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STEFFEN HARD-EDGE MODEL FOR QUADRUPOLES WITH EXTENDED FRINGE-FIELDS AT THE EUROPEAN XFEL

N. Golubeva*, V. Balandin, W. Decking, L. Fröhlich, M. Scholz,
DESY, Hamburg, Germany

Abstract

For modeling of linear focusing properties of quadrupole magnets the conventional rectangular model is commonly used for the design and calculations of the linear beam optics for accelerators. At the European XFEL the quadrupole magnets are described using a more accurate Steffen hard-edge model. In this paper we discuss the application of the Steffen approach for the European XFEL quadrupoles and present the examination of the model with the orbit response matrix technique.

INTRODUCTION

The 3.4 km long European XFEL facility is in operation since 2017 and currently serves three FEL beamlines simultaneously for user experiments [1, 2]. The facility is based on superconducting accelerator technology. After the injector which contains one standard superconducting accelerating module and a 3^{rd} harmonic module, the beam is accelerated in the superconducting linear accelerator. To achieve a high peak current, the bunches are longitudinally compressed in three magnetic chicanes which are located in the injector and after the first (Linac 1) and second (Linac 2) parts of the accelerator. Diagnostic sections are placed in the injector and after the second and third compression stages. After the final acceleration in Linac 3 the beam passes a collimation section, and then the bunches of one bunch train are distributed between two electron beamlines leading to the SASE1 and SASE3 undulators and to the SASE2 undulator. Individual bunches can be selectively directed to the linac dump.

In such complicated machine the design linear optics incorporates many constraints on the behaviour of optical functions. For example, in the bunch compressors a special choice of Twiss function reduces the emittance growth due to coherent synchrotron radiation, diagnostic sections have to provide the special conditions for measurements of beam parameters, the optical functions in the collimation section have to be suitable for collimation purposes, the distribution system based on the kicker-septum scheme presents challenges for the dispersion suppression in both planes.

To provide a good performance of such sophisticated design optics, a good knowledge of optical properties of all magnetic elements is required. Before the commissioning, on the stage of the manufacturing of magnets we have arranged the comprehensive set of magnetic measurements. The accurate measurements of longitudinal field and field-gradient profiles have been made for all types of the European XFEL magnets. But an analysis of measured data for quadrupole magnets has shown that for all quadrupoles the conventional hard-edge model does not provide a sufficient accuracy. For example, for the QI quadrupole type the conventional model can not provide more than two correct decimal digits in the transfer matrix elements (see Figure 1), that does not seem to be very satisfactory. This motivated us to consider other models for quadrupole magnets.

In general, having the field gradient profiles one may develop a special code calculating the focusing of quadrupoles with extended fringe-fields, but for practical purposes and especially for online optics calculations it is desirable to employ more practicable model. We have considered a Steffen approach [3] to quadrupole modeling. As it has been shown in the paper [4], for the European XFEL quadrupoles the Steffen parameters exist, and with sufficient for our purposes accuracy can be approximated using low order polynomials in the variable k_0 (mth-order Steffen model). And, for our luck, already the zeroth-order Steffen model can essentially improve the accuracy in comparison with the conventional hard-edge model (see Figure 1). In this paper we discuss the application of the zeroth-order Steffen model for the European XFEL quadrupoles. One should note that although the Steffen approach is not yet widely known, the interest seems to grow during last time (for example, see references in [4]).

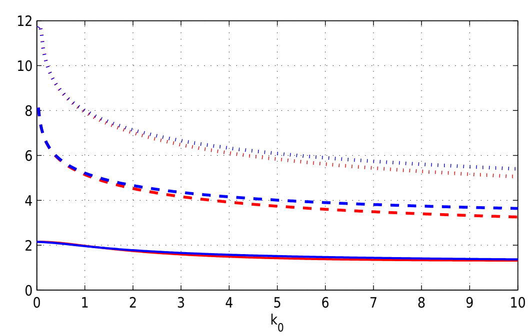

Figure 1: Quadrupole QI. The number of correct decimal digits in the approximation of the transport matrices $M^{\pm}(k_0)$ by conventional rectangular model (solid curves), and by zeroth (dashed curves) and first (dotted curves) order Steffen models. Red and blue colors refer to the focusing and defocusing matrices, respectively.

CONVENTIONAL AND STEFFEN HARD-EDGE MODELS

Optical linear properties of quadrupoles are determined by the quadrupole focusing strength k_0 and by the field gradient shape function $k(s)$ (the longitudinal field gradient profile). Figure 2 shows an example of the function $k(s)$

* nina.golubeva@desy.de

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for one of the European XFEL quadrupoles. Here we use the notation from the paper [4], and assume that the maximum value of $k(s)$ is equal to one, and it is zero outside the interval $[-l_f\,/\,2,\ l_f\,/\,2]$ of the longitudinal s-axis.

