

# UNDERSTANDING 1D TO 3D COHERENT SYNCHROTRON RADIATION EFFECTS

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## Abstract

Collective effects such as coherent synchrotron radiation (CSR) can have a strong influence of the properties of an electron bunch with respect to the quality of the FEL light that it produces. In particular, CSR experienced by a bunch on a curved trajectory can increase the transverse emittance of a beam. In this contribution, we present an extension to the well-established 1D theory of CSR by accounting fully for the forces experienced in the entrance and exit transients of a bending magnet. A new module of the General Particle Tracer (GPT) tracking code was developed for this study, showing good agreement with theory. In addition to this analysis, we present experimental measurements of the emittance growth experienced in the FERMI bunch compressor chicane as a function of bunch length. When the bunch undergoes extreme compression, the 1D theory breaks down and is no longer valid. A comparison between the 1D theory, experimental measurements and a number of codes which simulate CSR differently are presented, showing better agreement when the transverse properties of the bunch are taken into account.

## INTRODUCTION

Synchrotron radiation – the emission of radiation by a charged particle when travelling on a curved trajectory – can become coherent when the length of a particle bunch is shorter than the wavelength of the radiation emitted. This coherent synchrotron radiation (CSR) can degrade the quality of an electron bunch, causing an increase in projected and slice emittance, and energy spread [1–5]. The theoretical explanation of CSR has made significant progress since its initial formulation [6–11], and some experimental studies have demonstrated good agreement between experimental measurements and simulation results [3, 4].

Much of the existing literature on CSR, however, neglects to account for the transverse extent of the electron bunch, however, and this may become increasingly important for future free-electron laser (FEL) facilities and schemes which place increasingly stringent demands on high-brightness, high quality electron bunches. We present some new insights on the theory of the 1D CSR transient field at the edges of dipole magnets, based on the interactions between the velocity and acceleration components of the Liénard-Wiechert field. This work suggests novel compressor designs for the minimization of this instability. A new CSR feature of the General Particle Tracer (GPT) [12] tracking code was developed specifically for this study, which does not use the small-angle or ultrarelativistic approximations.

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A number of other codes exist which are capable of simulating the effects of CSR [13, 14], some of which utilise a 1D approximation, based on Refs. [15, 16], and others which extend the model to incorporate 2D and 3D effects [17–19]. While previous studies have shown good agreement between results from some of these simulation codes and experimental data [3, 4], there is a point at which the 1D approximation is no longer valid, as shown in [20], which suggests that projecting the bunch distribution onto a line may overestimate the level of coherent emission, particularly when the bunch has a large transverse-to-longitudinal aspect ratio. There are a number of effects that must be included for a simulation code to fully take account of 3D CSR effects, including (but perhaps not limited to):

1. Taking the transverse extent of the bunch into account for all particles, rather than assuming that all electrons emit and receive on-axis.
2. Self-consistently solving for the trajectory during emission rather than neglecting deviations from the nominal trajectory.
3. Taking the full Liénard-Wiechert field into account rather than only the term which arises during acceleration.
4. Including stochastic effects due to the long-range interaction between a discrete number of radiation cones.

One of the aims of this study is to determine if, during strong bunch compression, or for bunches with a large transverse-to-longitudinal aspect ratio, the limits of the 1D approximation could be found. This is achieved through comparing analytic results with simulation codes that incorporate the transverse bunch distribution, and with experimental data. The projected emittance of the electron beam was measured in parameter scans at the exit of the first bunch length compressor of the FERMI FEL [21, 22].

## NUMERICAL VALIDATION

An extension to the 1D theory of CSR was presented in [23], which demonstrated the importance of the so-called ‘velocity’ term of the Liénard-Wiechert field in cases when an electron bunch enters and exits a bending magnet. It was demonstrated that, while the radiation cone from a particle is reduced in volume as the particle becomes ultrarelativistic, there is a small spike in the field experienced by a receiving particle as it enters or exits the curved trajectory which is due to this term. For a full derivation of this effect, see [23]. In order to validate these analytic results, we have

numerically calculated the electromagnetic field distribution in an electron bunch in both the entrance and exit transient regimes using the GPT code [12]. GPT is a time-domain particle tracking code that integrates the equations of motion of a large number of charged particles in the presence of electromagnetic fields. A dedicated upgrade was made to the code in order to include the computation of the retarded Liénard-Wiechert fields of the tracked particles. Because this involves the storage of the trajectory of the particles and solution of retardation conditions, calculation of Liénard-Wiechert fields is computationally expensive. The cost of these computations can be reduced by approximating the emission of CSR through a longitudinal slicing of the bunch in a discrete number of time steps. From each of these slices, the transverse extent of the bunch is represented by a number of off-axis macroparticle emitters (either four or sixteen per slice), spaced regularly according to the transverse size of the slice. While integrating the equation of motion of a tracked particle, GPT evaluates the Liénard-Wiechert field resulting from the stored history of the past trajectory of each of the representative particles at the longitudinal position of the tracked particle. For more details on the code, see [24].

