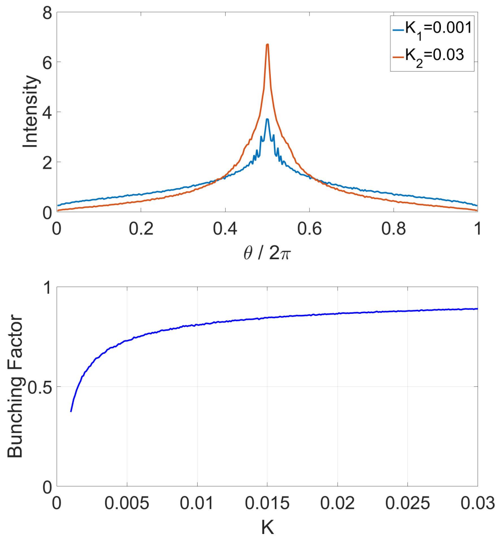

Figure 5: UP: steady-state current distribution with increase of K from 0.001 to 0.03 in 2 steps. Down: bunching factor on the modulation wavelength evolution with K increased from 0.001 to 0.03 in 300 steps. 10^6 particles simulated, 10^5 kicks for each step.

lattice parameters can manipulate the bunch length while preserving the longitudinal emittance. Similar ideas can also be applied to microbunching. The adiabatic trapping of microbunch simulation is shown in Fig. 6. Note the dramatic difference between Fig. 6 and Fig. 3. The spirit of adiabatic buncher [9, 10] is the same with adiabatic trapping for enhancing microbunching while causing as little emittance growth which is useful for FEL and IFEL.

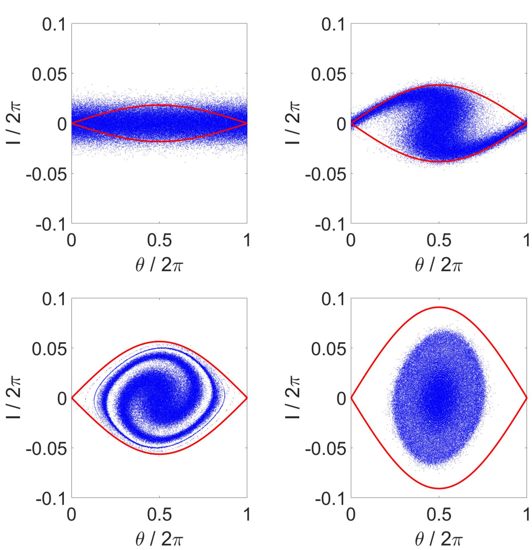

Figure 6: Adiabatic trapping. Red line: separatrix.

SUMMARY

In this paper, we study the particle trap and filamentation process after beam injected into an SSMB storage ring with modulation laser. One remarkable result is the steady-state current distribution after full filamentation has little dependence on bucket height as long as it is several times larger than the energy spread. Discrete and adiabatic change of bucket height is proposed for microbunching enhancement.

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