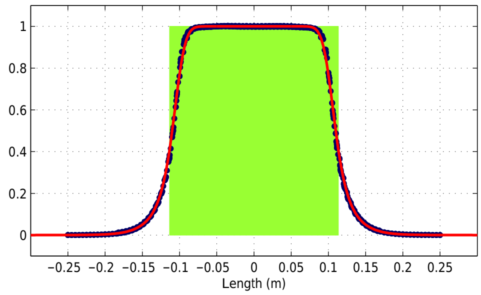

Figure 2: Quadrupole QI. Red curve shows its gradient shape function $k(s)$, which was obtained (in the form of some combination of the Enge functions) as a fit to the measured data represented in this figure by the blue dots. Green rectangle shows effective field boundary l_e.

Conventional Rectangular Model

In the conventional rectangular model the function $k(s)$ is approximated by the rectangular function which is unity inside and zero outside of an effective quadrupole length

$$l_e \;=\; \int_{-l_f/2}^{l_f/2} k(s)\, ds\,. \tag{1}$$

The linear beam transport through quadrupole can be described by 2×2 focusing and defocusing matrices $M^{\pm}(k_0)$ using the approximation

$$M^{\pm}(k_0) \approx D\left(\frac{l_f - l_e}{2}\right) Q^{\pm}(l_e, k_0)\, D\left(\frac{l_f - l_e}{2}\right), \tag{2}$$

where D is the 2×2 drift matrix, and Q^{+} and Q^{-} are the usual focusing and defocusing matrices of the hard-edge quadrupole model, respectively. Parameters l_e and k_0 are usually named as effective quadrupole parameters.

Steffen Hard-Edge Model

A basis of the Steffen approach is to find a rectangular type model with different effective parameters for focusing and defocusing planes whose transfer matrices are exactly equal to the matrices $M^{\pm}(k_0)$. As the Steffen quadrupole parameters, Steffen effective lengths $l^{\pm}$ and Steffen gradient reduction coefficients $r^{\pm}$ are introduced such that the following equations have to be hold

$$M^{\pm}(k_0) = D\left(\frac{l_f - l^{\pm}}{2}\right) Q^{\pm}(l^{\pm}, r^{\pm}k_0)\, D\left(\frac{l_f - l^{\pm}}{2}\right). \tag{3}$$

As mentioned in [4], the functional dependence $l^{\pm} = l^{\pm}(k_0)$ and $r^{\pm} = r^{\pm}(k_0)$ can be found only once at the stage of the analysis of magnetic measurement data and then stored as a property for each quadrupole magnet. For the European XFEL quadrupoles it is sufficient to approximate the effective parameters using low order polynomials in the variable k_0 and, as it has been shown in [4], already the zeroth-order Steffen model can essentially improve the accuracy in comparison with the conventional rectangular model. In this case as the Steffen quadrupole parameters the zeroth-order Steffen coefficients l_0 and r_0 can be used, which are given by the formulas [4]

$$l^{\pm} \approx l_0 = \left(\frac{12}{l_e} \int_{-l_f/2}^{l_f/2} s^2 k(s) ds\right)^{\frac{1}{2}}, \quad r^{\pm} \approx r_0 = \frac{l_e}{l_0}\,. \tag{4}$$

One should note that the zeroth-order Steffen model can be used in any standard beam dynamics program (for example, MAD and *Elegant*) because it is the same as conventional model but with modified effective parameters.

At the European XFEL there are in total about 500 quadrupole magnets which are grouped in 10 types, as reported in Table 1 (B_r is the quadrupole bore radius).

Table 1: European XFEL Quadrupole Types

Type	l_e (m)	l_0 (m)	r_0 (m)	B_r (m)	Count
Q	0.1973	0.2136	0.9240	0.040	98
QA	0.1098	0.1137	0.9655	0.008	103
QI	0.2268	0.2377	0.9541	0.025	58
QD	0.2252	0.2367	0.9516	0.025	34
QE	0.2254	0.2400	0.9393	0.025	42
QF	0.5261	0.5321	0.9887	0.025	89
QH	1.0262	1.0291	0.9972	0.025	34
QM	1.0515	1.0597	0.9923	0.050	12
QK	1.0450	1.0552	0.9903	0.050	21
QB	0.3215	0.3289	0.9775	0.025	2

It can be seen from Table 1 that for short quadrupoles (for example, the QI and QD types which are used in the injector and in the sections with chicanes for bunch compression, and the QA quadrupoles used in all three SASE undulator systems) the effective lengths in the conventional model and zeroth-order Steffen models differ for 3% − 5%. For long quadrupoles this difference becomes smaller.