It is important to note that GPT uses the exact expression for the Liénard-Wiechert fields based on the numerically obtained coordinates of particles in the bunch, and does not apply any analytic approximation or presumed trajectory of the bunch. The parameters used in the simulation are given in [23]. We deliberately chose artificially small energy spread and transverse bunch size, and used hard-edged magnet fringes in the exit transient simulations to match the analytic case as much as possible.

### Entrance Transient Effect

The CSR field was initially calculated by GPT at a point 24 cm into the magnet in order to simulate the entrance transient field. This distance is only half that of the steady-state condition  $D^{SS}$  [6], and so it is expected that the general expression of the steady-state CSR field will be required to calculate the fields. In this simulation, the drift before the magnet was set to 50 m. The results from the simulation are in good agreement with the expression for the steady-state field, as seen in the right-hand plot of Fig. 1. However, if the simulation is run again, but with the drift before the bend set to 10 cm, the GPT result effectively reduces to the full expression for the entrance transient field, and thereby differs from the usual approximation of Ref. [15]. The approximation of an infinitely long drift before the entrance to a bending magnet is not valid for some cases; as shown in Fig. 1, the GPT simulation reflects this behaviour.

### Exit Transient Effect

Next, we numerically validate the analytic results for the CSR forces in the exit transient regime (see [23] for details). Because the net CSR field experienced by the bunch involves cancellations between the radiation and velocity terms, it proves to be of interest to study in turn both the full CSR

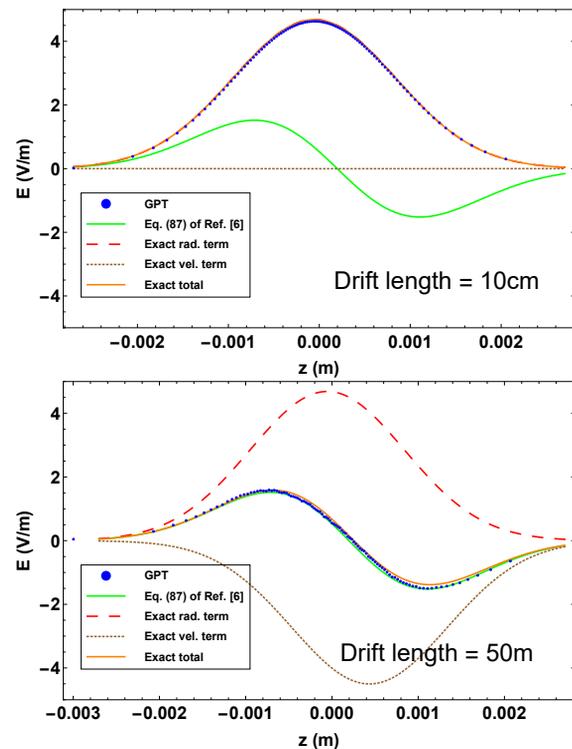


Figure 1: Longitudinal component of CSR electric field as a function of longitudinal position in the bunch and a drift before the magnet of: Above: 10 cm; and Below: 50 m, as simulated by GPT, against both Eq. 87 of Ref. [15] and Eq. 5 of Ref. [23] – both the velocity and radiation terms individually, and combined. Positive values of  $z$  refer to the head of the bunch.

field and the radiation term separately. Fig. 2 shows the longitudinal component of the electric field as a function of longitudinal position in the bunch, evaluated at 5 mm past the bending magnet. The full CSR field is represented by the orange line and the blue dots (GPT simulation), and the good agreement between these shows that the 1D, ultrarelativistic and small-angle approximations do not lead to any significant deviations from the exact CSR force.

However, an interesting 3D effect may be observed when studying the radiation term separately. In Fig. 2, this term is plotted according to the analytic result based on the 1D approximation (brown curve) and according to the GPT simulation (black dots). Clearly, the total CSR exit transient field significantly overestimates the magnitude of the radiation term. We found that this overestimation of the radiation field is due to the underestimation of the retarded distances between emitting and observing particles associated with the 1D approximation. Namely, the impact of the finite transverse bunch size could be roughly quantified by including a vertical offset of the emitting electron in the derivation of the CSR force. Due to the offset, the distance  $\sigma$  from emitter to observer is effectively increased.

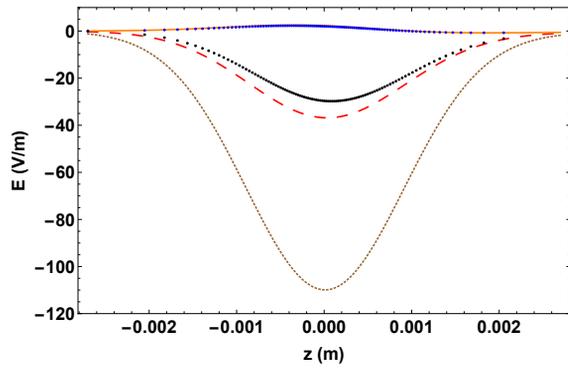


Figure 2: Longitudinal component of electric field as a function of longitudinal position in the bunch, at 5 mm from the exit of the dipole. Blue: GPT simulation of the full field; Black: GPT simulation of the radiation field only; Red: total CSR exit transient field with an offset in the y plane; Brown: radiation term only; Orange: total CSR exit transient field.

In addition, the angles between emitter and receiver are stretched somewhat, such that their cosines become smaller by a factor  $\cos \delta = \rho/\sigma$ , with  $\rho$  the longitudinal distance between the particles. Re-evaluating the Liénard-Wiechert field with these modifications shows that the electric field is still given by the total CSR exit transient field after the substitution  $\rho \rightarrow \sigma$ . By including this transverse offset, through the substitution  $\sqrt{\epsilon_y \beta_y} / \gamma = 4.5 \mu\text{m}$  (with  $\epsilon_y$  and  $\beta_y$  the vertical geometric emittance and beta function, respectively, and  $\gamma$  the Lorentz factor), the analytic expression for the radiation field is reduced to roughly the magnitude observed in the simulation. This means that the discrepancy between the 1D expression and the simulation can be accounted for by a bunch with a finite transverse extent, meaning that the 1D description cannot fully explain the radiation or velocity terms in the exit transient regime. However, this effect is masked by the fact that the radiation and velocity fields are of opposite sign, and partially cancel each other, resulting in an overall field that is relatively independent of the transverse bunch size.

## RESULTS

The emittance was measured at the exit of the first bunch compressor in the FERMI linac, BC1, as a function of Linac 1 RF phase and chicane bending angle, resulting in variations of the bunch compression factor in the range 20 – 64 and 8 – 60 for the two scans, respectively. During the phase scan, the accelerating gradient of Linac 1 was scaled in order to keep the mean bunch energy constant at the entrance to BC1. Measurements were taken using the single quad-scan technique [25], by varying the strength of one quadrupole magnet (Q\_BC01.07), located in the section directly after BC1. The machine was operated with a constant bunch charge of 100 pC, and a mean energy of approximately 300 MeV at BC1.

From an injector simulation in GPT, the bunch was then tracked using the ELEGANT code [26] up to the entrance of

BC1. From this point, three particle tracking codes have been used to compare the emittance measurement results with simulation: ELEGANT, CSRTRACK [27] and GPT using the CSR model outlined above in Sec. . In the 1D CSR simulations, ELEGANT applies the calculation of Saldin *et al* [15] to calculate the energy change due to coherent radiation in a bend, and the subsequent transient effect some time after the bunch exits the dipole, based on [16]. At the exit of the bunch compressor (including a drift to account for transient CSR effects), the output is tracked up to Q\_BC01.07, the measurement point, in ELEGANT. The relative bunch length was monitored using a pyroelectric detector at the exit of the bunch compressor, and online feedback was used to maintain the compression factor across all sets of measurements.

The emittance was measured by quad scan using the FERMI online emittance tool as a function of bunch compression factor during the experimental run. We compare these measurements of emittance with results from all three simulation codes, as shown in Figs. 3 and 4. The CSR-induced emittance growth in these regimes has also been calculated, based on the analytic theory given in [28], which presented an updated calculation that takes into account the cancellation effect between the transverse CSR and space-charge field in a bunched beam (for further discussion on this cancellation effect see [29–31]). The emittance growth corresponding to the longitudinal and transverse CSR wake with the entire bunch travelling on a circular orbit (i.e. the steady-state regime) are given as:

$$\Delta\epsilon_N^{long} = 7.5 \times 10^{-3} \frac{\beta_x}{\gamma} \left( \frac{r_e N L_b^2}{R^{5/3} \sigma_z^4} \right)^2 \quad (1a)$$