MODEL PREDICTIONS AND MEASUREMENTS

To examine the optics model a precise setup of the beam optics is needed, for that a good knowledge of the beam energy and a calibration of magnets are essential. As concerning the beam energy, for sections where the beam energy is not changed the pre-defined energy is used for the setting of magnets, and the real beam energy is adjusted relying on beam-based energy measurements in dispersive sections of the machine.

An issue of importance is the calibration function between the field gradient and the power supply current. For

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all quadrupole types the accurate measurements of full magnetic excitation curves have been made according the cycling procedures used in the machine operations, and two different calibration functions (in a form of a lookup table or a fifth-order polynomial) for the upward and downward current ramp are used. Using the Steffen quadrupole model the calibration functions are defined taking into account the Steffen gradient reduction coefficients r_0.

For the examination of quadrupole model, the measurements of orbit response matrices - which are widely employed to find and correct linear optics errors in accelerators - have been performed. The actual measurements from the machine, which are provided by an orbit response tool [5], have been compared with the prediction of online optics model [6]. To check the model for different types of quadrupoles and exclude other focusing contributions the measurements are performed locally in different sections.

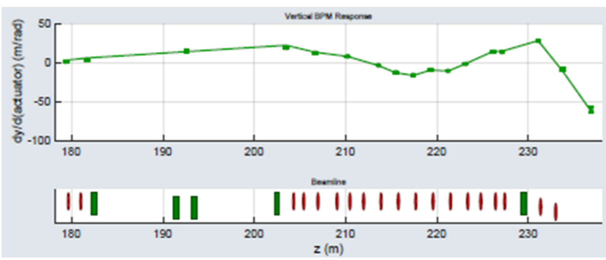

Figure 3: Section B1. Measured (solid curve) and predicted by model (dots) orbit response in the vertical plane. Rectangles mark dipole (green) and quadrupole (red) magnets.

Figures 3-5 present examples demonstrating the level of agreement between the model and measurements. The first example is for the section equipped with 20 short quadrupoles of the QI and QD types. The second example presents the post-linac collimation section which contains two types of quadrupoles (QF and QH), and dipole and sextupole magnets. For sextupole modeling the hard-edge model using the effective length is applied, but they are usually switched off for checking the quadrupole model.

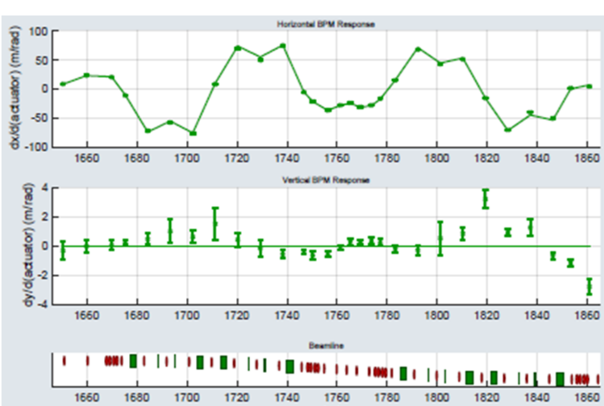

Figure 4: Collimation section. Measured (solid curve) and predicted by model (dots) orbit response in the horizontal plane (top). Response in other plane is also shown.

The Steffen model for superconducting quadrupoles also shows a good performance. Figure 5 shows the part of the accelerator (Linac 2) where the RF cavity focusing is negligible small.

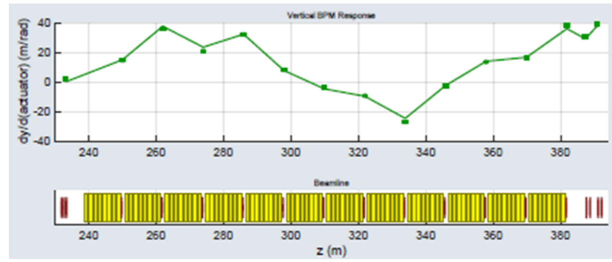

Figure 5: Linac 2. Acceleration from 700 MeV to 2.4 GeV. Measured (solid curve) and predicted by model (dots) orbit response in the vertical plane. Yellow rectangles mark the RF modules.

As can be seen, the agreement between measurements and model predictions is surprisingly good. About the same level of agreement is observed in all beamlines equipped with quadrupoles. We find that the zeroth-order Steffen model for quadrupoles is acceptably good, and currently we do not apply any corrections to the quadrupole model, neither to the quadrupole strengths nor to the calibration functions. Different design optics have been successfully used for the commissioning and for beam optics studies.

CONCLUSION

The zeroth-order Steffen model is used for the description of optical linear properties of all quadrupoles at the European XFEL from the beginning of operations and, together with the precise calibration of magnets, demonstrates a good accuracy for the application in the linear beam optics. Presently no any corrections based on beam-based measurements are applied to optical linear properties of quadrupoles.

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