$$\Delta\epsilon_N^{trans} = 2.5 \times 10^{-2} \frac{\beta_x}{\gamma} \left( \frac{r_e N L_b}{R \sigma_z} \right)^2, \quad (1b)$$

with  $\beta_x$  the horizontal beta function,  $N$  the number of particles,  $L_b$  the length of the dipole,  $R$  the bending radius,  $\sigma_z$  the bunch length, and  $r_e$  the classical electron radius. We have also calculated the ratio  $\sigma_v$  [20], indicating the validity of the 1D CSR approximation. For the analytical calculations to be valid, the following condition should be fulfilled:

$$\sigma_v = \sigma_\perp \sigma_z^{-2/3} R^{1/3} \ll 1, \quad (2)$$

with  $\sigma_\perp$  the transverse beam size. If this condition is not fulfilled, the 1D CSR approximation can be violated approaching maximal compression or in cases where the transverse beam size is large. The values for the  $\sigma_\perp$  and  $\sigma_z$  are taken from ELEGANT simulations with CSR switched off. In order to calculate  $\Delta\epsilon_N$ , we sum together both terms in Eq. 1 and add them to the initial value of the emittance at the entrance to the bunch compressor. The effect of the emittance from all four dipoles was calculated, but the largest impact by far is expected in the fourth dipole, when the majority of the actual compression takes place.

It can clearly be seen that there is a general agreement between the analytic calculations, the results from simulation

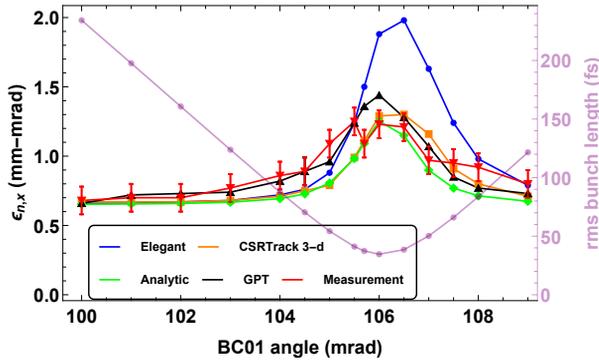


Figure 3: Horizontal emittance as a function of BC01 bending angle, with the corresponding bunch length as simulated by ELEGANT. The analytic results are calculated using Eq. 1.

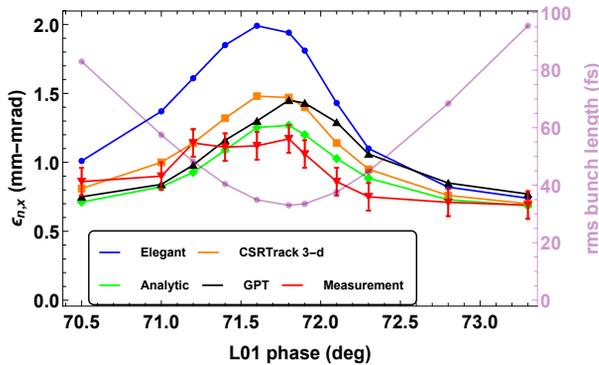


Figure 4: Horizontal emittance as a function of Linac 1 phase, with the corresponding bunch length as simulated by ELEGANT. The analytic results are calculated using Eq. 1.

and the measurement procedure. The discrepancy between simulation and experiment in the peak around 71.6–72.1° in Fig. 4 can be attributed to coherent OTR emission (COTR) [32, 33]. We observe a similar apparent overestimation of emittance growth for the bunch compressor angle scan in Fig. 3 for the ELEGANT simulation. GPT and CSRTRACK 3D are able to capture both the emittance trend and its absolute value more accurately over the entire range of bunch lengths.

By computing  $\sigma_v$  across both sets of compression scans, and comparing the results from simulation and experiment with this value, it is seen that the overestimation of the effect of CSR in the 1D simulation is largest when  $\sigma_v$  is greater than 2.5 at any point in the chicane. For more moderate values of the compression factor, this condition is not violated as strongly, and the agreement between all simulation results and the experimental measurements is good. The analytic estimation manages to reproduce the results generated through simulation and experiment throughout the range of bunch lengths.

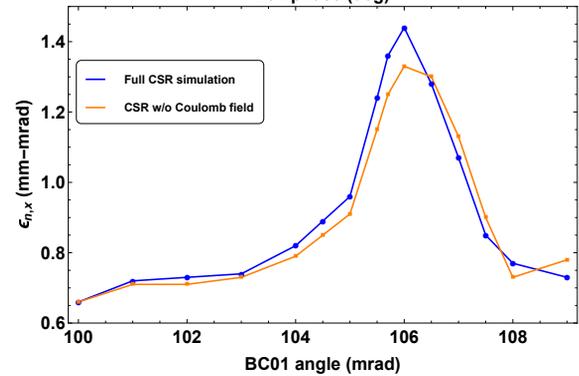
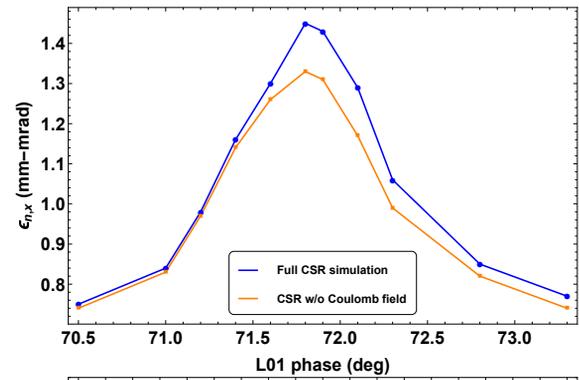


Figure 5: GPT simulation of emittance growth in the Linac 1 phase (top) and bunch compressor angle (bottom) scans with and without the velocity term of the Liénard-Wiechert field.

The differences between the ELEGANT results and those from CSRTRACK and GPT simulations are also noteworthy. It appears that, when the bunch undergoes maximum compression (as seen from the minimal bunch length in Figs. 3 and 4), the discrepancy between the 1D and 3D codes is largest, with ELEGANT returning an emittance value around 40% larger than CSRTRACK. GPT does return a slightly higher value for the emittance than CSRTRACK and the experimental data around maximal compression. Comparisons between CSR simulations and experimental data have been studied previously [2–4], but only for moderate compression factors (up to around 15 at a given bunch compressor). We also see relatively good agreement between the codes and experimental data to within 10% in this compression range, but the divergence at large compression suggests that there are limits to the applicability of the 1D CSR approximation when used in simulation.

The GPT code provides the functionality to include or to exclude the Coulomb term in simulation, thus demonstrating the importance of taking account of this term. In Fig. 5 we compare these two CSR simulations for the bunch compressor angle scan and the linac phase scan. It can be seen that, as the bunch approaches maximal compression, the projected emittance increases by around 10% in the case where the Coulomb field is taken into account as compared with only the simulation of the radiation field. This can be

understood as the relative distance between the Coulomb field and the receiving particle being relatively shorter in the case of a larger bending angle in the dipole, and therefore having a larger effect; analogously, a bunch with a minimal chirp based on the linac phase exhibits the same effect. In machines that have a larger number of compressive bending magnets, this effect will be compounded, and so these results provide further evidence of the importance of taking full account of CSR when designing future accelerators.

## CONCLUSIONS

An extension to the 1D theory of CSR has been presented. It was shown that, when considering the longitudinal electric field of a bunch as it enters and exits a bending magnet, it is important to consider the electrostatic term in the Liénard-Wiechert field. This term is sometimes neglected, as it is suppressed under ultrarelativistic conditions. However, since this field is always present, and travels along the direction of motion of the emitting particle, there is a brief crossover point at which the velocity field is able to influence the receiving particle in front of the emitter, and so this field should be taken into account in simulations of CSR. This is more significant when considering an accelerator lattice with multiple bends that are closer together, such as the transport for an ERL, or an FEL spreader line. It is also shown that there is a cancellation effect between these two fields, suggesting that it may be possible to design a magnet or a system of accelerator optics that take advantage of this cancellation, such that the CSR field is suppressed.

After benchmarking these new theoretical expressions using the GPT code, a comparative study between this simulation and two other codes – CSRTRACK and ELEGANT – was conducted. The effect of CSR on the projected emittance of a bunch after compression in the first compressor in the FERMI FEL was studied as a function of compression factor. These results were then compared with experimental measurements and theoretical predictions of the projected emittance. Good agreement was seen between all codes, experiment and theory when the compression factor was relatively low, but as maximum compression was reached, a greater divergence between the 1D simulation and the other results was observed, suggesting that the 1D approximation was no longer applicable. The breakdown of this condition has been studied experimentally; the theory suggested that the condition is valid only in the parameter regime  $\sigma_v \ll 1$ , whereas it has been demonstrated that up to  $\sigma_v \lesssim 2$ , the 1D CSR approximation remains valid, and so this condition can be relaxed. A promising result from this study, though, demonstrates that the theory produces good agreement with the other results. This study demonstrates the significance of taking as full an account of CSR as possible when designing future accelerators that place stringent requirements on the beam quality.

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